Malaya Journal of Matematik, Vol. 6, No. 2, 326-330, 2018
https://doi.org/10.26637/MJM0602/0004

A study on I-Cauchy sequences and I-divergence in S-metric spaces

Amar Kumar Banerjee[1]* and Apurba Banerjee[2]

Abstract
The notion of S-metric space was introduced by Sedghi et al. In this paper we study the ideas of I and I^*-Cauchy sequences in S-metric spaces and investigate their relation following the same approach as done by Das and Ghosal. We then study the ideas of I and I^*-divergent sequences in S-metric spaces and examine their relation under certain general assumption.

Keywords
Ideal, S-metric space, I-Cauchy, I^*-Cauchy, I-divergence, I^*-divergence, condition (AP).

AMS Subject Classification (2010)
Primary 54A20; Secondary 40A35, 54E15.

[1,2] *Department of Mathematics, The University of Burdwan, Burdwan-713104, West Bengal, India.*
***Corresponding author**: [1] akbanerjee1971@gmail.com, akbanerjee@math.buruniv.ac.in; [2]apurbamath12@gmail.com
Article History: Received **24** November **2017**; Accepted **21** February **2018**

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1. Introduction and background

The idea of statistical convergence of a sequence of real numbers was introduced by Fast ([11]) and Stienhaus ([24]). Lot of investigations have been done so far on such convergence and its topological consequences after the initial works by Šalát ([21]) (see [2], [19] where many more references can be found). The ideas of I and I^*-convergence which are interesting generalizations of statistical convergence were introduced by Kostyrko et al. ([13]), using the notion of ideals of the set $\mathbb{N}$ of positive integers. Later many works on I and I^*-convergence of sequences and also on double sequences have been done (see [17], [3], [4]). The idea of I-Cauchy condition was studied by Dems ([10]). The idea of I^*-Cauchy sequences in a linear metric space have been introduced by Nabiev et al. ([20]) where they showed that I^*-Cauchy sequences are I-Cauchy and they are equivalent if the ideal I satisfies the condition (AP). Later Das and Ghosal ([6]) studied further in this direction and they showed that under some general assumption, the condition (AP) is both necessary and sufficient for the equivalence of I and I^*-Cauchy conditions and cited an example in support of the fact that in general I-Cauchy sequences may not be I^*-Cauchy. They also introduced the notions of I-divergence and I^*-divergence of sequences in a metric space and discussed on certain basic properties. They also showed that condition (AP) is the necessary and sufficient condition for the equivalence of I and I^*-divergence under certain conditions. In 2014, P. Das, M. Sleziak, V. Toma ([8]) studied on I^K-Cauchy condition of functions defined on a nonempty set with values in a uniform space as a generalization of I^*-Cauchy sequences and I^*-Cauchy nets. They showed how this notion can be used to characterize complete uniform spaces. Also they showed the relationship between the condition AP(I,K) and the equivalence of I-Cauchy and I^K-Cauchy functions with values in a metric space. They also studied I^K-divergence of functions with values in a metric space.
Recently Sedghi et al. ([23]) have introduced the concept of S-metric spaces and proved some basic properties in S-metric spaces. In this paper we have studied the idea of I and I^*-convergence in S-metric spaces. In Section 2 we have studied the concepts of I and I^*-Cauchy conditions of sequences in S-metric spaces and find their relation following the same direction as in [6]. In section 3 we get acquainted with the ideas of I-divergence and I^*-divergence of sequences in S-metric spaces and investigate their relation under certain general

assumption.

Definition 1.1. *([15]) If X is a non-void set then a family of sets $I \subset 2^X$ is called an ideal if*
(i) $A, B \in I$ implies $A \cup B \in I$ and
(ii) $A \in I, B \subset A$ imply $B \in I$.

The ideal I is called *nontrivial* if $I \neq \{\emptyset\}$ and $X \notin I$. A nontrivial ideal I is said to be *admissible* if $\{x\} \in I$ for each $x \in X$.

Definition 1.2. *([15]) A non-empty family F of subsets of a non-void set X is called a filter if*
(i) $\emptyset \notin F$
(ii) $A, B \in F$ implies $A \cap B \in F$ and
(iii) $A \in F, A \subset B$ imply $B \in F$.

Lemma 1.3. *Let I be a nontrivial ideal of $X \neq \emptyset$. Then the family of sets $F(I) = \{A \subset X : X - A \in I\}$ is a filter on X. It is called the filter associated with the ideal.*

Definition 1.4. *([23]) Let X be a nonempty set. An S-metric on X is a function $S : X \times X \times X \to [0, \infty)$ that satisfies the following conditions*
(i) $S(x, y, z) \geq 0$ for all $x, y, z \in X$,
(ii) $S(x, y, z) = 0$ if and only if $x = y = z$,
(iii) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$ for all $x, y, z, a \in X$.

The pair (X, S) is called an *S-metric space*. Some familiar examples of S-metric spaces may be seen from [23].
In an S-metric space (X, S), $S(x, x, y) = S(y, y, x)$ holds for all $x, y \in X$.

2. I-convergence, I^*-convergence, I-Cauchy and I^*-Cauchy conditions

Throughout we assume that $I \subset 2^{\mathbb{N}}$ is a nontrivial ideal of the set of all positive integers $\mathbb{N}$ and (X, S) is an S-metric space unless otherwise stated. Below we introduce the following definitions in an S-metric space.

Definition 2.1. *(cf. [13]) A sequence $\{x_n\}$ of elements of X is said to be I-convergent to $x \in X$ if for each $\varepsilon > 0$, the set $A(\varepsilon) = \{n \in \mathbb{N} : S(x_n, x_n, x) \geq \varepsilon\} \in I$.*

Definition 2.2. *([17]) An admissible ideal I is said to satisfy the condition (AP) if for every countable family $\{A_1, A_2, A_3, ...\}$ of sets belonging to I there exists a countable family of sets $\{B_1, B_2, B_3, ...\}$ such that $A_j \Delta B j$ is a finite set for each $j \in \mathbb{N}$ and $\bigcup_{j=1}^{\infty} B_j \in I$.*

Note that $B_j \in I$ for all $j \in \mathbb{N}$.

Definition 2.3. *(cf. [13]) A sequence $\{x_n\}$ of elements of X is said to be I^*-convergent to $x \in X$ if and only if there exists a set $M \in F(I)$ (i.e., $\mathbb{N} \setminus M \in I$), $M = \{m_1 < m_2 < \cdots < m_k < \cdots\} \subset \mathbb{N}$ such that $lim_{k \to \infty} S(x_{m_k}, x_{m_k}, x) = 0$.*

It can be proved easily that I and I^*-convergence are equivalent for admissible ideals with property (AP).

Definition 2.4. *(cf. [20]) Let $I \subset 2^{\mathbb{N}}$ be an admissible ideal. A sequence $\{x_n\}$ of elements of X is called an I-Cauchy sequence in (X, S) if for every $\varepsilon > 0$ there exists a positive integer $n_0 = n_0(\varepsilon)$ such that the set*

$$A(\varepsilon) = \{n \in \mathbb{N} : S(x_n, x_n, x_{n_0}) \geq \varepsilon\} \in I$$

.

It can be shown that $\{x_n\}$ is I-Cauchy if for any given $\varepsilon > 0$, there exists $B = B(\varepsilon) \in I$ such that $m, n \notin B$ implies $S(x_m, x_m, x_n) < \varepsilon$.

Definition 2.5. *(cf. [20]) Let $I \subset 2^{\mathbb{N}}$ be an admissible ideal. A sequence $\{x_n\}$ of elements of X is called an I^*-Cauchy sequence in (X, S) if there exists a set $M = \{m_1 < m_2 < \cdots < m_k < \cdots\} \subset \mathbb{N}$, $M \in F(I)$ such that the subsequence $\{x_{m_k}\}$ is an ordinary Cauchy sequence in (X, S) i.e., for each preassigned $\varepsilon > 0$ there exists $k_0 \in \mathbb{N}$ such that $S(x_{m_k}, x_{m_k}, x_{m_r}) < \varepsilon$ for all $k, r \geq k_0$.*

Theorem 2.6. *Let I be an admissible ideal on $\mathbb{N}$. If $\{x_n\}$ is an I^*-Cauchy sequence in (X, S) then $\{x_n\}$ is I-Cauchy.*

Proof. Let $\{x_n\}$ be an I^*-Cauchy sequence in (X, S). Then by definition there exists a set $M = \{m_1 < m_2 < \cdots < m_k < \cdots\} \subset \mathbb{N}$, $M \in F(I)$ such that for every $\varepsilon > 0$ there exists a positive integer $k_0 = k_0(\varepsilon)$ such that $S(x_{m_k}, x_{m_k}, x_{m_r}) < \varepsilon$ for all $k, r > k_0 = k_0(\varepsilon)$. Let us take $n_0 = n_0(\varepsilon) = m_{k_0+1}$. Then for every $\varepsilon > 0$, we have $S(x_{m_k}, x_{m_k}, x_{n_0}) < \varepsilon$, for all $k > k_0$. Now let $H = \mathbb{N} \setminus M$. It is clear that $H \in I$ and

$$A(\varepsilon) = \{n \in \mathbb{N} : S(x_n, x_n, x_{n_0}) \geq \varepsilon\} \subset H \cup \{m_1, m_2, ..., m_{k_0}\} \in I$$

Hence we get that $A(\varepsilon) \in I$. Therefore, for every $\varepsilon > 0$ we can find a positive integer $n_0 = n_0(\varepsilon)$ such that $A(\varepsilon) \in I$ i.e., $\{x_n\}$ is I-Cauchy. □

In general I-Cauchy condition does not imply I^*-Cauchy condition. The following example is given in this direction.

Example 2.7. *Let $\mathbb{R}$ be the real number space with the usual metric d. Let $(\mathbb{R}, S)$ be an S-metric space where the S-metric is defined by $S(x, y, z) = d(x, z) + d(y, z)$ for all $x, y, z \in \mathbb{R}$. Let $\mathbb{N} = \bigcup_{j \in \mathbb{N}} \Delta_j$ be a decomposition of $\mathbb{N}$ such that each Δ_j is infinite and $\Delta_i \cap \Delta_j = \emptyset$ for $i \neq j$. Let I be the class of all those subsets A of $\mathbb{N}$ that can intersects only finite number of Δ_i's. Then I becomes an admissible ideal of $\mathbb{N}$.*
Now $\{\frac{1}{n}\}_{n \in \mathbb{N}}$ is a Cauchy sequence in $(\mathbb{R}, d)$. Let us define a sequence $\{x_n\}$ in $(\mathbb{R}, S)$ by $x_n = \frac{1}{j}$ if $n \in \Delta_j$. Let $\varepsilon > 0$ be given. Then $\{\frac{1}{n}\}_{n \in \mathbb{N}}$ being a Cauchy sequence there is $k \in \mathbb{N}$ such that $d(\frac{1}{m}, \frac{1}{n}) < \frac{\varepsilon}{4}$ whenever $m, n \geq k$. Now the set $B = \Delta_1 \cup \Delta_2 \cup ...... \cup \Delta_k \in I$ and clearly we see that $m, n \notin B$ implies $S(x_m, x_m, x_n) < \varepsilon$. Hence $\{x_n\}$ is I-Cauchy in $(\mathbb{R}, S)$. Next we shall show that $\{x_n\}$ is not I^*-Cauchy in $(\mathbb{R}, S)$. If possible assume that $\{x_n\}$ is I^*-Cauchy sequence in $(\mathbb{R}, S)$. Then there is

a set $M \in F(I)$ such that the subsequence $\{x_m\}_{m\in M}$ is Cauchy in $(\mathbb{R}, S)$. Since $\mathbb{N}\setminus M \in I$ so there exists a $p \in \mathbb{N}$ such that $\mathbb{N}\setminus M \subset \Delta_1 \cup \Delta_2 \cup ...... \cup \Delta_p$. But then it follows that $\Delta_i \subset M$ for all $i > p$. In particular, $\Delta_{p+1}, \Delta_{p+2} \subset M$. Let us choose a positive real $\varepsilon_0 = \frac{1}{4(p+1)(p+2)} > 0$. Now since $\{x_m\}_{m\in M}$ is Cauchy in $(\mathbb{R}, S)$ then for chosen ε_0 there exists $k \in \mathbb{N}$ such that $S(x_p, x_p, x_q) < \varepsilon_0$ for all $p, q \geq k$. From the construction of Δ_j's it clearly follows that given any $k \in \mathbb{N}$ there are $m \in \Delta_{p+1}$ and $n \in \Delta_{p+2}$ such that $m, n \geq k$. Then as defined earlier we have $x_m = \frac{1}{p+1}, x_n = \frac{1}{p+2}$ and $S(x_m, x_m, x_n) = 2d(x_m, x_n) = 2|\frac{1}{p+1} - \frac{1}{p+2}| = \frac{2}{(p+1)(p+2)} > \varepsilon_0$. Hence there is no $k \in \mathbb{N}$ for which the inequality $S(x_m, x_m, x_n) < \varepsilon_0$ holds whenever $m, n \in M$ with $m, n \geq k$. This contradicts the fact that $\{x_m\}_{m\in M}$ is Cauchy.

The definition of P-ideal is widely known as follows.

Definition 2.8. *An admissible ideal $I \subset 2^{\mathbb{N}}$ is called a P-ideal if for every sequence $\{A_n\}_{n\in\mathbb{N}}$ of sets in I there is a set $A_0 \in I$ with $A_n \setminus A_0$ finite for every $n \in \mathbb{N}$.*

If I is an admissible ideal satisfying the condition (AP) then I is a P-ideal and the converse is also true.
In consequence of this it can be shown that if I is an admissible ideal satisfying the condition (AP) then for every countable family $\{P_n\}_{n\in\mathbb{N}}$ of sets in $F(I)$ there exists a set $P \in F(I)$ such that $P \setminus P_n$ is finite for all $n \in \mathbb{N}$.

Theorem 2.9. *Let I be an admissible ideal satisfying the condition (AP). Then if $\{x_n\}$ is an I-Cauchy sequence in (X, S) it is I^*-Cauchy also.*

Proof. Let $\{x_n\}$ be an I-Cauchy sequence in (X, S). Then by definition, for every given $\varepsilon > 0$ there exists $n_0 = n_0(\varepsilon)$ such that $A(\varepsilon) = \{n \in \mathbb{N} : S(x_n, x_n, x_{n_0}) \geq \varepsilon\} \in I$. Let $P_k = \{n \in \mathbb{N} : S(x_n, x_n, x_{m_k}) < \frac{1}{k}\}$ for $k = 1, 2, 3, ...$, where $m_k = n_0(\frac{1}{k})$. It is clear that $P_k \in F(I)$ for every $k \in \mathbb{N}$. Since I satisfies the condition(AP) so there exists a set $P \in F(I)$ such that $P \setminus P_k$ is finite for all $k \in \mathbb{N}$. Now we show that $\{x_m\}_{m\in P}$ is I^*-Cauchy.

So, let $\varepsilon > 0$ and $j \in \mathbb{N}$ be such that $j > \frac{3}{\varepsilon}$. Since $P \setminus P_j$ is a finite set, so there exists $k = k(j)$ such that whenever $m, n \in P$ and $m, n > k_j$ we have $m, n \in P_j$. Hence it follows that

$$S(x_m, x_m, x_n) \leq 2S(x_m, x_m, x_{m_j}) + S(x_n, x_n, x_{m_j}) < \varepsilon$$

for $m, n > k(j)$. Thus for any $\varepsilon > 0$ there exists $k = k(\varepsilon) \in \mathbb{N}$ such that for $m, n > k(\varepsilon)$ and $m, n \in P \in F(I)$, $S(x_m, x_m, x_n) < \varepsilon$. This shows that the sequence $\{x_n\}$ in (X, S) is an I^*-Cauchy sequence. □

Theorem 2.10. *Let (X, S) be an S-metric space containing at least one accumulation point. If for every sequence $\{x_n\}$ I-Cauchy condition implies I^*-Cauchy condition then I satisfies the condition (AP).*

The proof of the above theorem follows the same approach as in [6].

3. I-divergence and I^*-divergence

The concept of divergent sequence of real numbers was generalized to statistically divergent sequence of real numbers by Macaj and Salat in [19]. Later Das and Ghosal in [6] introduced the concept of divergence of a sequence in a metric space and extended it with the help of ideals. Here following the same approach we introduce the idea of divergent sequence in an S-metric space and extend it with the help of ideals. Also we prove some results following the similar approach of [6].

Definition 3.1. *(cf. [6]) A sequence $\{x_n\}$ in an S-metric space (X, S) is said to be divergent (or properly divergent) if there exists an element $x \in X$ such that $S(x_n, x_n, x) \to \infty$ as $n \to \infty$.*

We note that a divergent sequence in an S-metric space cannot have any convergent subsequence.

Definition 3.2. *(cf. [6]) A sequence $\{x_n\}$ in an S-metric space (X, S) is said to be I-divergent if there exists an element $x \in X$ such that for any positive real number G, the set*

$$A(x, G) = \{n \in \mathbb{N} : S(x_n, x_n, x) \leq G\} \in I$$

.

Definition 3.3. *(cf. [6]) A sequence $\{x_n\}$ in an S-metric space (X, S) is said to be I^*-divergent if there exists $M \in F(I)$ i.e., $\mathbb{N}\setminus M \in I$ such that $\{x_m\}_{m\in M}$ is divergent i.e., there exists some $x \in X$ such that $lim_{m\to\infty} S(x_m, x_m, x) = \infty$ where $m \in M$.*

Theorem 3.4. *Let I be an admissible ideal. If $\{x_n\}$ in (X, S) is I^*-divergent then $\{x_n\}$ is I-divergent.*

Proof. Since $\{x_n\}$ is I^*-divergent so there exists $M \in F(I)$ i.e., $\mathbb{N}\setminus M \in I$ such that $\{x_m\}_{m\in M}$ is divergent i.e., there exists some $x \in X$ such that $lim_{m\to\infty} S(x_m, x_m, x) = \infty$ where $m \in M$. Then for any given positive real number G there exists $k \in \mathbb{N}$ such that $S(x_m, x_m, x) > G$ for all $m > k$ and $m \in M$. Hence we have $\{n \in \mathbb{N} : S(x_n, x_n, x) \leq G\} \subset \mathbb{N}\setminus M \cup \{1, 2, 3, ..., k\} \in I$. This implies that $\{x_n\}$ is I-divergent. □

The following example shows that the converse of the above theorem is not in general true.

Example 3.5. *Let $\mathbb{N} = \bigcup_{j\in\mathbb{N}} \Delta_j$ be a decomposition of $\mathbb{N}$ such that each Δ_j is infinite and $\Delta_i \cap \Delta_j = \emptyset$ for $i \neq j$. Let I be the class of all those subsets A of $\mathbb{N}$ that can intersects only finite number of Δ_i's. Then I becomes an admissible ideal of $\mathbb{N}$.Take the real line $\mathbb{R}$ with the usual metric d. Let $(\mathbb{R}, S)$ be an S-metric space where the S-metric is defined by $S(x, y, z) = d(x, z) + d(y, z)$ for all $x, y, z \in \mathbb{R}$. Let $\{x_n\}$ be a sequence in $(\mathbb{R}, S)$ defined by $x_i = n$ if $i \in \Delta_n$. Now for any given positive real number G there exists a natural number m such that $\frac{G}{2} < m$. Let us consider the set $\{i \in \mathbb{N} : S(x_i, x_i, 0) \leq G\}$. We assert that $\{i \in \mathbb{N} : S(x_i, x_i, 0) \leq G\} \cap \Delta_k = \phi$ for all $k \geq m$. If possible let $\{i \in \mathbb{N} : S(x_i, x_i, 0) \leq G\} \cap \Delta_k \neq \phi$ for some $k \geq m$ and $p \in \{i \in \mathbb{N} : S(x_i, x_i, 0) \leq G\} \cap \Delta_k$. Then*

$S(x_p, x_p, 0) = 2d(x_p, 0) = 2d(k, 0) = 2|k-0| = 2k$ *and since* $S(x_p, x_p, 0) \leq G$ *so we get* $2k \leq G$ *where* $k \geq m$ *which leads to a contradiction. Hence we conclude that* $\{i \in \mathbb{N} : S(x_i, x_i, 0) \leq G\} \subset \Delta_1 \cup \Delta_2 \cup \ldots\ldots \cup \Delta_{m-1} \in I$ *and consequently* $\{x_n\}$ *is* I*-divergent.*

Next we shall show that $\{x_n\}$ *is not* I^**-divergent in* $(\mathbb{R}, S)$*. If possible assume that* $\{x_n\}$ *is* I^**-divergent. Then there exists* $M \in F(I)$ *such that* $\{x_m\}_{m\in M}$ *is divergent in* $(\mathbb{R}, S)$*. Since* $\mathbb{N} \setminus M \in I$ *so there exists* $k \in \mathbb{N}$ *such that* $\mathbb{N} \setminus M \subset \Delta_1 \cup \Delta_2 \cup \ldots\ldots \cup \Delta_k$*. But then* $\Delta_i \subset M$ *for all* $i > k$*. In particular* $\Delta_{k+1} \subset M$*. But this implies that* $\{x_i\}_{i\in\Delta_{k+1}}$ *is a constant subsequence of* $\{x_m\}_{m\in M}$ *which is convergent to* $k+1$*. This contradicts the fact that* $\{x_m\}_{m\in M}$ *is divergent in* $(\mathbb{R}, S)$*.*

Theorem 3.6. *If I is an admissible ideal with property (AP) then for any sequence* $\{x_n\}$ *in* (X, S)*, I-divergence implies* I^**-divergence.*

Proof. First suppose that I satisfies the condition (AP). Since $\{x_n\}$ is I-divergent so there exists some $x \in X$ such that for any positive real number G, the set $A(x, G) = \{n \in \mathbb{N} : S(x_n, x_n, x) \leq G\} \in I$. Let us take $A_1 = \{n \in \mathbb{N} : S(x_n, x_n, x) \leq 1\}$, $A_2 = \{n \in \mathbb{N} : 1 < S(x_n, x_n, x) \leq 2\}$,...............,$A_k = \{n \in \mathbb{N} : k-1 < S(x_n, x_n, x) \leq k\}$ for all $k \geq 2$. Thus we get a countable collection of mutually disjoint sets $\{A_i\}$ with $A_i \in I$ for all $i \in \mathbb{N}$. By the condition (AP) there exists a family $\{B_i\}$ of subsets of $\mathbb{N}$ such that $A_i \Delta B_i$ is finite for all $i \in \mathbb{N}$ and $B = \bigcup_{i\in\mathbb{N}} B_i \in I$. Let $M = \mathbb{N} \setminus B$. Then $M \in F(I)$. Let $G > 0$ be any real. Then there exists $k \in \mathbb{N}$ such that $G < k$. Then $\{n \in \mathbb{N} : S(x_n, x_n, x) \leq G\} \subset A_1 \cup A_2 \cup \cdots \cup A_k$. Since $A_i \Delta B_i$ is finite for all $i \in \mathbb{N}$ so there exists $n_0 \in \mathbb{N}$ such that $(\bigcup_{i=1}^k B_i) \bigcap \{n \in \mathbb{N} : n \geq n_0\} = (\bigcup_{i=1}^k A_i) \bigcap \{n \in \mathbb{N} : n \geq n_0\}$. Clearly if $n \geq n_0$ and $n \in M$ then $n \notin \bigcup_{i=1}^k B_i$ which implies $n \notin \bigcup_{i=1}^k A_i$. Therefore $S(x_n, x_n, x) > k > G$. Hence we see there is a set $M = \mathbb{N} \setminus B \in F(I)$ such that the sequence $\{x_m\}_{m\in M}$ is a divergent sequence and consequently $\{x_n\}$ becomes I^*-divergent. □

Theorem 3.7. *Let* (X, S) *be an S-metric space containing at least one divergent sequence and let I be an admissible ideal. If for every sequence* $\{x_n\}$ *in* (X, S) *I-divergence implies* I^**-divergence then I satisfies the condition (AP).*

The proof of the above theorem follows the same approach as in [6].

4. Conclusion

Here we have studied the idea of I and I^*-Cauchy condition in a more general structure of an S-metric space. Also we have studied the notions of I-divergence and I^*-divergence in an S-metric space. As we know S-metric space is a generalization of a metric space, the same can be studied in a more general settings like Cone metric spaces, M-metric spaces etc. Also as a continuation of this work the idea of I and I^K-Cauchy conditions may be studied in such generalized spaces.

References

[1] V. Baláž, J. Červeňanský, P. Kostyrko, T. Šalát, I-convergence and I-continuity of real functions, *Acta Math. (Nitra),* 5 (2002), 43-50.

[2] M. Balcerzak, K. Dems, A. Komisarski, Statistical convergence and ideal convergence for sequences of functions, *J. Math. Anal. Appl.,* 328 (2007), 715-729.

[3] A.K. Banerjee, A. Banerjee, A note on I-convergence and I^*-convergence of sequences and nets in topological spaces, *Mat. Vesnik,* 67, 3 (2015), 212-221.

[4] A.K. Banerjee, R. Mondal, A note on convergence of double sequences in a topological space, *Mat. Vesnik,* 69, 2 (2017), 144-152.

[5] A.K. Banerjee, Anindya Dey, Metric spaces and complex analysis, *New age International(P) Limited Publishers,* 2008.

[6] P. Das, S.K. Ghosal, Some further results on I-Cauchy sequences and condition (AP), *Computers and Mathematics with Applications,* 59 (2010), 2597-2600.

[7] P. Das, S.K. Ghosal, On I-Cauchy nets and completeness, *Topology and its Applications,* 157 (2010), 1152-1156.

[8] P. Das, M. Sleziak, V. Toma, I^K-Cauchy functions, *Topology and its Applications,* 173 (2014), 9-27.

[9] K. Demirci, *I*-limit superior and limit inferior, *Mathematical Communications,* 6 (2001), 165-172.

[10] K. Dems: On I-Cauchy sequences, *Real Analysis Exchange,* 30(1) (2004/2005), 123-128.

[11] H. Fast, Sur la convergence statistique, *Colloq. Math.,* 2 (1951), 241-244.

[12] H. Halberstem, K.F. Roth, Sequences, *Springer, New York,* 1993.

[13] P. Kostyrko, T. Šalát, W. Wilczyński, I-convergence, *Real Analysis Exchange,* 26 (2)(2000/2001), 669-686.

[14] P. Kostyrko, M. Mačaj, T. Šalat, M. Sleziak, I-convergence and extremal I-limit points, *Math. Slovaca,* 55 (4) (2005), 443-464.

[15] K. Kuratowski, Topologie I, *PWN, Warszawa,* 1961.

[16] B.K. Lahiri, P. Das, Further results on I-limit superior and I-limit inferior, *Mathematical Communications,* 8 (2003), 151-156.

[17] B.K. Lahiri, P. Das, I and I^*-convergence in topological spaces, *Math. Bohemica,* 130 (2) (2005), 153-160.

[18] B.K. Lahiri, P. Das, I and I^*-convergence of nets, *Real Analysis Exchange,* 33 (2) (2007/2008), 431-442.

[19] M. Mačaj, T. Šalát, Statistical convergence of subsequences of a given sequence, *Math. Bohemica,* 126 (2001), 191-208.

[20] A. Nabiev, S. Pehlivan, M. Gurdal, On I-Cauchy sequences, *Taiwanese J. Math.,* 11 (2) (2007), 569-576.

[21] T. Šalát, On statistically convergent sequences of real numbers, *Math. Slovaca,* 30 (1980), 139-150.

[22] I.J. Schoenberg, The integrability of certain functions and related summability methods, *Amer. Math. Monthly,* 66 (1959), 361-375.

[23] S. Sedghi, N. Shobe, A. Aliouche, A generalization of

fixed point theorems in S-metric spaces, *Mat. Vesnik,* 64 (3) (2012), 258-266.

[24] H. Steinhaus, Sur la convergence ordinaire et la convergence asymptotique, *Colloq. Math.,* 2 (1951), 73-74.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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