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On nano semi pre neighbourhoods in nano topological spaces

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Abstract
The basic objective of this paper is to introduce and investigate the properties of nano semi pre-neighbourhoods, nano semi pre-interior, nano semi pre-frontier, nano semi pre-exterior, nano-dense, nano-submaximal and obtain relation between some of the existing sets.

Keywords
Nano β-interior, nano β-closure, nano β-neighbourhoods, nano β-frontier, nano β-exterior, nano-dense, nano-submaximal.

AMS Subject Classification
54B05.

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Contents

1. Introduction

In 1970, Levine[2] introduced the concept of generalized closed sets in topological spaces. In [1], the authors discussed a new form of generalized closed sets via regular local function in ideal topological spaces whereas in [8] the authors addressed the generalized star semi regular closed sets in topological spaces. The notion of Nano topology was introduced by Lellis Thivagar[3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano-interior and Nano-closure of a set. He also introduced weak form of Nano open sets namely Nano-α-open sets, NS-open sets and NP-open sets. This motivates the author to introduce and study the properties of Nano β-closed sets in Nano topological spaces.

2. Preliminaries

Definition 2.1. *[4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then,*
(i)The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = \bigcup_{x\in U} \{R(x){:}R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by $x \in U$.
(ii)The upper approximation of x with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = \bigcup_{x\in U} \{R(x){:}R(x) \cap X \neq \phi\}$
(iii)The boundary region of x with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. *[5] Let U be the universe, R be an equivalence relation on U and $\tau_R = \{U,\phi,L_R(X),U_R(X),B_R(X)\}$ where $X \subseteq U$. Then τ_R satisfies the following axioms*
(i) U and $\phi \in \tau_R$.

(ii) The union of the elements of any sub-collection of τ_R is in τ_R.
(iii) The intersection of the elements of any finite sub collection of τ_R is in $\tau_R(X)$.
Then τ_R is a topology on U called the nano topology on U with respect to X. We call (U, τ_R) as nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. The complement of the nano open sets are called nano closed sets.

Remark 2.3. *[4] If τ_R is the nano topology on U with respect to X , then the set B = $\{U, L_R(X), B_R(X)\}$ is the basis for τ_R.*

Definition 2.4. *[3] If (U, τ_R) is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$,then*
(i)The nano interior of A is defined as the union of all nano-open subsets of A and is denoted by NInt(A). That is, NInt(A) is the largest nano-open subset of A.
(ii)The nano closure of A is defined as the intersection of all nano-closed sets containing A and is denoted by Ncl(A). That is, Ncl(A) is the smallest nano-closed set containing A.

Definition 2.5. *[6]A subset U_R of a nano topological space U is called Nano semi pre open set if $U_R \subset$ Ncl(NInt(Ncl(A))). The complement of Nano semi pre open set is called Nano semi pre-closed . The family of Nano semi pre sets is denoted by Nβ(U).*

Lemma 2.6. *[6] NSO(U,X) $\cup$ NPO(U,X) $\subset$ NβO(U,X).*

3. Nano Semi Pre Neighbourhoods

Lemma 3.1. *NSO(U,X) $\subset$ NβO(U,X) and NPO(U,X) $\subset$ NβO(U,X).*

Proof. Follows from Lemma 2.6 □

Definition 3.2. *If (U, τ_R) is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$,then*
(i)The Nβ interior of the set A is defined as the union of all Nano semi pre open subsets contained in A and is denoted by NβInt(A). NβInt(A) is the largest Nano semi pre open subset of A.
(ii)The Nβ closure of the set A is defined as the intersection of all Nano semi pre closed sets containing A and is denoted by Nβcl(A). Nβcl(A) is the smallest Nano semi pre closed set containing A.

Lemma 3.3. *Let A and B be any two subsets of U in a nano topological spaces (U,τ_R) then following are true*
(i) NβInt(A) $\subset$ A
(ii) A $\subset$ B $\Rightarrow$ NβInt(A) $\subset$ NβInt(B)
(iii) NβInt(A) $\cup$ NβInt(B) $\subset$ NβInt(A $\cup$ B)
(iv) NβInt(A) $\cap$ NβInt(B) $\subset$ NβInt(A $\cap$ B)

Lemma 3.4. *For a subset A of U.*
i) Nβcl(A) $\subset$ Ncl(A).
ii) NInt(A) $\subset$ NβInt(A)).
iii) NβFr(A) = Ncl(A) $\cap$ Nβcl(U -A).
iv)Nβ Ext(A) = NβInt(U - A) = U - Nβcl(A).

Lemma 3.5. *A subset A of U is nano semi pre-closed iff A = Nβcl(A).*

Definition 3.6. *A subset $M_x \subset U$ is called a nano semi-neighbourhood*
(NS-nhd) of a point $x \in U$ iff there exists a $A \in$ NSO(U,X) such that $x \in A \subset M_x$ and a point x is called NS-nhd point of the set A.

Definition 3.7. *[7] A subset $M_x \subset U$ is called a nano pre-neighbourhood (NP-nhd) of a point $x \in U$ iff there exists a $A \in$ NPO(U,X) such that $x \in A \subset M_x$ and a point x is called NP-nhd point of the set A.*

Definition 3.8. *A subset $M_x \subset U$ is called a nano semi pre-neighbourhood (Nβ-nhd) of a point $x \in U$ iff there exists a $A \in$ NβO(U,X) such that $x \in A \subset M_x$ and a point x is called Nβ-nhd point of the set A.*

Definition 3.9. *A nano-subset of a nano topological space U is called nano-dense if Ncl(A) = U.*

Definition 3.10. *[3] A nano topological space (U, $\tau_R(X)$) is said to be nano extremally disconnected, if the nano-closure of each nano-open set is nano-open.*

Definition 3.11. *A space U is called nano-submaximal if each nano-dense subset of U is nano-open.*

Definition 3.12. *The family of all Nβ-nhds of the point $x \in U$ is called Nβ-nhd system of U and is denoted by Nβ-nhd x.*

Example 3.13. *Let U = $\{a,b,c,d\}$, U/R = $\{\{a,c\},\{b\},\{d\}\}$, X = $\{b,c\}$ and $\tau_R(X)$ = $\{U,\phi,\{b\},\{a,b,c\},\{a,c\},\}$.N$\beta$O(U,X) = $\{U,\phi,\{a\},\{b\},\{d\},\{a,b\},\{a,c\},\{a,d\}, \{b,c\},\{b,d\},\{c,d\},\{a,b,c\}, \{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Then*
Nβ-nhds(a) = $\{U,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$
Nβ-nhds(b) = $\{U,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$
Nβ-nhds(c) = $\{U,\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$
Nβ-nhds(d) = $\{U,\{d\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$

Lemma 3.14. *An arbitrary union of Nβ-nhds of a point x is again a Nβnhd of x.*

Proof. Let $\{A_\lambda\}_{\lambda \in I}$ be an arbitrary collection of Nβ-nhds of a point x $\in$ U. We have to prove that $\cup A_\lambda for \lambda \in$ I (where I denote index set) also a Nβ-nhd of x. For all $\lambda \in$ I there exists Nβ-open set M_x such that x $\in M_x \subset A_\lambda \subset \cup A_\lambda$ i.e x $\cup$ $M_x \subset \cup A_\lambda$ therefore $\cup A_\lambda$ for $\lambda \in$ I is a Nβ-nhd of x. That is arbitrary union of Nβ-nhds of x is again a Nβ-nhds of x. But intersection of Nβ-nhds of a point is not a Nβ-nhds of that point in general. For, □

Example 3.15. *Let U = $\{a,b,c,d\}$, U/R = $\{\{a\},\{b,d\},\{c\}\}$, X = $\{a,b\}$ and $\tau_R(X)$ = $\{U,\phi,\{a\},\{a,b,d\},\{b,d\}\}$ be a nano topology on U. Now NβO(U,X) = $\{U,\phi,\{a\},\{b\},\{d\},\{a,b\}, \{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Clearly $\{a,c\}$, $\{b,c\}$ are Nβ-nhds of $c \in U$ but $\{a,c\} \cap \{b,c\}$ = $\{c\}$ is not a Nβ-nhd of c.*

Theorem 3.16. *Every nano-nhd of x in U is a Nβ-nhd of x.*

Proof. Let A be an arbitrary nano-nhd of x $\in$ U then there exists an nano open set G such that x $\in$ G $\subset$ A. Since every nano open set is Nβ-open set. Therefore G $\in$ NβO(U,X), implies A is Nβ-nhd of x.
Converse of the above theorem is not true in general.For, □

Example 3.17. *From Example 3.15 Clearly $\{b,c,d\}$ is Nβ-nhds of b but not a nano nhd of b.*

Similarly every NS-nhd of x in U is a Nβ-nhd of x and every NP-nhd of x in U is a Nβ-nhd of x. But Nβ- nhd of x need not be NP-nhd or NS-nhd of x. For,

Example 3.18. *From Example 3.15 Now we have NSO(U,X) = $\{U,\phi,\{a\},\{a,c\},\{b,d\},\{a,b,d\},\{b,c,d\}\}$, NPO(U,X) = $\{U,\phi,$ $\{a\},\{b\},\{d\},\{a,b\},\{a,d\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\}$ and NβO(U,X) = $\{U,\phi,\{a\},\{b\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},$ $\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Clearly $\{c,d\}$ is Nβ-nhd of c but not a NS-nhd and NP-nhd of c.*

Theorem 3.19. *If the space U is nano submaximal and nano extremely disconnected space then every Nβ-nhd is NS-nhd and NP-nhd.*

Theorem 3.20. *The Nβ-nhd system Nβ-nhd(x) of a point x $\in$ U satisfies the following properties*
(a) if N $\in$ Nβ-N(x) then x $\in$ N
(b) if N $\in$ Nβ-N(x) and N $\subset$ M then M $\in$ Nβ-N(x)
(c) if N $\in$ Nβ-N(x) then there exists a G $\in$ Nβ-N(x) such that G $\subset$ N and G $\in$ Nβ-N(y), for all y $\in$ G.

Proof. (a) Let N $\in$ Nβ-N(x) implies N is the Nβ-nhd of x. Therefore x $\in$ N
(b) Let N $\in$Nβ-N(x) and N $\subset$ M. Therefore there exists G $\in$ NβO(U,X) such that x $\in$ G $\subset$ N $\subset$ M implies M is a Nβ-nhd of x and hence M $\in$ Nβ-N(x)
(c) Let N $\in$ Nβ-N(x) implies G $\in$ NβO(X) such that x $\in$ G $\subset$ N
G is Nβ-nhd of each of its points implies for all y $\in$ G, G is the Nβ-nhd of y and hence G $\in$ Nβ-N(y) for all y $\in$ G. □

4. Nano Semi Pre Interior of a Set

In the present section, we study some properties of Nβ interior of a set.

Definition 4.1. *The union of all nano semi pre-open sets which are contained in A is called nano semi pre-interior of A and is denoted by NβInt(A).*
And if x $\in$ NβInt(A) then x is called nano semi pre interior point of A.

Theorem 4.2. *For subsets A and B of space U, the following statements are true*
(a) NβInt(U-A) $\subset$ U-[NβInt(A)]
(b) Nβ Int(A-B) $\subset$ NβInt(A) - NβInt (B)

Proof. (a) Let y $\in$ NβInt(U-A), since NβInt(B) $\subset$ B by Lemma 3.3(i). Therefore y $\in$ U-A implies y $\notin$ A implies y $\notin$ NβInt(A) $\Rightarrow$ y $\in$ U-NβInt(A). Hence NβInt(U-A) $\subset$ U - [NβInt(A)].
(b) Now NβInt(A-B) = NβInt[(A $\cap$ (U-B)] = NβInt(A) $\cap$ NβInt(U-B) $\subset$ NβInt(A) $\cap$ (U-NβInt(B)) = NβInt(A) - NβInt(B).Therefore NβInt(A-B) $\subset$ NβInt(A) - NβInt (B) □

Theorem 4.3. *In a nano topological space U*
(a) NSInt(A) $\subset$ NβInt(A),
(b) NPInt(A) $\subset$ NβInt(A) for any subset A of U.

Proof. (a) Let x $\in$ NSInt(A) implies there exists nano semi open set G such that x $\in$ G $\subset$ A. Since every nano semi open set is Nβ - open and hence G is Nβ-open set such that x $\in$ G $\subset$ A implies x $\in$ NβInt(A). Therefore NSInt(A) $\subset$ NβInt(A).
(b) Let x $\in$ NPInt(A) implies there exists nano pre open set G such that x $\in$ G $\subset$ A. Since every nano pre open set is Nβ -open and hence G is Nβ-open set such that x $\in$ G $\subset$ A implies x $\in$ NβInt(A). Therfore NPInt(A) $\subset$ NβInt(A).
But NβInt(A) $\not\subset$ NS-Int (A) and NβInt(A) $\not\subset$ NP-Int (A). □

Example 4.4. *(i).From Example 3.18 Let A = $\{a,c\}$ then NβInt(A) = $\{a,c\}$ where as NPInt(A) = $\{a\}$ implies NPInt(A) $\subset$ NβInt(A) but NβInt(A) $\not\subset$ NPInt(A).*
(ii).Let A = $\{b,c\}$ then NβInt(A) = $\{b,c\}$ where as NSInt(A) =ϕ implies NSInt(A) $\subset$ BβInt(A) but NβInt(A) $\not\subset$ NSInt(A).

5. Nano Semi Pre-Frontier of a Set

Definition 5.1. *For a subset A of U, Nβcl(A) - NβInt (A) is said to be Nβ- frontier of A and is denoted by NβFr (A).*

Definition 5.2. *A point x $\in$ U is said to be Nβ-limit point of A iff for each X $\in$ NβO(U,X), X $\cap$ $\{A - \{x\}\} \neq \phi$.*

Definition 5.3. *The set of all Nβ-limit points of A is said to be the nano semi pre derived set and is denoted by NβD(A).*

Lemma 5.4. *Nβcl(A) = A $\cup$ NβD(A).*

Theorem 5.5. *For a subset A of space U the following statements hold;*
(a) NβFr(A) $\subset$ NFr(A)
(b) Nβcl(A) = NβInt (A) $\cup$ NβFr (A)
(c) NβInt(A) $\cap$ NβFr(A) = ϕ
(d) A is Nβ-open iff NβFr(A) = NβD(A)
(e) NβFr(A) = NβFr(U-A)
(f) NβFr(A) = Nβcl(A) $\cap$ Nβcl(U-A)
(g) NβFr(A) $\subset$ Nβclosed
(h) NβFr[NβFr(A)] $\subset$ NβFr(A)
(i) NβFr[Nβcl(A)) = NβFr(A)
(j) NβInt (A) = A - NβFr(A)

Proof. (a) Let x $\in$ NβFr(A) $\Rightarrow$ x $\in$ Nβcl(A) but x $\notin$ NβInt A By lemma 3.4 we have respectively Nβcl(A) $\subset$ Ncl(A) and NInt(A) $\subset$ NβInt (A). Therefore x $\in$ Ncl(A) and x $\notin$ NInt(A) $\Rightarrow$ x $\in$ Ncl(A) - NInt(A) i.e x $\in$ NFr(A). Hence NβFr(A) $\subset$

NFr(A).
Similarly (b), (c), (f), (i) and (j) are proved.
(d) Let A be Nβ-open then NβInt(A) = A. Since NβFr(A) = Nβcl(A) - NβInt(A) = Nβcl(A) - A. By Lemma 5.4 we have Nβcl(A) = A $\cup$ NβD(A). Therefore NβFr(A) = [A $\cup$ NβD(A)] - A = NβD(A).
Conversely Let NβFr(A) = NβD(A). i.e., Nβcl(A) - NβInt(A) = NβD(A) [A $\cup$ NβD(A)] - NβInt(A) = NβD(A) $\Rightarrow$ A - NβInt (A) =ϕ implies
A $\subset$ NβInt(A) ————(1)
and NβInt (A) $\subset$ A ———— (2)
Therefore from (1) and (2) we have NβInt(A) = A $\Rightarrow$ A is semi pre-open.
(e) By Lemma 3.4 we have NβFr(A) = Nβcl(A) $\cap$ Nβcl(U-A).
Therefore NβFr(U-A) = Nβcl(U-A) $\cap$ Nβcl(A) = NβFr(A).
(f) Now Nβcl(NβFr(A)) = Nβcl[Nβcl(U-A) $\cap$ Nβcl(A)] $\subset$ Nβcl(Nβcl(U-A)) $\cap$ Nβcl(Nβcl(A)) = [Nβcl(U-A) $\cap$ Nβcl(A)] = NβFr(A). Therefore Nβcl (NβFr(A)) $\subset$ NβFr(A) and NβFr(A) $\subset$ Nβcl(NβFr(A))
$\Rightarrow$ Nβcl(NβFr(A) = NβFr(A) $\Rightarrow$ NβFr(A) is semi pre closed.
(h) Now NβFr(NβFr(A)) = Nβcl(NβFr(A) $\cap$ Nβcl(U-NβFr(A)) $\Rightarrow$ NβFr(NβFr(A)) $\subset$ Nβcl(NβFr(A)) = (NβFr(A)) $\square$

6. Nano Semi Pre-Exterior of a Set

Definition 6.1. *A point x $\in$ U is called Nβ exterior point of a subset A of U if x is Nβ-interior point of U-A and set of all Nβ-exterior points of A is called Nβ-exterior of A and denoted by NβExt(A).Therefore NβExt(A) = NβInt(U-A).*

Theorem 6.2. *For a subset A of a space U the following statements holds*
(a) NβExt(A) $\subset$ Ext(A)
(b) NβExt(A) is Nβ-open
(c) NβExt(A) = U- Nβcl (A)
(d) NβExt[NβExt(A)] = NβInt[Nβcl(A)]
(e) If A $\subset$ B then NβExt(B) $\subset$ NβExt(A)
(f) NβExt(A $\cup$ B) $\subset$ NβExt(A) $\cup$ NβExt(B)
(g) NβExt(A) $\cap$ NβExt(B) $\subset$ NβExt(A $\cap$ B)
(h) NβExt(A $\cup$ B) = NβExt(A) $\cap$ NβExt(B)
(i) NβExt(U) =ϕ and NβExt(ϕ) = U
(j) NβExt(A) = NβExt [U-NβExt(A)]
(k) NβInt(A) $\subset$ NβExt [NβExt(A)]
(l) NβInt(A), NβExt(A) and βFr(A) are mutually disjoint and U = NβInt(A) $\cup$ NβExt(A) $\cup$ βFr(A).
(m) A $\cap$ NβExt(A) = ϕ

Proof. (a) Let x $\in$ NExt(A) $\Rightarrow$ x $\in$ NInt(U-A). There exists G $\in$ $\tau_R(X)$ such that x $\in$ G $\subset$ (U-A). G also $\in$ NβO(U,X). Therefore x $\in$ G $\subset$ (U-A) for Nβ-open set G $\Rightarrow$ (U-A) is Nβ-interior of x, x $\in$NβInt(U-A) i.e x $\in$ NβExt(A). Hence NExt A $\subset$ NβExt(A)
(b) Now NβInt [NβExt(A)] = NβInt [NβExt(U-A)] = NβInt(U-A)
= NβExt(A) $\Rightarrow$ NβExt(A) is Nβ-open.
(c) NβExt(A) = NβInt(U-A) = U-Nβcl(A).
(d) NβExt(A)[NβExt(A)] = NβExt[U-Ncl(A)] by (c)
= NβInt[U-[U-Nβcl(A)]] = NβInt[Nβcl(A)].
(e) If A $\subset$ B then U-B $\subset$ U-A $\Rightarrow$ NβInt(U-B) $\subset$ NβInt(U-A) i.e NβExt(B) $\subset$ NβExt(A)
(f) Since A $\subset$ A $\cup$ B and B $\subset$ A $\cup$ B $\Rightarrow$ NβExt(A $\cup$ B) $\subset$ NβExt(A)
NβExt(A $\cup$ B) $\subset$ NβExt(B).
Therefore NβExt(A $\cup$ B) $\subset$ NβExt(A) $\cup$ NβExt(B).
(g) We have A $\cap$ B $\subset$ A, A $\cap$ B $\subset$ B
$\Rightarrow$ NβExt(A) $\subset$ NβExt(A $\cap$ B) and NβExt(A) $\subset$ NβExt(A $\cap$ B)
$\Rightarrow$ NβExt(A) $\cap$ NβExt(B) $\subset$ NβExt(A $\cap$ B)
(h) NβExt(A $\cup$ B) = NβInt[U-(A $\cup$ B)] = NβInt([U-A] $\cap$ [U-B])
$\supset$ NβInt[U-A] $\cap$ NβInt([U-B] = NβExt(A) $\cap$ NβExt(B)
NβExt(A $\cup$ B) $\supset$ NβExt(A) $\cap$ NβExt(B)—————-(1).
Therefore we have A $\subset$ A $\cup$ B and B $\subset$ A $\cup$ B
$\Rightarrow$ NβExt(A $\cup$ B) $\subset$ NβExt(A) and NβExt(A $\cup$ B) $\subset$ NβExt(B)
$\Rightarrow$ NβExt(A $\cup$ B) $\subset$ NβExt(A) $\cap$ NβExt(B).............(2).
From (1) and (2) we have NβExt(A $\cup$ B) = NβExt(A) $\cap$ NβExt(B)
(i) NβInt(U) = NβInt(ϕ) = ϕ and NβExt(ϕ) = NβInt(U) = U
NβExt(U- NβExt(A)) = NβInt[U-(U-NβExt(A)]
= NβInt(NβExt(A))= NβInt[NβInt(U-A)]
= NβInt (U-A) = NβExt(A). Therefore NβExt(U-NβExt(A)) = NβExt(A)
(j) NβExt[U-NβExt(A)] = NβInt[U-(U-NβExt(A)]
= NβInt(NβExt(A))
= NβInt[NβInt(U-A)]
= NβInt(U-A) = NβExt(A)
(k) By the definition NβExt(A) $\subset$ U-A then from (e) NβExt(U-A) $\subset$ NβExt[NβExt(A)]
i.e NβInt(A) $\subset$ NβExt[NβExt(A)].
(l) Let us assume that NβExt(A) $\cap$ NβInt(A) $\neq$ ϕ therefore there exists
x $\in$ NβExt(A) $\cap$ NβInt(A) $\Rightarrow$ x $\in$ NβExt(A) and x $\in$ NβInt(A) $\Rightarrow$ x $\in$ U-A and x $\in$ A which is not possible. Therefore our assumption is wrong.
Hence NβExt(A) $\cap$ NβInt(A) = ϕ similarly other two results.
We have NβExt(A) = U-Ncl(A) = U-[NβInt(A) $\cup$ NβFr(A)] that implies
X = NβInt(A) $\cup$ NβExt(A) $\cup$ NβFr(A).
(m) Obvious.
In general, the converses of (f) and (g) are not true.
i.e NβExt(A) $\cup$ NβExt(B) $\not\subset$ NβExt(A $\cup$ B) and
NβExt(A $\cap$ B) $\not\subset$ NβExt(A) $\cap$ NβExt(B). $\square$

Example 6.3. *From Example 3.15 Let A = $\{a\}$ and B = $\{b,d\}$ then NβExt(A) = $\{b,c,d\}$, NβExt(B) = $\{a,c\}$, NβExt(A $\cup$ B) = NβInt$\{c\}$ =ϕ*
$\Rightarrow$ NβExt(A) $\cup$ NβExt(B) $\not\subset$ NβExt(A $\cup$ B).
Next NβExt(A $\cap$ B) = U and NβExt(A) $\cap$ NβExt(B) = $\{c\}$. Therefore NβExt(A $\cap$ B) $\not\subset$ NβExt(A) $\cap$ NβExt(B).

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