Malaya Journal of Matematik, Vol. 6, No. 2, 310-320, 2018

https://doi.org/10.26637/MJM0602/0002

Convergence properties on C^* -algebra valued fuzzy soft metric spaces and related fixed point theorems

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Abstract

In this article we introduce the notion of C^* - algebra valued fuzzy soft metric space and we prove convergence properties and some related fixed point results. We also give supported examples to our results.

Keywords

C^*-algebra, Fuzzy soft points, C^*-algebra-valued Fuzzy soft metric, contractive mappings, Fixed point theorems.

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

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Article History: Received **24** November **2017**; Accepted **09** March **2018**

Contents

1. Introduction

In daily life, the problems in many fields deal with uncertain data and are not successfully modeled in classical mathematics.There are two types of mathematical tools to deal with uncertainties namely fuzzy set theory introduced by Zadeh [26] and the theory of soft sets initiated by Molodstov [14] which helps to solve problems in all areas. Maji et al.[12] [13] introduced several operations in soft sets and as also coined fuzzy soft sets. In [21] Thangaraj Beaula et. al defined fuzzy soft metric space in terms of fuzzy soft points and proved some results. On the other hand several authors proved so many results in fuzzy soft sets and fuzzy soft metric spaces (see [6, 7, 9, 10],[17, 19, 21–23]).

Recently Ma et.al in [11] introduced the concept of C^*-algebra valued metric space and established some fixed point results for mapping under contractive conditions. This line of research was continued in (see [1], [2],[3], [4], [5], [8], [16], [18],[20], [24],[25],[27],[28]).

In this setting we are motivated their ideas and results, we will introduce definition of C^*- algebra valued fuzzy soft metric space in terms of fuzzy soft points and defined converges, Cauchy sequence. Further some basic fixed point theorems for self mapping under different contractions conditions relating to these concepts are established. Some suitable example and an applications to integral equations are given here to illustrate the usability of the obtained results.

2. Preliminaries

In this section we recollect some basic definitions and notations.

Throughout our discussion, U refers to an initial universe, E the set of all parameters for U and $P(\tilde{U})$ the set of all fuzzy set of U. (U,E) means the universal set U and parameter set E, $\tilde{C}$ refer to C^*-algebras. The details on C^*-algebras are available in [15]. An algebra '$\tilde{C}$'together with a conjugate linear involution map $*: \tilde{C} \to \tilde{C}$, defined by $\tilde{a} \to \tilde{a}^*$ such that for all $\tilde{a}, \tilde{b} \in \tilde{C}$, we have $(\tilde{a}\tilde{b})^* = \tilde{b}^*\tilde{a}^*$ and $(\tilde{a}^*)^* = \tilde{a}$, is called a $\star$ - algebra. Moreover , if $\tilde{C}$ an identity element $\tilde{I}_{\tilde{C}}$, then the pair $(\tilde{C},\star)$ is called a unital $\star$ - algebra. A unital $\star$ - algebra $(\tilde{C},\star)$ together with a complete sub multiplicative norm satisfying $\tilde{a} = \tilde{a}^*$ for all $\tilde{a} \in \tilde{C}$ is called a Banach $\star$ - algebra. A C^* - algebra is a Banach $\star$-algebra $(\tilde{C},\star)$ such that

$\tilde{a}^*\tilde{a} = \tilde{a}^2$ for all $\tilde{a} \in \tilde{C}$, An element $\tilde{a} \in \tilde{C}$is called a positive element if $\tilde{a} = \tilde{a}^*$ and $\sigma(\tilde{a}) \subset \Re(C)^*$ is set of non-negative fuzzy soft real numbers, where $\sigma(\tilde{a}) = \{\lambda \in \Re(C)^* \ \lambda\tilde{I} - \tilde{a}$, is non-invertible$\}$. If $\tilde{a} \in \tilde{C}$ is positive, we write it as $\tilde{a} \geq \tilde{0}_{\tilde{C}}$. Using positive elements, one can define partial ordering on $\tilde{C}$ as follows;$\tilde{a} \preceq \tilde{b}$ if and only if $\tilde{0}_{\tilde{C}} \preceq \tilde{b} - \tilde{a}$. Each positive element '$\tilde{a}$'of a C^*-algebra $\tilde{C}$ has a unique positive square root. Subsequently, $\tilde{C}$ will denote a unital C^*-algebra with the identity element $\tilde{I}_{\tilde{C}}$. Further, $\tilde{C}_+$ is the set $\{\tilde{a} \in \ \tilde{0}_{\tilde{C}} \preceq \tilde{a}\}$ of positive element of $\tilde{C}$. a C^*-algebra-valued Fuzzy soft metric space is defined in the following .

Definition 2.1. *([14] A Fuzzy set A in U is characterized by a function with domain as U and values in* $[0,1]$*. The collection of all fuzzy set U is* $P(\tilde{U})$*.*

Definition 2.2. *([12]) A pair* (F,E) *is called a soft set over U if and only if* $F: E \to P(U)$*is mapping from E into* $P(U)$ *the set of all sub set of U.*

Definition 2.3. *([17]) Let* $C \subseteq E$ *then the mapping* $F_E: C \to P(\tilde{U})$*, defined by* $F_E(e) = \mu^e F_E$ *(a fuzzy sub set of U), is called fuzzy soft set over* (U,E) *where ,*$\mu^e F_E = \tilde{0}$ *if* $e \in E - C$ *and* $\mu^e F_E \neq \tilde{0}$ *if* $e \in C$*. The set of all fuzzy soft set over* (U,E) *is denoted by* $FS(U,E)$*.*

Definition 2.4. *([17]) Let* $F_E \in FS(U,E)$ *and* $F_E(e) = \tilde{1}$ *for all* $e \in E$*. Then* F_E *is called absolute fuzzy soft set. it is denoted by* $\tilde{E}$*.*

3. Main Results

Here we introduce the notion of C^*-algebra valued fuzzy soft metric space.

Definition 3.1. *Let* $C \subseteq E$ *and* $\tilde{E}$ *be the absolute fuzzy soft set that is* $F_E(e) = \tilde{1}$ *for all* $e \in E$*. Let* $\tilde{C}$ *denote the* C^**-algebra. The* C^**-algebra valued fuzzy soft metric using fuzzy soft points is defined as a mapping* $\tilde{d_{c^*}}: \tilde{E} \times \tilde{E} \to \tilde{C}$ *satisfying the following conditions*

(M_0) $\tilde{0}_{\tilde{C}} \preceq \tilde{d}(F_{e_1}, F_{e_2})$ *for all* $F_{e_1}, F_{e_2} \in \tilde{E}$*.*

(M_1) $\tilde{d_{c^*}}(F_{e_1}, F_{e_2}) = \tilde{0}_{\tilde{C}} \Leftrightarrow F_{e_1} = F_{e_2}$

(M_2) $\tilde{d_{c^*}}(F_{e_1}, F_{e_2}) = \tilde{d_{c^*}}(F_{e_2}, F_{e_1})$

(M_3) $\tilde{d_{c^*}}(F_{e_1}, F_{e_3}) \preceq \tilde{d_{c^*}}(F_{e_1}, F_{e_2}) + \tilde{d_{c^*}}(F_{e_2}, F_{e_3})$ $\forall \ F_{e_1}, F_{e_2}, F_{e_3} \in \tilde{E}$

The fuzzy soft set $\tilde{E}$ *with the* C^**-algebra valued fuzzy soft metric* $\tilde{d_{c^*}}$ *is called the* C^**-algebra valued fuzzy soft metric space. It is denoted by* $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$*.*

It is obvious that C^*-algebra valued fuzzy soft metric generalize the concept of fuzzy soft metric spaces, replacing the set of fuzzy soft real numbers by $\tilde{C}_+$.

Definition 3.2. *A sequence* $\{F_{e_n}\}$ *in a* C^**-algebra valued fuzzy soft metric space* $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ *is said to converges to* $F_{e^{\prime}}$ *in* $\tilde{E}$ *with respect to* $\tilde{C}$*.*
If $||\tilde{d_{c^*}}(F_{e_n}, F_{e^{\prime}})||_{\tilde{C}} \to \tilde{0}_{\tilde{C}}$ *as* $n \to \infty$ *that is for every* $\tilde{0}_{\tilde{C}} \prec \tilde{\varepsilon}$ *there exists* $\tilde{0}_{\tilde{C}} \prec \tilde{\delta}$ *and a positive integer* $N = N(\tilde{\varepsilon})$ *such that* $||\tilde{d_{c^*}}(F_{e_n}, F_{e^{\prime}})|| < \tilde{\delta}$
implies that $||\mu^a_{F_{e_n}}(s) - \mu^a_{F_{e^{\prime}}}(s)|| < \tilde{\varepsilon}$ *whenever* $n \geq N$*. It is usually denoted as* $\lim_{n\to\infty} F_{e_n} = F_{e^{\prime}}$*.*

Definition 3.3. *A sequence* $\{F_{e_n}\}$ *in a* C^* *- algebra valued fuzzy soft metric space* $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ *is said to be Cauchy sequence. If to every* $\tilde{0}_{\tilde{C}} \prec \tilde{\varepsilon}$ *there exist* $\tilde{0}_{\tilde{C}} \prec \tilde{\delta}$ *and a positive integer* $N = N(\tilde{\varepsilon})$ *such that* $||\tilde{d_{c^*}}(F_{e_n}, F_{e_m})|| < \tilde{\delta}$ *implies that* $||\mu^a_{F_{e_n}}(s) - \mu^a_{F_{e_m}}(s)|| < \tilde{\varepsilon}$ *whenever* $n, m \geq N$*. That is*
$||\tilde{d_{c^*}}(F_{e_n}, F_{e_m})||_{\tilde{C}} \to \tilde{0}_{\tilde{C}}$*as* $n, m \to \infty$

Definition 3.4. *A* C^**-algebra valued fuzzy soft metric space* $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ *is said to be complete . If every Cauchy sequence in* $\tilde{E}$ *converges to some fuzzy soft point of* $\tilde{E}$*.*

Example 3.5. *Let* $C \subseteq R$ *and* $E \subseteq R$*, let* $\tilde{E}$ *be absolute fuzzy soft set, that is* $\tilde{E}(e) = \tilde{1}$ *for all* $e \in E$*, and* $\tilde{C} = M_2(R(A)^*)$*, define* $\tilde{d_{c^*}}: \tilde{E} \times \tilde{E} \to \tilde{C}$ *by*

$$\tilde{d_{c^*}}(F_{e_1}, F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

where $i = \inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)|/s \in C\}$ *and* $F_{e_1}, F_{e_2} \in \tilde{E}$*. Then* $\tilde{d_{c^*}}$ *is a* C^**-algebra valued fuzzy soft metric and* $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ *is a complete* C^* *- algebra valued fuzzy soft metric space by the completeness of* $R(C)^*$*.*

Proof. i) Let $F_{e_1}, F_{e_2} \in \tilde{E}$, then $\inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)|\} \geq 0$.
So $\tilde{0}_{\tilde{C}} \preceq \tilde{d_{c^*}}(F_{e_1}, F_{e_2})$
ii) Suppose $\tilde{d_{c^*}}(F_{e_1}, F_{e_2}) = \tilde{0}_{\tilde{C}} \Rightarrow \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \tilde{0}_{\tilde{C}}$
then $i = \inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)|\} = \tilde{0}_{\tilde{C}}$
implies $\mu^a_{F_{e_1}}(s) = \mu^a_{F_{e_2}}(s)$ that is $F_{e_1} = F_{e_2}$.
Similarly the reverse process.
Hence $\tilde{d_{c^*}}(F_{e_1}, F_{e_2}) = \tilde{0}_{\tilde{C}} \Leftrightarrow F_{e_1} = F_{e_2}$
iii) Clearly, $\tilde{d_{c^*}}(F_{e_1}, F_{e_2}) = \tilde{d_{c^*}}(F_{e_2}, F_{e_1})$

$$\begin{aligned} iv)\tilde{d_{c^*}}(F_{e_1}, F_{e_3}) &= \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \\ &\text{where } k = \inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_3}}(s)|/s \in C\} \\ &\preceq \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} + \begin{bmatrix} j & 0 \\ 0 & j \end{bmatrix} \\ &\text{where } i = \inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)|/s \in C\} \\ &\text{and } j = \inf\{|\mu^a_{F_{e_2}}(s) - \mu^a_{F_{e_3}}(s)|/s \in C\} \\ &\preceq \tilde{d_{c^*}}(F_{e_1}, F_{e_2}) + \tilde{d_{c^*}}(F_{e_2}, F_{e_3}) \end{aligned}$$

Hence from all above $\tilde{d_{c^*}}$ is C^* - algebra valued fuzzy soft metric and $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ is C^* - valued fuzzy soft metric space. Now we have to verify that the completeness.

Let $\{F_{e_n}\}$ in $\tilde{E}$ be a Cauchy sequence with respect to $\tilde{C}$. Then for given $\tilde{0}_{\tilde{C}} \prec \tilde{\varepsilon}$, $\exists$ a natural number $N = N(\tilde{\varepsilon}) \ni$ for all $n,m \geq N(\tilde{\varepsilon})$

$$\tilde{d_{c^*}}(F_{e_n}, F_{e_m}) = \begin{bmatrix} \eta & 0 \\ 0 & \eta \end{bmatrix}$$

where $\eta = \inf\{|\mu^a_{F_{en}}(s) - \mu^a_{F_{em}}(s)|/s \in C\}$

$$||\tilde{d_{c^*}}(F_{e_n}, F_{e_m})||_{\tilde{C}} = ||\inf\{|\mu^a_{F_{en}}(s) - \mu^a_{F_{em}}(s)|/s \in C\}|| < \tilde{\varepsilon}$$

Then $\{F_{e_n}\}$ is a Cauchy sequence in the space $\tilde{E}$. Thus there is a $F_{e^{\shortmid}} \in \tilde{E}$ and natural number $N_1 = N_1(\tilde{\varepsilon})$ such that $||\inf\{|\mu^a_{F_{en}}(s) - \mu^a_{F_{e^{\shortmid}}}(s)|/s \in C\}|| < \tilde{\varepsilon}$ if $n \geq N_1$
It follows

$$\tilde{d_{c^*}}(F_{e_n}, F_{e^{\shortmid}}) = \begin{bmatrix} \kappa & 0 \\ 0 & \kappa \end{bmatrix}$$

where $\kappa = \inf\{|\mu^a_{F_{en}}(s) - \mu^a_{F_{e^{\shortmid}}}(s)|/s \in C\}$
$||\tilde{d_{c^*}}(F_{e_n}, F_{e^{\shortmid}})||_{\tilde{C}} = ||\inf\{|\mu^a_{F_{en}}(s) - \mu^a_{F_{e^{\shortmid}}}(s)|/s \in C\}|| < \tilde{\varepsilon}$
$\forall \ n \geq N_1$
Therefore, the sequence $\{F_{e_n}\}$ converges to $F_{e^{\shortmid}} \in \tilde{E}$ with respect to $\tilde{C}$, that is $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ is a complete C^* - algebra valued fuzzy soft metric space with respect to $\tilde{C}$. □

Lemma 3.6. *A C^*-algebra valued fuzzy soft metric space $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ is complete . If every Cauchy sequence in $\tilde{E}$ has a converges sub sequence.*

Proof. Let $\{F_{e_n}\}_{n=1}{}^{\infty}$ be a Cauchy sequence in $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$
We show that if $\{F_{e_n}\}_{n=1}{}^{\infty}$ has a sub sequence $\{F_{e_{n_k}}\}_{n=1}{}^{\infty}$ that converges to a point $F_{e^{\shortmid}}$ then the sequence $\{F_{e_n}\}_{n=1}{}^{\infty}$ converges to $F_{e^{\shortmid}}$.
Given $\tilde{0}_{\tilde{C}} \prec \tilde{\varepsilon}$, there exists $\tilde{0}_{\tilde{C}} \prec \tilde{\delta} \ni ||\tilde{d_{c^*}}(F_{e_n}, F_{e_m})|| < \frac{\tilde{\delta}}{2}$ for all $n,m \geq N$ which implies $||\mu^a_{F_{en}}(s) - \mu^a_{F_{em}}(s)|| < \frac{\tilde{\varepsilon}}{2}$.
Then choose $n_i \geq N$ and $||\tilde{d_{c^*}}(F_{e_{n_i}}, F_{e^{\shortmid}})|| < \frac{\tilde{\delta}}{2}$ implies $||\mu^a_{F_{en_i}}(s) - \mu^a_{F_{e^{\shortmid}}}(s)|| < \frac{\tilde{\varepsilon}}{2}$
Using fact that $n_1 < n_2 <$ .... is an increasing sequence of integers and $\{F_{e_{n_i}}\}$ converges to $F_{e^{\shortmid}}$,
Therefore, $n \geq N$, we have

$$\begin{aligned} ||\tilde{d_{c^*}}(F_{e_n}, F_{e^{\shortmid}})|| &\leq ||\tilde{d_{c^*}}(F_{e_n}, F_{e_{m_i}}) + \tilde{d_{c^*}}(F_{e_{m_i}}, F_{e^{\shortmid}})|| \\ &\leq ||\tilde{d_{c^*}}(F_{e_n}, F_{e_{m_i}})|| + ||\tilde{d_{c^*}}(F_{e_{m_i}}, F_{e^{\shortmid}})|| \\ &< \frac{\tilde{\delta}}{2} + \frac{\tilde{\delta}}{2} = \tilde{\delta} \end{aligned}$$

$$\begin{aligned} \textit{and} ||\mu^a_{F_{en}}(s) - \mu^a_{F_{e^{\shortmid}}}(s)|| &\leq ||\mu^a_{F_{en}}(s) - \mu^a_{F_{en_i}}(s)|| \\ &\quad + ||\mu^a_{F_{en_i}}(s) - \mu^a_{F_{e^{\shortmid}}}(s)|| \\ &< \frac{\tilde{\varepsilon}}{2} + \frac{\tilde{\varepsilon}}{2} = \tilde{\varepsilon} \end{aligned}$$

Hence $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ is complete C^* - algebra valued fuzzy soft metric space. □

Lemma 3.7. *Let $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ be a C^*-algebra valued fuzzy soft metric space. Then every C^*-algebra valued fuzzy soft convergent sequence is a C^*-algebra valued fuzzy soft Cauchy sequence .*

Proof. Let $\{F_{e_n}\}_{n=1}{}^{\infty}$ be a C^*-algebra valued fuzzy soft convergent sequence in $\tilde{E}$ converging to $F_{e^{\shortmid}}$,
Since $||\tilde{d_{c^*}}(F_{e_n}, F_{e^{\shortmid}})|| \to \tilde{0}_{\tilde{C}}$ as $n \to \infty$
That is given $\tilde{0}_{\tilde{C}} \prec \tilde{\varepsilon}$, there exist a $\tilde{0}_{\tilde{c}} \prec \tilde{\delta}$, choose $N = N(\tilde{\varepsilon})$ such that $||\tilde{d_{c^*}}(F_{e_n}, F_{e^{\shortmid}})|| < \frac{\tilde{\delta}}{2}$ for all $n \geq N$ which implies $||\mu^a_{F_{en}}(s) - \mu^a_{F_{e^{\shortmid}}}(s)|| < \frac{\tilde{\varepsilon}}{2}$.
Then for all $n,m \geq N$, we have

$$\begin{aligned} ||\tilde{d_{c^*}}(F_{e_n}, F_{e_m})|| &\leq ||\tilde{d_{c^*}}(F_{e_n}, F_{e^{\shortmid}})|| + ||\tilde{d_{c^*}}(F_{e^{\shortmid}}, F_{e_m})|| \\ &< \frac{\tilde{\delta}}{2} + \frac{\tilde{\delta}}{2} = \tilde{\delta} \end{aligned}$$

and

$$\begin{aligned} ||\mu^a_{F_{en}}(s) - \mu^a_{F_{em}}(s)|| &\leq ||\mu^a_{F_{en}}(s) - \mu^a_{F_{e^{\shortmid}}}(s)|| + ||\mu^a_{F_{e^{\shortmid}}}(s) - \mu^a_{F_{em}}(s)|| \\ &< \frac{\tilde{\varepsilon}}{2} + \frac{\tilde{\varepsilon}}{2} = \tilde{\varepsilon} \end{aligned}$$

Hence $\{F_{e_n}\}$ is Cauchy sequence in $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$. □

Definition 3.8. *Let $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ be C^* - algebra valued fuzzy soft metric space. A mapping $\mathbf{T}\colon \tilde{E} \to \tilde{E}$ is said to be a C^* - algebra valued fuzzy soft contraction mapping. If there exists $\tilde{a} \in \tilde{C}$ with $||\tilde{a}|| < 1$ and*
$\tilde{d_{c^*}}(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2}) \preceq \tilde{a}\ \tilde{d_{c^*}}(F_{e_1}, F_{e_2})$ for all $F_{e_1}, F_{e_2} \in \tilde{E}$.

Lemma 3.9. *Let $\tilde{C}$ be C^* - algebra with the identity element $\tilde{I}_{\tilde{C}}$ and $\tilde{x}$ be positive element of $\tilde{C}$. If $\tilde{a} \in \tilde{C}$ is such that $||\tilde{a}|| < 1$, then for $m < n$,*
we have

$$\lim_{n\to\infty} \sum_{k=m}^{n} (\tilde{a}^{\star})^k \tilde{x} (\tilde{a})^k \preceq \tilde{I}_{\tilde{C}} ||\widetilde{(x)^{\frac{1}{2}}}||^2 \left(\frac{||\tilde{a}||^m}{1 - ||\tilde{a}||}\right) \tag{3.1}$$

and

$$\sum_{k=m}^{n} (\tilde{a}^{\star})^k \tilde{x} (\tilde{a})^k \to \tilde{0}_{\tilde{C}} \text{ as } m \to \infty \tag{3.2}$$

Proof. Since $\tilde{x}$ be a positive element of $\tilde{C}$, we have

$$\begin{aligned} \textstyle\sum_{k=m}^n (\tilde{a}^{\star})^k \tilde{x} (\tilde{a})^k &= \textstyle\sum_{k=m}^n (\tilde{a}^{\star})^k (\tilde{x})^{\frac{1}{2}} (\tilde{x})^{\frac{1}{2}} (\tilde{a})^k \\ &= \textstyle\sum_{k=m}^n (\tilde{x}^{\frac{1}{2}} \tilde{a}^k)^{\star} (\tilde{x}^{\frac{1}{2}} \tilde{a}^k) \\ &= \textstyle\sum_{k=m}^n |\tilde{x}^{\frac{1}{2}} \tilde{a}^k|^2 \\ &\preceq ||\textstyle\sum_{k=m}^n |\tilde{x}^{\frac{1}{2}} \tilde{a}^k|^2|| \\ &\preceq \tilde{I}_{\tilde{A}} \textstyle\sum_{k=m}^n ||\tilde{x}^{\frac{1}{2}}||^2 ||\tilde{a}^2||^k \\ &= \tilde{I}_{\tilde{A}} ||\tilde{x}^{\frac{1}{2}}||^2 \textstyle\sum_{k=m}^n ||\tilde{a}^2||^k \\ &= \tilde{I}_{\tilde{C}} ||\widetilde{(x)^{\frac{1}{2}}}||^2 \left(\frac{||\tilde{a}||^m}{1 - ||\tilde{a}||}\right) \end{aligned}$$

Moreover, $||\tilde{a}||^m \to 0$ as $m \to \infty$. □

Theorem 3.10. *Let $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ be a C^* - algebra valued fuzzy soft metric space. Suppose the mapping $\mathbf{T}\colon \tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1}, F_{e_2} \in \tilde{E}$,*
$\tilde{d_{c^*}}(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2}) \preceq \tilde{a}^{\star} \tilde{d_{c^*}}(F_{e_1}, F_{e_2}) \tilde{a}$ where $\tilde{a} \in \tilde{C}$ with $||\tilde{a}|| < 1$.
Then $\mathbf{T}$ has a unique fixed point in $\tilde{E}$.

Proof. It is clear that if $\tilde{C} = \tilde{0}_C$, $\mathbf{T}$ map the $\tilde{E}$ into a single point.
Thus without loss of gentility, one can suppose $\tilde{C} \neq \tilde{0}_C$, choose fuzzy soft point $F_{e_0} \in \tilde{E}$, Define the sequence $\{F_{e_n}\}_{n=1}{}^{\infty}$ in a way $F_{e_1} = \mathbf{T}F_{e_0}$, $F_{e_2} = \mathbf{T}F_{e_1} = \mathbf{T}^2F_{e_0}$ and so on then the follows $F_{e_n} = \mathbf{T}^nF_{e_0}$ and $F_{e_{n+1}} = \mathbf{T}^{n+1}F_{e_0}$, n=1, 2,3,........
Notices that in C^*-algebra, if $\tilde{a}, \tilde{b} \in \tilde{C_+}$ and $\tilde{a} \preceq \tilde{b}$, then for any $\tilde{x} \in \tilde{C_+}$ both $\tilde{x}^\star\tilde{a}\tilde{x}$ and $\tilde{x}^\star\tilde{b}\tilde{x}$ are positive elements and $\tilde{x}^\star\tilde{a}\tilde{x} \preceq \tilde{x}^\star\tilde{b}\tilde{x}$.
Thus

$$\begin{aligned}\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_n}) &= \tilde{d_{c^*}}(\mathbf{T}F_{e_n}, \mathbf{T}F_{e_{n-1}}) \preceq \tilde{a}^\star \tilde{d_{c^*}}(F_{e_n}, F_{e_{n-1}})\tilde{a} \\ &\preceq \tilde{a}^\star \tilde{d_{c^*}}(\mathbf{T}F_{e_{n-1}}, \mathbf{T}F_{e_{n-2}})\tilde{a} \\ &\preceq (\tilde{a}^\star)^2 \tilde{d_{c^*}}(F_{e_{n-1}}, F_{e_{n-2}})\tilde{a}^2 \\ &\vdots \\ &\preceq (\tilde{a}^\star)^n \tilde{d_{c^*}}(F_{e_1}, F_{e_0})\tilde{a}^n\end{aligned}$$

For $n+1 > m$, we have

$$\begin{aligned}\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_m}) &\preceq \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_n}) + \tilde{d_{c^*}}(F_{e_n}, F_{e_{n-1}}) \\ &\quad + ... + \tilde{d_{c^*}}(F_{e_{m+1}}, F_{e_m}) \\ &\preceq (\tilde{a}^\star)^n \tilde{d_{c^*}}(F_{e_1}, F_{e_0})\tilde{a}^n \\ &\quad + (\tilde{a}^\star)^{n-1} \tilde{d_{c^*}}(F_{e_1}, F_{e_0})\tilde{a}^{n-1} \\ &\quad + \cdots + (\tilde{a}^\star)^m \tilde{d_{c^*}}(F_{e_1}, F_{e_0})\tilde{a}^m \\ &\preceq \textstyle\sum_{k=m}^n (\tilde{a}^\star)^k \tilde{d_{c^*}}(F_{e_1}, F_{e_0})(\tilde{a})^k \\ &\preceq \textstyle\sum_{k=m}^n (\tilde{a}^\star)^k \tilde{D}(\tilde{a})^k \text{ where } \tilde{D} = \tilde{d_{c^*}}(F_{e_1}, F_{e_0}) \\ &\preceq \textstyle\sum_{k=m}^n (\tilde{a}^\star)^k (\tilde{D})^{\frac{1}{2}} (\tilde{D})^{\frac{1}{2}} (\tilde{a})^k \\ &= \textstyle\sum_{k=m}^n (\tilde{D}^{\frac{1}{2}}\tilde{a}^k)^\star (\tilde{D}^{\frac{1}{2}}\tilde{a}^k) \\ &= \textstyle\sum_{k=m}^n |\tilde{D}^{\frac{1}{2}}\tilde{a}^k|^2 \\ &\preceq ||\textstyle\sum_{k=m}^n |\tilde{D}^{\frac{1}{2}}\tilde{a}^k|^2|| \\ &\preceq \tilde{I_{\tilde{C}}} \textstyle\sum_{k=m}^n ||\tilde{D}^{\frac{1}{2}}||^2 ||\tilde{a}^2||^k \\ &\preceq \tilde{I_{\tilde{C}}} ||\tilde{D}^{\frac{1}{2}}||^2 \textstyle\sum_{k=m}^n ||\tilde{a}^2||^k\end{aligned}$$

Therefore, $||\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_m})|| \preceq \tilde{I_{\tilde{C}}} ||\tilde{d_{c^*}}^{\frac{1}{2}}||^2 \left(\frac{||\tilde{a}||^{2m}}{1-||\tilde{a}||}\right) \to \tilde{0}_{\tilde{A}}$ as $m \to \infty$.
Thus, $\{F_{e_n}\}_{n=1}{}^{\infty}$ is a Cauchy sequence with respect to $\tilde{C}$. By the completeness of $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$, there is an $F_e^{|} \in \tilde{E}$ such that $\lim_{n\to\infty} \mathbf{T}F_{e_n} = F_{e^{|}}$. That is $||\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e^{|}})|| \to \infty$ as $m \to \infty$
We now claim that $F_{e^{|}}$ is a fixed point of $\mathbf{T}$.

$$\begin{aligned}\tilde{0}_{\tilde{C}} \preceq \tilde{d_{c^*}}(\mathbf{T}F_{e^{|}}, F_{e^{|}}) &\preceq \tilde{d_{c^*}}(\mathbf{T}F_{e^{|}}, \mathbf{T}F_{e_n}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_n}, F_{e^{|}}) \\ &\preceq \tilde{a}^\star \tilde{d_{c^*}}(F_{e^{|}}, F_{e_n})\tilde{a} + \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e^{|}})\end{aligned}$$

So that

$$\begin{aligned}0 &\leq ||\tilde{d_{c^*}}(\mathbf{T}F_{e^{|}}, F_{e^{|}})|| \\ &\leq ||\tilde{a}||^2 ||\tilde{d_{c^*}}(F_{e^{|}}, F_{e_n})|| + ||\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e^{|}})|| \\ &< ||\tilde{d_{c^*}}(F_{e^{|}}, F_{e_n})|| + ||\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e^{|}})|| \to 0 \text{ as } n \to \infty\end{aligned}$$

Hence $\mathbf{T}F_{e^{|}} = F_{e^{|}}$, that is $F_{e^{|}}$ is fixed point of $\mathbf{T}$.
Now suppose $F_{e^{||}}$ be anther fixed point of $\mathbf{T}$
Since $\tilde{0}_{\tilde{A}} \preceq \tilde{d_{c^*}}(F_{e^{|}}, F_{e^{||}}) \preceq \tilde{d_{c^*}}(\mathbf{T}F_{e^{|}}, \mathbf{T}F_{e^{||}}) \preceq \tilde{a}^\star \tilde{d_{c^*}}(F_{e^{|}}, F_{e^{||}})\tilde{a}$.
Then we have

$$\begin{aligned}0 &\leq ||\tilde{d_{c^*}}(F_{e^{|}}, F_{e^{||}})|| \\ &\leq ||\tilde{a}^\star \tilde{d_{c^*}}(F_{e^{|}}, F_{e^{||}})\tilde{a}|| \\ &\leq ||\tilde{a}||^2 ||\tilde{d_{c^*}}(F_{e^{|}}, F_{e^{||}})|| \\ &< ||\tilde{d_{c^*}}(F_{e^{|}}, F_{e^{||}})||\end{aligned}$$

it is impossible, so $\tilde{d_{c^*}}(F_{e^{|}}, F_{e^{||}}) = \tilde{0}$ implies $F_{e^{|}} = F_{e^{||}}$.
Hence The fixed point is unique. □

Example 3.11. *Let $E = \{e_1, e_2, e_3, e_4\}$, $R = U = \{a, b, c, d\}$ and*
$C = \{e_1, e_2, e_3,\}$, define fuzzy soft sets as

$$(F_E, C) = \left\{ \begin{array}{c} e_1 = \{a_{0.3}, b_{0.2}, c_{0.4}, d_{0.6}\}, \\ e_2 = \{a_{0.1}, b_{0.4}, c_{0.5}, d_{0.2}\}, \\ e_3 = \{a_{0.6}, b_{0.7}, c_{0.8}, d_{0.9}\} \end{array} \right\}$$

and
$F_{e_1} = \mu_{F_{e_1}} = \{a_{0.3}, b_{0.2}, c_{0.4}, d_{0.6}\}$,$F_{e_2} = \mu_{F_{e_2}} = \{a_{0.1}, b_{0.4}, c_{0.5}, d_{0.2}\}$,
$F_{e_3} = \mu_{F_{e_3}} = \{a_{0.6}, b_{0.7}, c_{0.8}, d_{0.9}\}$
therefore, $FSC(F_E) = \{F_{e_1}, F_{e_2}, F_{e_3}\}$ and $F_E(e) = \tilde{1}\ \ \forall\ \ e \in E$
We define $\tilde{d_{c^*}} : \tilde{E} \times \tilde{E} \to \tilde{C}$ by

$$\tilde{d_{c^*}}(F_{e_1}, F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

where $i = inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)|/s \in C\}$ and also define
$\mathbf{T}$*: $\tilde{E} \to \tilde{E}$ by $\mathbf{T}F_e(a) = \frac{F_e}{3}$ for all $a \in U$ and $F_{e_1}, F_{e_2} \in \tilde{E}$*
where $\tilde{c} = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \in \tilde{C}$ and $||\tilde{c}|| = \frac{\sqrt{2}}{2} < 1$
Then $\mathbf{T}$ has a unique fixed point.

Proof. It follows from Theorem (3.10), $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ is a complete metric space. Moreover, $\mathbf{T}$ is a self mapping on $\tilde{E}$, exactly as in the proof of Theorem (3.10). Notice that the Fuzzy soft real number $\tilde{c}$ generating the constant C^*- algebra valued fuzzy soft number satisfies $||\tilde{c}|| = \frac{\sqrt{2}}{2} < 1$. Finally, we obtain the following contraction condition, for each $F_{e_1}, F_{e_2} \in \tilde{E}$
For $a \in U, |\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)| = |0.3 - 0.1| = 0.2$,
$b \in U, |\mu^b_{F_{e_1}}(s) - \mu^b_{F_{e_2}}(s)| = |0.2 - 0.2| = 0.2$,
$c \in U, |\mu^c_{F_{e_1}}(s) - \mu^c_{F_{e_2}}(s)| = |0.4 - 0.5| = 0.1$,
$d \in U, |\mu^d_{F_{e_1}}(s) - \mu^d_{F_{e_2}}(s)| = |0.6 - 0.2| = 0.4$.
Now

$\inf\{|\mu^a_{F_{e_1}}(s)-\mu^a_{F_{e_2}}(s)|/s\in C\}=inf\{0.2,0.2,0.1,0.4\}=0.1$

therefore, $\tilde{d_{c^*}}(F_{e_1},F_{e_2})=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}$ then

$\tilde{d_{c^*}}(\mathbf{T}F_{e_1},\mathbf{T}F_{e_2})=\begin{bmatrix}0.03 & 0\\ 0 & 0.03\end{bmatrix}$

So one can verify that

$$\begin{aligned}\tilde{d_{c^*}}(\mathbf{T}F_{e_1},\mathbf{T}F_{e_2}) &= \begin{bmatrix}0.03 & 0\\ 0 & 0.03\end{bmatrix}\\ &\preceq \begin{bmatrix}\frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2}\end{bmatrix}\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}\begin{bmatrix}\frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2}\end{bmatrix}\\ &\preceq \tilde{c}^{\star}\tilde{d_{c^*}}(F_{e_1},F_{e_2})\tilde{c}.\end{aligned}$$

Therefore, $\tilde{d_{c^*}}(\mathbf{T}F_{e_1},\mathbf{T}F_{e_2})\preceq\tilde{c}^*\tilde{d_{c^*}}(F_{e_1},F_{e_2})\tilde{c}$.
Hence $\mathbf{T}$ has a unique fixed point. □

Definition 3.12. *Let $\tilde{E}$ be a absolute fuzzy soft set. We call a mapping $\mathbf{T}$ is a C^* - algebra valued fuzzy soft expansion mapping on $\tilde{E}$, if $\mathbf{T}\colon\tilde{E}\to\tilde{E}$ satisfies:*
1) $\mathbf{T}(\tilde{E})=\tilde{E}$
2) $\tilde{d_{c^*}}(\mathbf{T}F_{e_1},\mathbf{T}F_{e_2})\succeq\tilde{a}^{\star}\tilde{d_{c^*}}(F_{e_1},F_{e_2})\tilde{a}$ for all $F_{e_1},F_{e_2}\in\tilde{E}$ where $\tilde{a}\in\tilde{C}$ is an invertible element and $||\tilde{a}^{-1}||<1$.

Remark 3.13. *Let $(\tilde{E},\tilde{C},\tilde{d_{c^*}})$ be complete a C^*-algebra valued fuzzy soft metric space. Then there is an expansion mapping $\mathbf{T}$ has no fixed point.*

Example 3.14. *Let $C=E=\{e_1,e_2,e_3,\},U=R=\{a,b,c,d\}$, let $\tilde{E}$ be absolute fuzzy soft set, that is $\tilde{E}(e)=\tilde{1}$ for all $e\in E$, and $\tilde{C}=(R(A)^*)$, define $\tilde{d_{c^*}}:\tilde{E}\times\tilde{E}\to\tilde{C}$ by*

$$\tilde{d_{c^*}}(F_{e_1},F_{e_2})=\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}$$

where $i=\inf\{|\mu^a_{F_{e_1}}(s)-\mu^a_{F_{e_2}}(s)|/s\in C\}$ and define the mapping $\mathbf{T}\colon\tilde{E}\to\tilde{E}$ by $\mathbf{T}F_e(a)=\frac{F_e}{2}$ for all $a\in U$ and $F_{e_1},F_{e_2}\in\tilde{E}$, where $\tilde{c}=\begin{bmatrix}\frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2}\end{bmatrix}\in\tilde{C}$ and $||\tilde{c}||=\frac{\sqrt{2}}{2}<1$
Then $\mathbf{T}$ has no fixed point.

Proof. It follows from Theorem (3.10), $(\tilde{E},\tilde{C},\tilde{d_{c^*}})$ is a complete metric space. Moreover, $\mathbf{T}$ is a self expansion mapping on $\tilde{E}$, exactly as in the Definition (3.12). Notice that the Fuzzy soft real number $\tilde{c}$ generating the constant C^*- algebra valued fuzzy soft number satisfies $||\tilde{c}||=\frac{\sqrt{2}}{2}<1$. Finally, we obtain the following contraction condition, for each $F_{e_1},F_{e_2}\in\tilde{E}$
For $a\in U,|\mu^a_{F_{e_1}}(s)-\mu^a_{F_{e_2}}(s)|=|0.3-0.1|=0.2$,
$b\in U,|\mu^b_{F_{e_1}}(s)-\mu^b_{F_{e_2}}(s)|=|0.2-0.2|=0.2$,
$c\in U,|\mu^c_{F_{e_1}}(s)-\mu^c_{F_{e_2}}(s)|=|0.4-0.5|=0.1$,
$d\in U,|\mu^d_{F_{e_1}}(s)-\mu^d_{F_{e_2}}(s)|=|0.6-0.2|=0.4$.
Now
$inf\{|\mu^a_{F_{e_1}}(s)-\mu^a_{F_{e_2}}(s)|/a\in U\}=inf\{0.2,0.2,0.1,0.4\}=0.1$.
Therefore,

$$\tilde{d_{c^*}}(F_{e_1},F_{e_2})=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}$$

then

$$\tilde{d_{c^*}}(\mathbf{T}F_{e_1},\mathbf{T}F_{e_2})=\begin{bmatrix}0.05 & 0\\ 0 & 0.05\end{bmatrix}$$

one can verify that

$$\begin{aligned}\tilde{d_{c^*}}(\mathbf{T}F_{e_1},\mathbf{T}F_{e_2}) &= \begin{bmatrix}0.05 & 0\\ 0 & 0.05\end{bmatrix}\\ &\succeq \begin{bmatrix}\frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2}\end{bmatrix}\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}\begin{bmatrix}\frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2}\end{bmatrix}\\ &\succeq \tilde{c}^{\star}\tilde{d_{c^*}}(F_{e_1},F_{e_2})\tilde{c}.\end{aligned}$$

we have, $\tilde{a}=\begin{bmatrix}\frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2}\end{bmatrix}\in\tilde{C}$ $and||\tilde{c}||=\frac{\sqrt{2}}{2}<1$
but $||\tilde{c}^{-1}||\not<1$.
Hence $\mathbf{T}$ has no fixed point.

□

Before introducing another fixed point theorem, we give a lemma first.Such results can be found in [15].

Lemma 3.15. *Suppose $\tilde{C}$ is a unital C^*-algebra with unit $\tilde{1}$.*
i)If $\tilde{a}\in\tilde{C_+}$ with $||\tilde{a}||<\frac{1}{2}$ then $\tilde{I}-\tilde{a}$ is invertible and $||\tilde{a}(\tilde{I}-\tilde{a})^{-1}||<1$
ii) suppose that $\tilde{a},\tilde{b}\in\tilde{C}$ with $\tilde{a},\tilde{b}\succeq\tilde{0}_{\tilde{C}}$ and $\tilde{a}\tilde{b}=\tilde{b}\tilde{a}$ then $\tilde{a}\tilde{b}\succeq\tilde{0}_{\tilde{C}}$
iii) $\tilde{C}'$ denote the set $\{\tilde{a}\in\tilde{C}/\tilde{a}\tilde{b}=\tilde{b}\tilde{a}\ \forall\ \tilde{b}\in\tilde{C}\}$. Let $\tilde{a}\in\tilde{C}'$, if $\tilde{b},\tilde{c}\in\tilde{C}$ with $\tilde{b}\succeq\tilde{c}\succeq\tilde{0}$ and $\tilde{I}-\tilde{a}\in\tilde{C}'_+$ is an invertible operator, then $(\tilde{I}-\tilde{a})^{-1}\tilde{b}\succeq(\tilde{I}-\tilde{a})^{-1}\tilde{c}$.
Notices that in C^* - algebra , if $\tilde{0}\preceq\tilde{a},\tilde{b}$, one can't conclude that $\tilde{0}\preceq\tilde{a}\tilde{b}$. Indeed, consider the c^* - algebra $(R(C)^*)$ and set
$\tilde{a}=\begin{bmatrix}F_{e_1}(a) & F_{e_2}(a)\\ F_{e_2}(a) & F_{e_1}(b)\end{bmatrix}=\begin{bmatrix}0.3 & 0.1\\ 0.1 & 0.2\end{bmatrix}$
and $\tilde{b}=\begin{bmatrix}F_{e_1}(c) & F_{e_2}(c)\\ F_{e_2}(c) & F_{e_1}(d)\end{bmatrix}=\begin{bmatrix}0.4 & 0.5\\ 0.5 & 0.6\end{bmatrix}$
then clearly $\tilde{a}\succeq\tilde{0}$ and $\tilde{b}\succeq\tilde{0}$ but $\tilde{a},\tilde{b}\in(R(C)^*)_+$ while $\tilde{a}\tilde{b}$ is not.

Theorem 3.16. *Let $(\tilde{E},\tilde{C},\tilde{d_{c^*}})$ be a C^*-algebra valued fuzzy soft metric space. Suppose the mapping $\mathbf{T}\colon\tilde{E}\to\tilde{E}$ satisfies for all $F_{e_1},F_{e_2}\in\tilde{E}$,*

$$\tilde{d_{c^*}}(\mathbf{T}F_{e_1},\mathbf{T}F_{e_2})\preceq\tilde{a}^{\star}\left(\tilde{d_{c^*}}(\mathbf{T}F_{e_1},F_{e_2})+\tilde{d_{c^*}}(\mathbf{T}F_{e_2},F_{e_1})\right),$$

where $\tilde{a}\in\tilde{C}$ with $||\tilde{a}||<\frac{1}{2}$. Then $\mathbf{T}$ has a unique fixed point in $\tilde{E}$.

Proof. It is clear that if $\tilde{C}=\tilde{0}_C$, $\mathbf{T}$ map the $\tilde{E}$ into a single point.
Thus without loss of gentility, one can suppose $\tilde{C}\neq\tilde{0}_C$,Notices that
$\tilde{a}\in\tilde{C}_+$, $\tilde{a}^{\star}\left(\tilde{d_{c^*}}(\mathbf{T}F_{e_1},F_{e_2})+\tilde{d_{c^*}}(\mathbf{T}F_{e_2},F_{e_1})\right)$ is also positive, choose fuzzy soft point $F_{e_0}\in\tilde{E}$, Define the sequence $\{F_{e_n}\}_{n=1}^{\infty}$ in a way $F_{e_1}=\mathbf{T}F_{e_0}$ $F_{e_2}=\mathbf{T}F_{e_1}=\mathbf{T}^2F_{e_0}$ and so on then the

follows $F_{e_n} = \mathbf{T}^n F_{e_0}$ and $F_{e_{n+1}} = \mathbf{T}^{n+1} F_{e_0}$, n=1, 2,3,........ By D denote the element $\tilde{d_{c^*}}(F_{e_1}, F_{e_0})$ in $\tilde{C}$, then we have

$$\begin{aligned}
\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_n}) &= \tilde{d_{c^*}}(\mathbf{T}F_{e_n}, \mathbf{T}F_{e_{n-1}}) \\
&\preceq \tilde{a}\left(\tilde{d_{c^*}}(\mathbf{T}F_{e_n}, F_{e_{n-1}}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_{n-1}}, F_{e_n})\right) \\
&\preceq \tilde{a}\left(\tilde{d_{c^*}}(\mathbf{T}F_{e_n}, \mathbf{T}F_{e_{n-2}}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_{n-1}}, \mathbf{T}F_{e_{n-1}})\right) \\
&\preceq \tilde{a}\left(\tilde{d_{c^*}}(\mathbf{T}F_{e_n}, \mathbf{T}F_{e_{n-1}}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_{n-1}}, \mathbf{T}F_{e_{n-2}})\right) \\
&\preceq \tilde{a}\tilde{d_{c^*}}(\mathbf{T}F_{e_n}, \mathbf{T}F_{e_{n-1}}) + \tilde{a}\tilde{d_{c^*}}(\mathbf{T}F_{e_{n-1}}, \mathbf{T}F_{e_{n-2}})
\end{aligned}$$

Thus $(\tilde{I} - \tilde{a})\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_n}) \preceq \tilde{a}\tilde{d_{c^*}}(F_{e_n}, F_{e_{n-1}})$
Since $\tilde{a} \in C'_+$ with $||\tilde{a}|| < \frac{1}{2}$ and furthermore, $\tilde{a}(\tilde{I} - \tilde{a})^{-1} \in C'_+$ with $||\tilde{a}(\tilde{I} - \tilde{a})^{-1}|| < 1$ by Lemma (3.15). Therefore,

$$\begin{aligned}
\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_n}) &\preceq \tilde{a}(\tilde{I} - \tilde{a})^{-1}\tilde{d_{c^*}}(F_{e_n}, F_{e_{n-1}}) \\
&\preceq \tilde{t}\tilde{d_{c^*}}(F_{e_n}, F_{e_{n-1}}) \text{ where } \tilde{t} = \tilde{a}(\tilde{I} - \tilde{a})^{-1}
\end{aligned}$$

For $n+1 > m$

$$\begin{aligned}
&\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_m}) \\
&\preceq \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e_n}) + \tilde{d_{c^*}}(F_{e_n}, F_{e_{n-1}}) + \cdots + \tilde{d_{c^*}}(F_{e_{m+1}}, F_{e_m}) \\
&\preceq (\tilde{t}^n + \tilde{t}^{n-1} + \tilde{t}^{n-2} + \ldots\ldots.. + \tilde{t}^m)\tilde{d_{c^*}}(F_{e_1}, F_{e_0}) \\
&\preceq \textstyle\sum_{k=m}^{n} D\tilde{t}^k \\
&\preceq \textstyle\sum_{k=m}^{n} D^{\frac{1}{2}} D^{\frac{1}{2}} \tilde{t}^{\frac{k}{2}} \tilde{t}^{\frac{k}{2}} \\
&\preceq \textstyle\sum_{k=m}^{n} (D^{\frac{1}{2}} \tilde{t}^{\frac{k}{2}})^\star (D^{\frac{1}{2}} \tilde{t}^{\frac{k}{2}}) \\
&\preceq \textstyle\sum_{k=m}^{n} |D^{\frac{1}{2}} \tilde{t}^{\frac{k}{2}}|^2 \\
&\preceq \tilde{I}||\textstyle\sum_{k=m}^{n} |D^{\frac{1}{2}} \tilde{t}^{\frac{k}{2}}|^2|| \\
&\preceq \tilde{I}\textstyle\sum_{k=m}^{n} ||D^{\frac{1}{2}}||^2 ||\tilde{t}^{\frac{k}{2}}||^2 \\
&\preceq \tilde{I}||D^{\frac{1}{2}}||^2 \textstyle\sum_{k=m}^{n} ||\tilde{t}||^k \\
&\preceq \tilde{I}||D^{\frac{1}{2}}||^2 \left(\frac{||\tilde{t}||^m}{1 - ||\tilde{t}||}\right) \to \tilde{0} \text{ as } m \to \infty
\end{aligned}$$

This implies $\{F_{e_n}\}_{n=1}{}^{\infty}$ is a Cauchy sequence with respect to $\tilde{C}$. By the completeness of $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$, there exists $F_{e'} \in \tilde{E}$ such that $\lim_{n\to\infty} F_{e_n} = F_{e'}$. That is $\lim_{n\to\infty} \mathbf{T}F_{e_{n-1}} = F_{e'}$.That is
$||\tilde{d_{c^*}}(F_{e_n}, F_{e'})|| \to \tilde{0}$ as $n \to \infty$
Now we show that $F_{e'}$ is fixed point of $\mathbf{T}$

$$\begin{aligned}
\tilde{0}_{\tilde{C}} &\preceq \tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_{e'}) \\
&\preceq \tilde{d_{c^*}}(\mathbf{T}F_{\mathbf{e}'}, \mathbf{T}F_{e_n}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_n}, F_{e'}) \\
&\preceq \tilde{a}\left(\tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_{e_n}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_n}, F_{e'})\right) + \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'}) \\
&\preceq \tilde{a}\left(\tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_{e'}) + \tilde{d_{c^*}}(F_{e'}, F_{e_n}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_n}, F_{e'})\right) \\
&\quad + \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'})
\end{aligned}$$

$$\begin{aligned}
&(\tilde{I} - \tilde{a})\tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_{e'}) \\
&\preceq \tilde{a}\left(\tilde{d_{c^*}}(F_{e'}, F_{e_n}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_n}, F_{e'})\right) + \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'})
\end{aligned}$$

$$\begin{aligned}
\tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_{e'}) &\preceq \tilde{a}(\tilde{I} - \tilde{a})^{-1}\left(\tilde{d_{c^*}}(F_{e'}, F_{e_n}) + \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'})\right) \\
&\quad + (\tilde{I} - \tilde{a})^{-1}\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'})
\end{aligned}$$

$$\begin{aligned}
\tilde{0} &\leq ||\tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_{e'})|| \leq ||\tilde{a}(\tilde{I} - \tilde{a})^{-1}||\,||\tilde{d_{c^*}}(F_{e'}, F_{e_n}) + \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'})|| \\
&\quad + ||(\tilde{I} - \tilde{a})^{-1}||\,||\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'})||
\end{aligned}$$

$$\begin{aligned}
\tilde{0} &\leq ||\tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_{e'})|| < ||\tilde{d_{c^*}}(F_{e'}, F_{e_n}) + \tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'})|| \\
&\quad + ||\tilde{d_{c^*}}(F_{e_{n+1}}, F_{e'})|| \\
&\to \tilde{0} \quad \text{as} \quad n \to \infty
\end{aligned}$$

Therefore, $\tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_{e'}) = \tilde{0}$ implies $\mathbf{T}F_{e'} = F_{e'}$
Hence, $F_{e'}$ is fixed point of $\mathbf{T}$.
Now $F_e' \neq F_e''$ is another fixed point of $\mathbf{T}$. Then we have

$$\begin{aligned}
\tilde{0}_C &\leq \tilde{d_{c^*}}(F_e', F_e'') \\
&= \tilde{d_{c^*}}(\mathbf{T}F_e', \mathbf{T}F_e'') \\
&\preceq \tilde{a}^\star\left(\tilde{d_{c^*}}(\mathbf{T}F_{e'}, F_e'') + \tilde{d_{c^*}}(\mathbf{T}F_e'', F_e')\right) \\
&\preceq \tilde{a}^\star\left(\tilde{d_{c^*}}(F_e', F_e'') + \tilde{d_{c^*}}(F_e'', F_e')\right).
\end{aligned}$$

That is

$$\begin{aligned}
(\tilde{I} - \tilde{a}^\star)\tilde{d_{c^*}}(F_e', F_e'') &\preceq \tilde{a}^\star\tilde{d_{c^*}}(F_e'', F_e') \\
&\preceq \tilde{a}^\star(\tilde{I} - \tilde{a}^\star)^{-1}\tilde{d_{c^*}}(F_e'', F_e')
\end{aligned}$$

Since $||\tilde{a}^\star(\tilde{I} - \tilde{a}^\star)^{-1}|| < 1$ so $\tilde{0} \leq ||\tilde{d_{c^*}}(F_e', F_e'')|| < ||\tilde{d_{c^*}}(F_e', F_e'')||$ which means $\tilde{d_{c^*}}(F_e', F_e'') = \tilde{0}_{\tilde{A}} \Leftrightarrow F_e' = F_e''$.
Therefore, the fixed point is unique and the proof is complete. □

Example 3.17. *Let $E = C = U = \{\frac{1}{n} : n \in N\}$. and define $\tilde{d_{c^*}} : \tilde{E} \times \tilde{E} \to \tilde{C}$ by $\tilde{d_{c^*}}(F_{e_1}(a), F_{e_2}(a))(s) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ where $i = \inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)|/s \in C\}$ is a C^*-algebra valued fuzzy soft metric on $\tilde{E}$. Furthermore, C^* - algebra valued fuzzy soft metric space $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ is complete.*
Let $\mathbf{T}: \tilde{E} \to \tilde{E}$ by $\mathbf{T}F_e(a) = \frac{F_e(a)}{4}$ $\forall$ $F_e \in \tilde{E}$ and $a \in U$. We show that $\mathbf{T}$ satisfies the conditions of Theorem (3.16) where $\tilde{c} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \in \tilde{C}$ with $||\tilde{c}|| = \frac{1}{3}$. In fact,

given $F_{e_1}, F_{e_2} \in \tilde{E}$ *and* $a \in U, s \in C$ *then we have*

$$\tilde{d_{c^*}}(\mathbf{T}F_{e_1}(a), \mathbf{T}F_{e_2}(a))(s) = \tilde{d_{c^*}}(\tfrac{F_{e_1}(a)}{4}, \tfrac{F_{e_2}(a)}{4})(s)$$

$$= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$
where $\lambda = \inf\{|\frac{\mu^a_{F_{e_1}}(s)}{4} - \frac{\mu^a_{F_{e_2}}(s)}{4}| / s \in C\}$

$$= \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$
where $i = \inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)| / s \in C\}$

$$\preceq \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix}$$
where $\gamma = \inf\{|\mu^a_{F_{e_1}}(s) + \mu^a_{F_{e_2}}(s)| / s \in C\}$

$$\preceq \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \left(\begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix} + \begin{bmatrix} \nu & 0 \\ 0 & \nu \end{bmatrix} \right)$$
where $\mu = \inf\{|\mu^a_{F_{e_1}}(s) - \frac{\mu^a_{F_{e_2}}(s)}{4}| / s \in C\}$
and $\nu = \inf\{|\mu^a_{F_{e_2}}(s) - \frac{\mu^a_{F_{e_1}}(s)}{4}| / s \in C\}$

$$\preceq \tilde{c}\left(\tilde{d_{c^*}}(F_{e_1}(a), \mathbf{T}F_{e_2}(a))(s) + \tilde{d_{c^*}}(\mathbf{T}F_{e_1}(a), F_{e_2}(a))(s)\right)$$

Hence $\tilde{d_{c^*}}(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2}) \preceq \tilde{c}\left(\tilde{d_{c^*}}(F_{e_1}, \mathbf{T}F_{e_2}) + \tilde{d_{c^*}}(\mathbf{T}F_{e_1}, F_{e_2})\right)$
However, $\mathbf{T}$ *has no fixed point.*

Now present an example where we can apply Theorem 3.16 but not Theorem 3.10.

Example 3.18. *Let* $U = R^+$ *and* $E = C = \{0, 1\}$, *define* $\tilde{d_{c^*}} : \tilde{E} \times \tilde{E} \to \tilde{C}$ *by* $\tilde{d_{c^*}}(F_{e_1}(a), F_{e_2}(a))(s) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ *where* $i = \inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)| / s \in C\}$ *and for all* $a \in U$ *and* $F_{e_1}, F_{e_2} \in \tilde{E}$ *is a* C^* *- algebra valued fuzzy soft metric on* $\tilde{E}$. *Since* R^+ *is complete for the Euclidean metric, we deduce that* $(\tilde{E}, \tilde{C}, \tilde{d_{c^*}})$ *is complete* C^* *- algebra valued fuzzy soft metric space.*

Let $\mathbf{T}: \tilde{E} \to \tilde{E}$ *such that* $\mathbf{T}F_e(a)(s) = \begin{cases} 0, & \text{if } a \in [0,2) \\ \frac{1}{2}, & \text{if } a \in [2,\infty) \end{cases}$

for all $F_e \in \tilde{E}$. *Let* $\tilde{c}$ *be a constant* C^* *-algebra valued fuzzy soft real number such that* $||\tilde{c}|| < 1$, *choose* $b \in [0,2)$*and* $\tilde{b} = \begin{bmatrix} \frac{3}{2} & 0 \\ 0 & \frac{3}{2} \end{bmatrix}$ *such that* $||\tilde{a}(2I - \tilde{b})|| < \frac{1}{2}$. *Then for each* $s \in C$,

we have

$$\tilde{d_{c^*}}(\mathbf{T}F_{e_1}(2), \mathbf{T}F_{e_1}(b))(s)$$
$$= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \succ \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} (2I - \tilde{b})$$

$$\succ \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} (2I - \tilde{b}) \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\succ \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} \frac{3}{2} & 0 \\ 0 & \frac{3}{2} \end{bmatrix} \right) \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\succ \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\succ \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \tilde{d_{c^*}}(F_{e_1}(2), F_{e_1}(b)) \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\succ \tilde{c}^* \tilde{d_{c^*}}(F_{e_1}(2), F_{e_1}(b)) \tilde{c}.$$

Therefore, $\mathbf{T}$ *does not satisfy contraction condition of Theorem 3.10 for any* $||\tilde{c}|| < 1$. *However, taking,* $a \in [0,2)$ *with* $\tilde{a} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ *and* $b \in [2, \infty)$, *with* $\tilde{b} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ *we obtain, for* $s \in C$,

$$\tilde{d_{c^*}}(\mathbf{T}F_{e_1}(a), \mathbf{T}F_{e_1}(b))(s)$$
$$= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \right)$$

$$\preceq \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \right)$$

$$\preceq \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right)$$

$$\preceq \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \left(\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right)$$

Hence,
$\tilde{d_{c^*}}(\mathbf{T}F_{e_1}(a), \mathbf{T}F_{e_1}(b))(s)$
$\preceq \tilde{c}\left(\tilde{d_{c^*}}(F_{e_1}(a), \mathbf{T}F_{e_1}(b))(s) + \tilde{d_{c^*}}(\mathbf{T}F_{e_1}(a), F_{e_1}(b))(s)\right)$
Therefore, $\mathbf{T}$ *satisfies contraction condition of Theorem (3.16) for* $||\tilde{c}|| = \frac{1}{3}$. *In fact,* $F_\phi = \tilde{0}$ *is a unique fixed point of* $\mathbf{T}$.

4. On Caristi type contraction

We begin this section by introducing the notion of lower semi continuity in the context of C^*-algebra valued fuzzy soft metric space. And we proved that many of the known fixed point theorems can be deduced from caristi's mapping.

Definition 4.1. *Let $(\tilde{E},\tilde{C},\tilde{d}_{c^*})$ be a C^*-algebra valued fuzzy soft metric space. Let $\tau : \tilde{E} \to \tilde{C}$ be a mapping, we say that τ is lower semi continuous at F_{e_0} with respect to $\tilde{C}$ if*

$$||\tau(F_{e_0})|| \leq \lim_{F_e \to F_{e_0}} \inf ||\tau(F_e)||$$

Example 4.2. *Let $E = \{e_1, e_2, e_3\}, U = \{a,b,c,d\}$ and C and D are two subset of E where $C = \{e_1,e_2,e_3\}, D = \{e_1,e_2,\}$. Define fuzzy soft set as,*

$$(F_E, C) = \left\{ \begin{array}{c} e_1 = \{a_{0.1}, b_{0.3}, c_{0.4}, d_{0.6}\}, \\ e_2 = \{a_{0.3}, b_{0.4}, c_{0.6}, d_{0.8}\}, \\ e_3 = \{a_{0.6}, b_{0.7}, c_{0.8}, d_{0.9}\} \end{array} \right\}$$

$$(G_E, D) \\ = \{e_1 = \{a_{0.4}, b_{0.5}, c_{0.2}, d_{0.5}\}, e_2 = \{a_{0.5}, b_{0.6}, c_{0.3}, d_{0.7}\}\}$$

$$F_{e_1} = \mu_{F_{e_1}} = \{a_{0.1}, b_{0.3}, c_{0.4}, d_{0.6}\}$$

$$F_{e_2} = \mu_{F_{e_2}} = \{a_{0.3}, b_{0.4}, c_{0.6}, d_{0.8}\}$$

$$F_{e_3} = \mu_{F_{e_3}} = \{a_{0.6}, b_{0.7}, c_{0.8}, d_{0.9}\}$$

$$G_{e_1} = \mu_{G_{e_1}} = \{a_{0.4}, b_{0.5}, c_{0.2}, d_{0.5}\}$$

$$G_{e_2} = \mu_{G_{e_2}} = \{a_{0.5}, b_{0.6}, c_{0.3}, d_{0.7}\}$$

and $FSC(F_E) = \{F_{e_1}, F_{e_2}, F_{e_3}, G_{e_1}, G_{e_2}\}$, let $\tilde{E}$ be absolute fuzzy soft set, that is $\tilde{E}(e) = \tilde{1}$ for all $e \in E$, and $\tilde{C} = M_2(R(C)^*)$, be the C^*-algebra with $||(F_{e_1}, F_{e_2})|| = \sqrt{|F_{e_1}|^2 + |F_{e_2}|^2}$. Define an order $\preceq$ on $\tilde{C}$ as follows ;

$$(F_{e_1}, G_{e_1}) \preceq (F_{e_2}, G_{e_2}) \Leftrightarrow F_{e_1} \leq F_{e_2} \text{ and } G_{e_1} \leq G_{e_2},$$

where $\leq$ is the usual order on the element of ***R***. *It is easy to verify that $\preceq$ is a partial order on $\tilde{C}_+$. Consider $\tilde{d}_{c^*} : \tilde{E} \times \tilde{E} \to \tilde{C}$ by $\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) = (Inf\{|F_{e_1}(a) - F_{e_2}(a)|/a \in C\}, 0)$, then obviously $(\tilde{E},\tilde{C},\tilde{d}_{c^*})$ be a C^*-algebra valued fuzzy soft metric space. Define a mapping*

$$\tau : \tilde{E} \to \tilde{C}, \tau(F_e) = \left\{ \begin{array}{ll} (\frac{F_e(a)}{2}, 0) & if\ F_e(a) \geq 0 \\ (1,0) & otherwise \end{array} \right.$$

then it is easy to verify that τ is lower semi continuous at $F_{e_0} = 0$.

It is straight forward to prove the following lemma.

Lemma 4.3. *$(\tilde{E},\tilde{C},\tilde{d}_{c^*})$ be a complete C^*-algebra valued fuzzy soft metric space and $\tau : \tilde{E} \to \tilde{C}_+$ be the lower semi continuous mapping. The order relation $\preceq_\tau$ on $\tilde{E}$, for each $F_{e_1}, F_{e_2} \in \tilde{E}$ defined by*

$$F_{e_1} \preceq_\tau F_{e_2} \Leftrightarrow \tilde{d}_{c^*}(F_{e_1}, F_{e_2}) \preceq \tau(F_{e_2}) - \tau(F_{e_1}) \tag{4.1}$$

then $\preceq_\tau$ is a partial order on $\tilde{E}$.

Theorem 4.4. *Let $(\tilde{E},\tilde{C},\tilde{d}_{c^*})$ be a complete C^*-algebra valued fuzzy soft metric space. Suppose that $\tau : \tilde{E} \to \tilde{C}_+$ be a lower semi continuous mapping and $\preceq_\tau$ on $\tilde{E}$ defined by (4.1). Then $(\tilde{E}, \preceq_\tau)$ has a minimal element.*

Proof. Let us consider the non-increasing sequence in $\tilde{E}$ as follows
$F_{e_1} \succeq_\tau F_{e_2} \succeq_\tau F_{e_3} \succeq_\tau$ ...., then from (4.1), we have
$\tilde{0}_{\tilde{C}} \preceq \tilde{d}_{c^*}(F_{e_2}, F_{e_1}) \preceq \tau(F_{e_1}) - \tau(F_{e_2})$,
$\tilde{0}_{\tilde{C}} \preceq \tilde{d}_{c^*}(F_{e_3}, F_{e_2}) \preceq \tau(F_{e_2}) - \tau(F_{e_3})$,
which implies $\tau(F_{e_1}) \succeq \tau(F_{e_2}) \succeq \tau(F_{e_3})$
Therefore, $\{\tau(F_{e_\beta}) : \beta \in \Delta\}$ is a decreasing sequence in $\tilde{C}_+$, where Δ is an indexing set. Let $\{\beta_n\}$ be an increasing sequence of elements from the indexing set Δ such that

$$\lim_{n\to\infty} \tau(F_{e_\beta}) = \inf\{\tau(F_{e_\beta}) : \beta \in \Delta\} \tag{4.2}$$

take $m > n$ then $F_{e_{\beta_n}} \succeq_\tau F_{e_{\beta_m}}$. It follows from (4.1),

$$\tilde{d}_{c^*}(F_{e_{\beta_m}}, F_{e_{\beta_n}}) \preceq \tau(F_{e_{\beta_n}}) - \tau(F_{e_{\beta_m}}) \\ \tilde{|}|\tilde{d}_{c^*}(F_{e_{\beta_m}}, F_{e_{\beta_n}})|| \preceq ||\tau(F_{e_{\beta_n}}) - \tau(F_{e_{\beta_m}})||$$

Letting $n \to \infty$ then together with (4.2), it further, implies that

$$\lim_{n\to\infty} \tilde{|}|\tilde{d}_{c^*}(F_{e_{\beta_m}}, F_{e_{\beta_n}})|| \\ \leq \lim_{n\to\infty} ||\tau(F_{e_{\beta_n}}) - \tau(F_{e_{\beta_m}})|| \\ \leq ||\inf\{\tau(F_{e_\beta}) : \beta \in \Delta\} - \inf\{\tau(F_{e_\beta}) : \beta \in \Delta\}|| = 0$$

therefore, $\{F_{e_{\beta_n}}\}$ is a Cauchy sequence in $\tilde{E}$ by Definition 3.3.
As $\tilde{E}$ is complete, there exist $F_{e^{\shortmid}} \in \tilde{E}$ such that $F_{e_{\beta_n}} \to F_{e^{\shortmid}}$. Since $\{F_{e_{\beta_n}}\}$ is decreasing sequence in $\tilde{E}$, it follows $F_{e^{\shortmid}} \preceq_\tau F_{e_{\beta_n}}$ for all $n \geq 1$. Which implies $F_{e^{\shortmid}}$ is lower bound for $\{F_{e_{\beta_n}}\}_{n\geq 1}$.
We prove that $F_{e^{\shortmid}}$ is a lower bound for $\{F_{e_\beta}\}_{\beta\in\Delta}$.
Let $\gamma \in \Delta$ be such that $F_{e_\gamma} \preceq_\tau F_{e_{\beta_n}}$ for all $n \geq 1$ then

$$\tilde{0}_{\tilde{C}} \preceq \tilde{d}_{c^*}(F_{e_{\beta_n}}, F_{e_\gamma}) \preceq \tau(F_{e_{\beta_n}}) - \tau(F_{e_\gamma})$$

taking limit $n \to \infty$ which implies

$$\tau(F_{e_\gamma}) \preceq \inf\{\tau(F_{e_\beta}) : \beta \in \Delta\} \tag{4.3}$$

since $\gamma \in \Delta$, we have

$$\inf\{\tau(F_{e_\beta}) : \beta \in \Delta\} \preceq \tau(F_{e_\gamma}) \tag{4.4}$$

Combining (4.3) and (4.4) we get

$$`\tau(F_{e_\gamma}) = \inf\{\tau(F_{e_\beta}) : \beta \in \Delta\} \tag{4.5}$$

As $F_{e_\gamma} \preceq_\tau F_{e_{\beta_n}}$, it follows from (4.1),
$\tilde{d}_{c^*}(F_{e_\gamma}, F_{e_{\beta_n}},) \preceq \tau(F_{e_{\beta_n}}) - \tau(F_{e_\gamma})$, using (4.5) and the fact that $\{F_{e_{\beta_n}}\}$ is decreasing chain in $\tilde{E}$, we obtain

$$\begin{aligned} \tilde{0}_{\tilde{C}} &\preceq \tilde{d}_{c^*}(\lim_{n\to\infty} F_{e_{\beta_n}}, F_{e_\gamma}) \\ &\preceq \lim_{n\to\infty} \tau(F_{e_{\beta_n}}) - \tau(F_{e_\gamma}) \\ &= \tau(F_{e_\gamma}) - \tau(F_{e_\gamma}) = \tilde{0}_{\tilde{C}}. \end{aligned}$$

Therefore, $\tilde{d}_{c^*}(\lim_{n\to\infty} F_{e_{\beta_n}}, F_{e_\gamma}) = \tilde{0}_{\tilde{C}}$.
Hence $\lim_{n\to\infty} F_{e_{\beta_n}} = F_{e_\gamma}$. It follows from the uniqueness of limit $F_{e^{\shortmid}} = F_{e_\gamma}$. That is, $F_{e^{\shortmid}}$ is a lower bound of $\{F_{e_\beta} : \beta \in \Delta\}$. Thus by using Zorn's lemma we conclude that $\tilde{E}$ has a minimal element. □

As a consequence of above theorem we have the following fixed point results.

Theorem 4.5. *Let $(\tilde{E}, \tilde{C}, \tilde{d}_{c^*})$ be a complete C^*-algebra valued fuzzy soft metric space and Suppose $\tau : \tilde{E} \to \tilde{C}_+$ be a lower semi continuous mapping. Let the self mapping $\mathbf{T}\colon \tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1} \in \tilde{E}$,*

$$\tilde{d}_{c^*}(F_{e_1}, \mathbf{T}F_{e_1}) \preceq \tau(F_{e_1}) - \tau(\mathbf{T}F_{e_1}) \tag{4.6}$$

Then $\mathbf{T}$ has at least one fixed point in $\tilde{E}$.

Proof. Let $F_{e^{\shortmid}} \in \tilde{E}$ be a minimal element of $\tilde{E}$. Since $\mathbf{T}F_{e^{\shortmid}} \in \tilde{E}$, it follows $F_{e^{\shortmid}} \preceq_\tau F_{e_1}$ for all $F_{e_1} \in \tilde{E}$
In particular,

$$F_{e^{\shortmid}} \preceq_\tau \mathbf{T}F_{e^{\shortmid}} \tag{4.7}$$

By combining (4.7) and the condition (4.6) we have $\mathbf{T}F_{e^{\shortmid}} = F_{e^{\shortmid}}$, that is $\mathbf{T}$ has a fixed point. □

Example 4.6. *Let $E = C = [0,1]$ and let $\tilde{E}$ be absolute fuzzy soft set, that is $\tilde{E}(e) = \tilde{1}$ for all $e \in E$, and $\tilde{C} = M_2(R(C)^*)$ be a C^*-algebra with the partial order as given in Example 4.2. Define $\tilde{d}_{c^*} : \tilde{E}\times\tilde{E} \to \tilde{C}$ by $\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ where $i = \inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)| : s \in C\}$ and $F_{e_1}, F_{e_2} \in \tilde{E}$. And $\tau : \tilde{E} \to \tilde{C}_+$, $\tau(F_{e_1}) = \begin{bmatrix} F_{e_1} & 0 \\ 0 & F_{e_1} \end{bmatrix}$ be a continuous mapping, and $\mathbf{T}\colon \tilde{E} \to \tilde{E}$ is given by the $\mathbf{T}F_{e_1} = F^2_{e_1}$. Then it is easy to see that all the conditions of Theorem 4.5 are satisfied and $\mathbf{T}$ has a fixed point.*

Corollary 4.7. *Let $(\tilde{E}, \tilde{C}, \tilde{d}_{c^*})$ be a C^*- algebra valued fuzzy soft metric space. Suppose the mapping $\mathbf{T}\colon \tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1}, F_{e_2} \in \tilde{E}$,*

$$\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) \preceq \tau(F_{e_1}, F_{e_2}) - \tau(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2}) \tag{4.8}$$

where $\tau : \tilde{E}\times\tilde{E} \to \tilde{C}_+$ is lower semi continuous with respect to the first variable. Then $\mathbf{T}$ has a unique fixed point in $\tilde{E}$.

Proof. For each $F_{e_1} \in \tilde{E}$, let we define $F_{e_2} = \mathbf{T}F_{e_1}$ and $\tau(F_{e_1}) = \tau(F_{e_1}, \mathbf{T}F_{e_1})$, then for each $F_{e_1} \in \tilde{E}$, we have
$\tilde{d}_{c^*}(F_{e_1}, \mathbf{T}F_{e_1}) \preceq \tau(F_{e_1}) - \tau(\mathbf{T}F_{e_1})$,
since τ is lower semi continuous mapping. Thus, we can applying Theorem 4.5 lead us to conclude the appropriate result. To see the uniqueness of the fixed point, suppose $F_{e^{\shortmid}}, F_{e^{\shortmid\shortmid}}$ are two distinct fixed points of $\mathbf{T}$. Then

$$\begin{aligned} \tilde{0}_{\tilde{C}} \preceq \tilde{d}_{c^*}(F_{e^{\shortmid}}, F_{e^{\shortmid\shortmid}}) &\preceq \tau(F_{e^{\shortmid}}, F_{e^{\shortmid\shortmid}}) - \tau(\mathbf{T}F_{e^{\shortmid}}, \mathbf{T}F_{e^{\shortmid\shortmid}}) \\ &= \tau(F_{e^{\shortmid}}, F_{e^{\shortmid\shortmid}}) - \tau(F_{e^{\shortmid}}, F_{e^{\shortmid\shortmid}}) = \tilde{0}_{\tilde{C}} \end{aligned}$$

Therefore, we have $F_{e^{\shortmid}} = F_{e^{\shortmid\shortmid}}$. □

Corollary 4.8. *Let $(\tilde{E}, \tilde{C}, \tilde{d}_{c^*})$ be a C^*-algebra valued fuzzy soft metric space. Suppose the mapping $\mathbf{T}\colon \tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1}, F_{e_2} \in \tilde{E}$,*

$$\tilde{d}_{c^*}(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2}) \preceq \tilde{a}^{\star}\tilde{d}_{c^*}(F_{e_1}, F_{e_2})\tilde{a}$$

where $\tilde{a} \in \tilde{C}$ with $||\tilde{a}|| < 1$. Then $\mathbf{T}$ has a unique fixed point in $\tilde{E}$.

Proof. Define $\tau(F_{e_1}, F_{e_2}) = (I - \tilde{a}\tilde{a}^{\star})^{-1}\tilde{d}_{c^*}(F_{e_1}, F_{e_2})$ then (2.1) show that

$$(I - \tilde{a}\tilde{a}^*)\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) \preceq \tilde{d}_{c^*}(F_{e_1}, F_{e_2}) - \tilde{d}_{c^*}(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2})$$

which means

$$\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) \preceq (I - \tilde{a}\tilde{a}^*)^{-1}\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) - (I - \tilde{a}\tilde{a}^*)^{-1}\tilde{d}_{c^*}(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2})$$

and so we have

$$\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) \preceq \tau(F_{e_1}, F_{e_2}) - \tau(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2})$$

Therefore, by applying corollary 1, one can conclude that $\mathbf{T}$ has a unique fixed point in $\tilde{E}$. □

Corollary 4.9. *Let $(\tilde{E}, \tilde{C}, \tilde{d}_{c^*})$ be a C^*-algebra valued fuzzy soft metric space. Suppose the mapping $\mathbf{T}\colon \tilde{E} \to \tilde{E}$ satisfies for all $F_{e_1}, F_{e_2}, F_{e_3} \in \tilde{E}$,*

$$\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) \preceq \tau(F_{e_1}, F_{e_2}) - \tau(F_{e_2}, F_{e_3}) \tag{4.9}$$

where $\tau : \tilde{E}\times\tilde{E} \to \tilde{C}_+$ is lower semi continuous with respect to the first variable. Then $\mathbf{T}$ has a fixed point in $\tilde{E}$.

Proof. For each $F_{e_1} \in \tilde{E}$, let we define $F_{e_2} = \mathbf{T}F_{e_1}$, $F_{e_3} = \mathbf{T}F_{e_2} = \mathbf{T}^2F_{e_1}$ and $\tau(F_{e_1}) = \tau(F_{e_1}, \mathbf{T}F_{e_1})$, then for each $F_{e_1} \in \tilde{E}$, we have
$\tilde{d}_{c^*}(F_{e_1}, \mathbf{T}F_{e_1}) \preceq \tau(F_{e_1}) - \tau(\mathbf{T}F_{e_1})$,
since τ is lower semi continuous mapping. Thus, we can applying Theorem 4.5 lead us to conclude the appropriate result. □

5. Applications to integral equations

As applications of contractive mapping theorem on complete C^* - algebra valued fuzzy soft metric spaces, existence and uniqueness theorems for a type of integral equation and operator equation are given.
Following example shows that a C^*-algebra valued fuzzy soft metric space.

Example 5.1. *Let $E = C = [0,1]$, and the absolute fuzzy soft set $\tilde{E} = L^\infty(C)$ and $H = L^2(C)$, where the parameter set C is a Lebesgue measurable set. By $L(H)$ we denote the set of bounded linear operators on Hilbert space H. Clearly $L(H)$ is a C^*-algebra with usual operator norm. Define $\tilde{d}_{c^*} : \tilde{E}\times\tilde{E} \to L(H)$ by $\tilde{d}_{c^*}(F_{e_1}, F_{e_2}) = M_{\inf\{|\mu^a_{F_{e_1}}(s) - \mu^a_{F_{e_2}}(s)|/s\in C\}}$ for all $F_{e_1}, F_{e_2} \in \tilde{E}$, where $M_h : H \to H$ is the multiplication operator defined by $M_h(\tau) = h.\tau$ for $\tau \in H$ Then $\tilde{d}_{c^*}$ is a*

C^* - algebra valued fuzzy soft metric and $(\tilde{E}, L(H), \tilde{d_{c^*}})$ is a complete C^* - algebra valued fuzzy soft metric space.
Indeed, it suffices to verify the completeness. Let $\{F_{e_n}\} \in \tilde{E}$ be a Cauchy sequence with respect to $L(H)$. Then for a given $\varepsilon > 0$, there is a natural numbers $N(\varepsilon)$ such that for all $n, m \geq N(\varepsilon)$,

$$\tilde{d_{c^*}}(F_{e_n}, F_{e_m}) = M_{\inf\{|\mu^a_{F_{e_n}}(s) - \mu^a_{F_{e_m}}(s)|/s \in C\}}$$

$$\begin{aligned} ||\tilde{d_{c^*}}(F_{e_n}, F_{e_m})|| &= ||M_{\inf\{|\mu^a_{F_{e_n}}(s) - \mu^a_{F_{e_m}}(s)|/s \in C\}}|| \\ &= ||\inf\{|\mu^a_{F_{e_n}}(s) - \mu^a_{F_{e_m}}(s)|/s \in C\}||_\infty < \varepsilon \end{aligned}$$

then $\{F_{e_n}\}$ is a Cauchy sequence in the space $\tilde{E}$. Thus, there is a $F_{e^l} \in \tilde{E}$ and natural number $N_1(\varepsilon)$ such that $||\inf\{|\mu^a_{F_{e_n}}(s) - \mu^a_{F_{e^l}}(s)|/s \in C\}||_\infty < \varepsilon$ if $n > N_1$. Its follows,

$$\begin{aligned} ||\tilde{d_{c^*}}(F_{e_n}, F_{e^l})|| &= ||M_{\inf\{|\mu^a_{F_{e_n}}(s) - \mu^a_{F_{e^l}}(s)|/s \in C\}}|| \\ &= ||\inf\{|\mu^a_{F_{e_n}}(s) - \mu^a_{F_{e^l}}(s)|/s \in C\}||_\infty < \varepsilon \end{aligned}$$

Therefore, the sequence $\{F_{e_n}\}$ converges to the function $F_{e^l} \in \tilde{E}$ with respect to $L(H)$, that is, $(\tilde{E}, L(H), \tilde{d_{c^*}})$ is complete with respect to $L(H)$.

Theorem 5.2. *Consider the integral equation*

$$F_e(t) = \int_C G(t, s, F_e(s))ds + f(t), t \in E$$

Where C is a Lebesgue measurable set. Suppose

(*i*) *$G : C \times C \times R(C)^* \to R(C)^*$ and $f \in L^\infty(C)$.*

(*ii*) *there exist a continuous function $\tau : C \times C \to R(C)^*_+$ and $r \in (0,1)$ such that*

$$\inf\{|G(t,s,u) - G(t,s,v)|\} \leq r \inf\{|\tau(t,s)|\}.\inf\{|(u-v)|\},$$

for $t, s \in C$ and $u, v \in R(C)^*$

(*iii*) $\sup_{t \in C} \int_C \inf\{|\tau(t,s)|\}ds \leq 1$

then the integral equation has a unique solutions $F_{e^l} \in L^\infty(C)$.

Proof. Let $\tilde{E} = L^\infty(C)$ and $H = L^2(C)$. Set $\tilde{d_{c^*}}$ as a above example, then $\tilde{d_{c^*}}$ is a C^* - algebra valued fuzzy soft metric and $(\tilde{E}, L(H), \tilde{d_{c^*}})$ is a complete C^* - algebra valued fuzzy soft metric space with respect to $L(H)$.
Let $\mathbf{T} : L^\infty(C) \to L^\infty(C)$ be such that

$$\mathbf{T}F_e(t) = \int_C G(t, s, F_e(s))ds + f(t), t \in E$$

Let $\tilde{C} = r\tilde{I_C}$ then $\tilde{C} \in L(H)_+$ and $||\tilde{C}|| = r < 1$. For any $h \in H$,

$$\tilde{d_{c^*}}(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2}) = \mathrm{M}_{\inf\{|\mu^a_{\mathbf{T}F_{e_1}}(s) - \mu^a_{\mathbf{T}F_{e_2}}(s)|/s \in C\}}$$

$$||\tilde{d_{c^*}}(\mathbf{T}F_{e_1}, \mathbf{T}F_{e_2})|| = \sup_{||h||=1} (\mathrm{M}_{\inf\{|\mu^a_{\mathbf{T}F_{e_1}}(s) - \mu^a_{\mathbf{T}F_{e_2}}(s)|/s \in C\}} h, h)$$

$$= \sup_{||h||=1} \int_C \left[\inf\{| \int_C (G(t,s,F_{e_1}(s)) - G(t,s,F_{e_2}(s)))|\}ds \right] h(t)\overline{h(t)}dt$$

$$\leq \sup_{||h||=1} \int_C \left[\int_C \inf\{|G(t,s,F_{e_1}(s)) - G(t,s,F_{e_2}(s))|\}ds \right] |h(t)|^2 dt$$

$$\leq \sup_{||h||=1} \int_C \left[\int_C r \inf\{|\tau(t,s)(F_{e_1}(s) - F_{e_2}(s))|\}ds \right] |h(t)|^2 dt$$

$$\leq r \sup_{||h||=1} \int_C \left[\int_C \inf\{|\tau(t,s)|\} \inf\{|F_{e_1}(s) - F_{e_2}(s)|\}ds \right] |h(t)|^2 dt$$

$$\leq r \sup_{||h||=1} \int_C \left[\int_C \inf\{|\tau(t,s)|\}ds \right] |h(t)|^2 dt.||\inf\{|F_{e_1}(s) - F_{e_2}(s)|\}||_\infty$$

$$\leq r \sup_{||h||=1} \int_C \inf\{|\tau(t,s)|\}ds. \sup_{||h||=1} \int_C |h(t)|^2 dt.||\inf\{|F_{e_1}(s) - F_{e_2}(s)|\}||_\infty$$

$$\leq r.||\inf\{|F_{e_1}(s) - F_{e_2}(s)|\}||_\infty$$

$$\begin{aligned} &\leq r||\tilde{d_{c^*}}(F_{e_1}, F_{e_2})|| \\ &\leq ||\tilde{a}||||\tilde{d_{c^*}}(F_{e_1}, F_{e_2})|| \end{aligned}$$

Since $||\tilde{a}|| < 1$, the integral equation has a unique solution $F_{e^l} \in L^\infty(C)$. □

6. Conclusion

In this paper we introduce C^* - algebra valued fuzzy soft metric space, under the restriction that a set of parameters is finite. We also studied some essential properties of the induced metric thus obtained. In this paper we conclude some fixed point results in C^* -algebra valued fuzzy soft metric spaces and suitable examples that supports the main results. Also, applications to integral equations are provided.

7. Declaration

- **Competing Interesting:** The authors declare that they have no competing interest.
- **Funding:** No funding
- **Author contributions :** All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.
- **Acknowledgement:** The authors are very thanks to the reviewers and editors for valuable comments, remarks and suggestions which improved the paper in good form.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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