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On new class of continuous functions in nano topological spaces

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Abstract
The aim of this paper is to define and study the new class of functions, namely nano g^*-continuous functions, nano g^*s-continuous functions in nano topological spaces and study some of their properties. Also we investigate the relationships between the other existing continuous functions. Further, we have given an appropriate to understand the abstract concept clearly.

Keywords
nano g^*-closed sets, nano g^*s-closed sets, nano g^*-continuous, nano g^*s-continuous.

AMS Subject Classification
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Contents

1. Introduction

Topology is the branch of mathematics through which we elucidate and investigate the ideas of continuity, within the framework of mathematics. The study of topological spaces, their continuous mappings and general properties make up one branch of topologies known as general topology. In 1970, Levine[7] introduced the concept of generalized closed sets in topological spaces. In 1987, Bhattacharya [1] introduced the notion of semi generalized closed sets in topological spaces. In 1993, Devi et.al[4] introduced the concept of semi generalized mappings in topological spaces. In 2000, Veerakumar[11] introduced the notion of generalized star closed sets in topological spaces.

In 2017, Sekar[8], defined a new class of sets, namely generalized b star-closed (briefly. gb^*-closed) set in topological spaces. The concept of nano in topological spaces is treated in the classic text by Lellis Thivagar[5]. He also defined nano closed sets, nano-interior and nano-closure of a set and in 2013, introduced the concept of nano continuous functions in nano topological spaces. In 2014, Bhuvaneshwari[2] defined the concept of nano g-closed sets (briefly. Ng-closed) and nano gs-closed sets (briefly. Ngs-closed) in nano topological spaces. In 2015, Rajendran et.al[10] introduced the concept of nano g star-closed sets in nano topological spaces. The purpose of this paper is to define and study the new class of functions, namely nano g^*-continuous functions, nano g^*s-continuous functions in nano topological spaces and study some of their properties. The structure of this manuscript is as follows,
In section 2, we recall some fundamental definition and results which are very useful to prove our main result.
In section 3, we define and study the concept of nano g^*-continuous functions, nano g^*s-continuous functions in nano topological spaces and study some of their properties.

2. Preliminaries

Definition 2.1. [5] *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The* *pair (U,R) is said to be the approximation space. Let $X \subseteq U$, then*

(i) The lower approximation of X with respect to R is the set of all objects which can be certain classified as X with respect to R and is denoted by $L_R(X)$.
$L_R(X) = \cup\{R(x) : R(x) \subseteq X, x \in U\}$, *where $R(x)$ denotes the equivalence class determined by $x \in U$.*

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.
$U_R(X) = \cup\{R(x) : R(x) \cap X \neq \phi, x \in U\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X)$-$L_R(X)$.

Property 2.2. [5] *If (U,R) is an approximation space and $X,Y \subseteq U$, then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3. [5] *Let U the universe, R be an equivalence relation on U and the nano topology $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then by property 2.2, $\tau_R(X)$ satisfies the following axioms.*

1. U and $\phi \in \tau_R(X)$.

2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
Then $\tau_R(X)$ is a topology on U is called the nano topology on U with respect to X. $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets.

Remark 2.4. [5] *If $\tau_R(X)$ is the nano topology on U with respect to X, the set $B = \{U, L_R(X), U_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. [5] *If $(U, \tau_R(X))$ is a nano topological space with respect to X, where $X \subseteq U$ and if $A \subseteq U$, then*

1. *The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by $Nint(A)$. $Nint(A)$ is the largest nano open subset of A.*
2. *The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest nano closed set containing A.*

Remark 2.6. [6] *Throughout this paper, U and V are nonempty finite universe; $X \subseteq U$ and $Y \subseteq V$, U/R and V/$R^{'}$ denote the families of equivalence classes by equivalence relations R and $R^{'}$ respectively on U and V. $(U, \tau_R(X))$ and $(V, \tau_{R^{'}}(X))$ are the nano topological spaces with respect to X and Y.*

Definition 2.7. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be*

1. *Ng-closed[2], If $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano open set in U.*
2. *Ngs-closed[3], If $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano open set in U.*
3. *Ng^*-closed[10], If $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano g-open set in U.*
4. *Ng^*s-closed[10], If $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano g-open set in U.*

Definition 2.8. [8, 9]

1. *A subset A of a topological space (X, τ), is called gb^*-closed set if $bcl(A) \subseteq U$ whenever $A \subset U$ and U is g^*-open in X.*
2. *A function $f : (X, \tau) \to (Y, \sigma)$ is called almost contra pgb-continuous if $f^{-1}(V)$ is pgb-closed in (X, τ) for every regular open set V in (Y, σ).*

Definition 2.9. [6] *Let $(U, \tau_R(X))$ and $(V, \tau_{R^{'}}(Y))$ be the nano topological spaces. Then the function $f : (U, \tau_R(X)) \to (V, \tau_{R^{'}}(Y))$ is nano continuous on U, If the inverse image of every nano open set in V is nano open set in U.*

3. Nano g^*s-continuous functions

In this section, we define and study the new class of functions, namely nano g^*-continuous functions, nano g^*s-continuous functions in nano topological spaces and study some of their properties. Also we investigate the relationships between the other existing continuous functions.

Definition 3.1. *Let $(U, \tau_R(X))$ and $(V, \tau_{R^{'}}(Y))$ be a nano topological spaces. Then the function $f : (U, \tau_R(X)) \to (V, \tau_{R^{'}}(Y))$ is a In section 3, we define and study the concept of nano g^*-continuous functions, nano g^*s-continuous functions in nano topological spaces and study some of their properties.*

1. *nano g^*-continuous on U, if the inverse image of every nano open set in V is nano g^*-open set in U.*
2. *nano g^*s-continuous on U, if the inverse image of every nano open set in V is nano g^*s-open set in U.*

Example 3.2. *Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b,d\}, \{a,b,d\}\}$ which are nano open sets*

- *nano g-closed sets are $\{U,\phi,\{c\},\{a,c\},\{b,c\},\{c,d\}, \{a,b,c\},\{a,c,d\},\{b,c,d\}\}$*
- *nano gs-closed sets are $\{U,\phi,\{a\},\{c\},\{a,c\},\{b,c\}, \{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$*
- *nano g^*-closed sets are $\{U,\phi,\{c\},\{a,c\},\{c,d\}, \{a,b,c\},\{a,c,d\},\{b,c,d\}\}$*
- *nano g^*s-closed sets are $\{U,\phi,\{a\},\{c\},\{a,c\}, \{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$*

Let $V = \{x,y,z,w\}$ with $V/R' = \{\{x\},\{w\},\{y,z\}\}$ and $Y = \{x,z\}$. Then $\tau_{R'}(Y) = \{V,\phi,\{x\},\{y,z\},\{x,y,z\}\}$ which are nano open sets

- *nano g-closed sets are $\{V,\phi,\{w\},\{x,w\},\{y,w\},\{z,w\}, \{x,y,w\},\{x,z,w\},\{y,z,w\}\}$*
- *nano gs-closed sets are $\{V,\phi,\{x\},\{y\},\{z\},\{w\},\{x,w\}, \{y,z\},\{y,w\},\{z,w\},\{x,y,w\},\{x,z,w\},\{y,z,w\}\}$*
- *nano g^*-closed sets are $\{V,\phi,\{w\},\{x,w\},\{y,w\}, \{z,w\},\{x,y,w\},\{x,z,w\},\{y,z,w\}\}$*
- *nano g^*s-closed sets are $\{V,\phi,\{x\},\{w\},\{x,w\}, \{y,z\},\{y,w\},\{z,w\},\{x,y,w\},\{x,z,w\},\{y,z,w\}\}$*

Then we define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x$, $f(b) = y$, $f(c) = w$, $f(d) = z$. Then $f^{-1}(V) = U$, $f^{-1}(\phi) = \phi$, $f^{-1}(x) = \{a\}$, $f^{-1}(y,z) = \{b,d\}$, $f^{-1}(w) = \{c\}$. Thus the inverse image of every nano open set in V is nano g^*-open set and nano g^*s-open set in U. Hence $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano g^*-continuous and nano g^*s-continuous.

Theorem 3.3. *A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano g^*-continuous if and only if the inverse image of every nano closed set in V is nano g^*-closed set in U.*

Proof. : Let f be nano g^*-continuous and F be nano closed set in V. That is V - F is nano open set in V. Since f is nano g^*-continuous, $f^{-1}(V - F)$ is nano g^*-open set in U. That is, $f^{-1}(V)$ - $f^{-1}(F) = U$ - $f^{-1}(F)$ is nano g^*-open in U. Hence $f^{-1}(F)$ is nano g^*-closed in U, if f is nano g^*-continuous on U.

Conversely, Let the inverse image of every nano closed set in V is nano g^*-closed in U. Let G be a nano open set in V. Then V - G is nano closed set in V. Then $f^{-1}(V - G)$ is nano g^*-closed set in U. That is, $f^{-1}(V)$ - $f^{-1}(F) = U$ - $f^{-1}(G)$ is nano g^*-closed set in U. Therefore, $f^{-1}(G)$ is nano g^*-open set in U. Thus the inverse image of every nano open set in V is nano g^*-open set in U. That is, f is nano g^*-continuous on U. □

Theorem 3.4. *A function $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano g^*s-continuous if and only if the inverse image of every nano closed set in V is nano g^*s-closed set in U.*

Proof. : Proof follows from the Theorem 3.3. □

Remark 3.5. *A function $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$, then the following hold,*

1. *Every nano continuous function is nano g^*-continuous*
2. *Every nano continuous function is nano g^*s-continuous*
3. *Every nano g^*-continuous function is nano g^*s-continuous*
4. *Every nano g^*s-continuous function is nano gs-continuous*

Proof*: 1) Let $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ be nano continuous on U. We have every nano closed set is nano g^*-closed set. Since f is nano continuous on U, the inverse image of every nano closed set in V is nano closed set in U. Hence the inverse image of every nano closed set in V is nano g^*-closed set in U. Hence f is nano g^*-continuous.*

Proof of (2) to (4) is as follows from the (1).

Remark 3.6. *The converse of the above remarks need not be true as seen from the following examples.*

Example 3.7. *1). Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b\}, \{c,d\}\}$ and $X = \{a,d\}$. Then $\tau_R(X) = \{U,\phi,\{a\},\{c,d\},\{a,c,d\}\}$. Let $V = \{x,y,z,w\}$ with $V/R' = \{\{x\},\{w\},\{y,z\}\}$ and $Y = \{x,z\}$. Then $\tau_{R'}(Y) = \{V,\phi,\{x\},\{y,z\},\{x,y,z\}\}$. Define a function $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ as $f(a) = y$, $f(b)= w$, $f(c) = z$ and $f(d) = x$. Then f is nano g^*-continuous but not nano continuous.*

2). In example 3.2, Let us define $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ as $f(a) = z$, $f(b)= x$, $f(c) = w$ and $f(d) = y$. Then f is nano g^*s-continuous but not nano continuous.

3). Let $U = \{a,b,c,d,\}$ with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$. Let $V = \{x,y,z,w,\}$ with $V/R = \{\{x\},\{y\},\{z,w\}\}$ and $Y = \{x,z\}$. Then $\tau_R(Y) = \{U,\phi,\{x\},\{z,w\},\{x,z,w\}\}$. Define a function $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ as $f(a) = y$, $f(b)= z$, $f(c) = x$, $f(d) = w$. Then f is nano g^*s-continuous but not nano g^*-continuous.

4). From the above example, Define a function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x$, $f(b) = y$, $f(c) = z$, $f(d) = w$. Then f is nano gs-continuous but not nano g^*s-continuous.

Theorem 3.8. *: A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano g^*s continuous if and only if $f(Ng^*scl(A)) \subseteq Ncl(f(A))$ for every subset A of U.*

Proof. Let f be nano g^*s-continuous and $A \subseteq U$. Then $f(A) \subseteq V$. Since f is nano g^*s-continuous and $Ncl(f(A))$ is nano closed in V, $f^{-1}(Ncl(f(A)))$ is nano g^*s-closed in U. Since $f(A) \subseteq Ncl(f(A))$, $f^{-1}(f(A)) \subseteq f^{-1}(Ncl(f(A)))$, then $Ng^*scl(A) \subseteq Ng^*scl(f^{-1}(Ncl(f(A))))= f^{-1}(Ncl(f(A)))$. Thus $Ng^*scl(A) \subseteq f^{-1}(Ncl(f(A)))$. Therefore, $f(Ng^*scl(A)) \subseteq$

$Ncl(f(A))$ for every subset A of U.

Conversely, Let $f(Ng^*scl(A)) \subseteq Ncl(f(A))$ for every subset A of U. If F is nano closed set in V, since $f^{-1}(F) \subseteq$ U, $f(Ng^*scl(f^{-1}(F)) \subseteq Ncl(f(f^{-1}(F)))= Ncl(F)$. That is, $Ng^*scl(f^{-1}(F)) \subseteq f^{-1}(Ncl(F)) = f^{-1}(F)$. Since F is nano closed. Thus $Ng^*scl(f^{-1}(F)) \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq Ng^*scl(f^{-1}(F))$. Thus $Ng^*scl(f^{-1}(F)) = f^{-1}(F)$. Therefore $f^{-1}(F)$ is nano g^*s-closed in U for every nano closed set F in V. That is, f is nano g^*s-continuous. □

Remark 3.9. *If f : $(U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano g^*s continuous, then $f(Ng^*scl(A))$ is not necessarily equal to $Ncl(f(A))$.*

Example 3.10. *In Example 3.2, Let us define f : $(U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ as $f(a) = x$, $f(b) = y$, $f(c) = z$, $f(d) = w$. Here f is nano g^*s-continuous. Since the inverse image of every nano open set in V is nano g^*s-open in U. Let $B = \{z\} \subseteq V$. Then $Ncl(B) \subseteq \{y,z,w\}$. Hence $f^{-1}(Ncl(B)) = f^{-1}\{y,z,w\} = \{b,c,d\}$. Also $f^{-1}(B) = \{c\}$. Hence $Ng^*scl(f^{-1}(b)) = Ng^*scl(\{c\}) = \{c\}$. Thus $Ng^*scl(f^{-1}(B)) \neq f^{-1}(Ncl(B))$, even though f is nano g^*s-continuous.*

Theorem 3.11. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces, where $X \subseteq U$ and $Y \subseteq V$. Then $\tau_{R'}(Y) = \{V,\phi,L_{R'}(Y),U_{R'}(Y),B_{R'}(Y)\}$ and its basis is given by $B_{R'} = \{V,L_{R'}(Y),B_{R}{'}(Y)\}$. A function f : $(U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano g^*s-continuous if and only if the inverse image of every member of $B_{R'}$, is nano g^*s-open in U.*

Proof. Let $f : (U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ be nano g^*s-continuous on $(U, \tau_R(X))$. Let B $\in B_{R'}$. Then B is nano open in $(V,\tau_{R'}(Y))$. Since f is nano g^*s-continuous. f^{-1}(B) is nano g^*s-open in U and $f^{-1}(B) \in \tau_R(X)$. Hence the inverse image of every member of $B_{R'}$, is nano g^*s-open in U.

Conversely, Let the inverse image of every member of $B_{R'}$, be nano g^*s-open in U. Let G be nano open in V. Now $G = \{B : B \in B_1\}$ where $B_1 \subset B_{R'}$,. Then $f^{-1}(G) = f^{-1}[\cup\{B : B \in B_1\}] = \cup\{f^{-1}(B) : B \in B_1\}$ where each $f^{-1}(B)$ is nano g^*s-open in U and their union which is $f^{-1}(G)$ is also nano g^*s-open set in U. By definition, f : $(U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano g^*s-continuous on $(U, \tau_R(X))$. □

Theorem 3.12. *A function f : $(U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano g^*s-continuous if and only if $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$.*

Proof. If f is nano g^*s-continuous and $B \subseteq V$, then $Ncl(B)$ is nano closed in V and hence $f^{-1}(Ncl(B))$ is nano g^*s-closed set in U. Therefore, $Ng^*scl(f^{-1}(Ncl(B))) = f^{-1}(Ncl(B))$. Since $B \subseteq Ng^*scl(B)$, $f^{-1}(B) \subseteq f^{-1}(Ncl(B))$. Therefore, $Ng^*scl(f^{-1}(B)) \subseteq Ng^*scl(f^{-1}(Ncl(B))) = f^{-1}(Ncl(B))$. That is $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$.

Conversely, Let $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$ for every subset B of V. If B is nano closed in V, then $Ncl(B) = B$. By assumption, $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))) = f^{-1}(B)$. Thus $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq Ng^*scl(f^{-1}(B))$. Therefore, $Ng^*scl(f^{-1}(B)) = f^{-1}(B)$. That is, $f^{-1}(B)$ is nano g^*s-closed in U for every nano closed set B in V. Therefore f is nano g^*s-continuous on U. □

Theorem 3.13. *If $(U, \tau_R(X))$ and $(V,\tau_{R'}(Y))$ are nano topological spaces with respect to $X \subseteq U$ and $Y \subseteq V$ respectively, then any function f : $(U, \tau_R(X)) \to (V,\tau_{R'}(Y))$, the following conditions are equivalent.*

1. *f is nano g^*s-continuous.*
2. *$f(Ng^*scl(A)) \subseteq Ncl(f(A))$ for every subset A of U.*
3. *$Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$ for every subset B of V.*

Proof. (1) ⇒ (2) : Let f be nano g^*s-continuous and $A \subseteq U$. Then $f(A) \subseteq V$. Since f is nano g^*s- continuous and $Ncl(f(A))$ is nano closed in V, $f^{-1}(Ncl(f(A))$ is nano g^*s-closed in U. Since $f(A) \subseteq Ncl(f(A))$, $f^{-1}(f(A)) \subseteq f^{-1}(Ncl (f(A)))$, then $Ng^*scl(A) \subseteq Ng^*scl(f^{-1}(Ncl(f(A)))) = f^{-1}(Ncl(f(A)))$. Thus $Ng^*scl(A) = f^{-1}(Ncl(f(A)))$. Therefore $f(Ng^*scl(A)) \subseteq Ncl(f(A))$ for every subset A of U.

(2) ⇒ (3) : Let $f(Ng^*scl(A)) \subseteq Ncl(f(A))$ and A= $f^{-1}(B) \subseteq U$ for every subset $B \subseteq V$. Since $f(Ng^*scl(A)) \subseteq Ncl(f(A))$ we have $f(Ng^*scl(f^{-1}(B))) \subseteq Ncl(f(f^{-1}(B))) \subseteq Ncl(B)$. That is, $f(Ng^*scl(f^{-1}(B))) \subseteq Ncl(B)$. which ⇒ that $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$ for every $B \subseteq V$.

(3) ⇒ (1) : Let $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$ for every subset B of V. If B is nano closed in V, then $Ncl(B) = B$. By assumption, $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B)) = f^{-1}(B)$. Thus $Ng^*scl(f^{-1}(B)) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq Ng^*scl(f^{-1}(B))$. Therfore $Ng^*scl(f^{-1}(B)) = f^{-1}(B)$. That is $f^{-1}(B)$ is nano g^*s-closed U for every nano closed set B in V. Therefore f is nano g^*s-continuous on U. □

Theorem 3.14. *A function f :$(U, \tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano g^*s-continuous if and only if $f^{-1}(Nint(B)) \subseteq Ng^*s\ int (f^{-1}(B))$ for every subset B of V.*

Proof. If f is nano g^*s - continuous and $B \subseteq V$, then $Nint(B)$ is nano-open in V and hence f^{-1}(Nint(B)) is nano g^*s-open in U. Therefore, $Ng^*sint[f^{-1}(Nint(B))] = f^{-1}(Nint(B))$. Also $Nint(B) \subseteq B$ implies that $f^{-1}(Nint(B)) \subseteq f^{-1}(B)$. Therefore $Ng^*sint[f^{-1}(Nint(B))] \subseteq Ng^*sint[f^{-1}(B)$. That is $f^{-1}(Nint(B)) \subseteq Ng^*sint(f^{-1}(B))$.

Conversely, Let $f^{-1}(Nint(B)) \subseteq Ng^*sint(f^{-1}(B))$ for every $B \subseteq V$, If B is nano-open in V, then $Nint(B) = B$. By assumption, $f^{-1}(Nint(B)) \subseteq Ng^*sint(f^{-1}(B))$. Thus $f^{-1}(B) \subseteq Ng^*sint(f^{-1}(B))$. But $Ng^*sint(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B) = Ng^*sint(f^{-1}(B))$. That is $f^{-1}(B)$ is nano g^*s-open in U for every nano open set B in V. Therefore, f is nano g^*s-continuous on U. □

Remark 3.15. *Equality of the above Theorems 3.12 and 3.14 does not holds in general that can be seen from the following example.*

Example 3.16. *In Example 3.2, Let us define $f:(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as $f(a) = x$, $f(b) = y$, $f(c) = w$ and $f(d) = z$. Here f is nano g^*s-continuous. Since the inverse image of every nano open set in V is nano g^*s-open set in U. Let $B = \{x\} \subset V$. Then $Ncl(B) = \{x,w\}$. Hence $f^{-1}(Ncl(B)) = f^{-1}(\{x,w\}) = \{a,c\}$. Also $f^{-1}(B) = \{a\}$. Hence $Ng^*scl(f^{-1}(B)) = Ng^*scl(\{a\}) = \{a\}$. Thus $Ng^*scl(f^{-1}(B)) \neq f^{-1}(Ncl(B))$. Also $A = \{y,z,w\} \subset V$, $f^{-1}(Nint(A)) = f^{-1}(\{y,z\}) = \{b,d\}$. But $Ng^*sint(f^{-1}(A)) = Ng^*sint(\{b,c,d\}) = \{b,c,d\}$. That is, $Ng^*sint(f^{-1}(A)) \neq f^{-1}(Nint(A))$.*

Conclusion

In this paper, we defined and studied the notions of nano g^*-continuous and nano g^*s-continuous functions in nano topological spaces and discussed their properties . Also we discussed the relationships between the other existing continuities. In future, we extend this work in various topological fields.

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