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On TL-bi-ideals of ternary semigroups

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Abstract
We introduce the notions of TL-ternary subsemigroup and TL-bi-ideals of a ternary semigroup. We redefine TL-ternary subsemigroup and TL-bi-ideals using T-product on L-sets .We introduce the notion of T-intersection of L-sets. We establish that T-intersection of two TL-bi-ideals is again a TL-bi-ideal. We establish necessary and sufficient conditions for a pre-image of L-set under homomorphism to be a TL-ideal. We introduce the notion of TL-level sets. We characterize TL-bi-ideal by TL-level sets.

Keywords
T-norm, L-set, TL-subsemigroup, TL-bi-ideal, TL-level set.

AMS Subject Classification
03E72, 06B23, 16D25, 47A66.

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Contents

1. Introduction

J.A.Goguen [4] introduced L-sets in 1967. After the introduction of the concept of L-ideals in semigroups by Neggers et al [16],[17]. Ronnason Chinram[18] studied L-ideals in ternary semirings. S.Kar and Palutu Sarakar[8] studied the concept of fuzzy quasi-ideals and fuzzy bi-ideals of ternary semigroups. Dheena and Mohanraj [2] introduced T-fuzzy ideals of a ring using triangular norm. Mohanraj and Prabu[15] devoleped redefined T-fuzzy right h-ideals of hemirings. Basic definition and mathematical facts about lattices and T-norm can be found in Birkhoff[1] and [7]Klement.E.P.

In this paper, by the introduction of the notions of TL-ternary subsemigroup and TL-bi-ideals of a ternary semigroup, the TL-ternary subsemigroup and TL-bi-ideals are redefined using T-product on L of ternary semigroup. We establish that T-intersection of two TL-bi-ideals is again a TL-bi-ideal. It is established that homomorphism pre-image of a TL-bi-ideal is again a TL-bi-ideal. Using TL-level sets, we characterstice TL-bi-ideal of S

2. Preliminaries

Definition 2.1. *A non-empty set S is called ternary semigroup if there exist a mapping $S\times S\times S\to S$ denoted by juxtaposition that satisfies the following condition: $(abc)de = a(bcd)e = ab(cde)$ for all $a,b,c,d,e \in S$.*

Example 2.2. *Let $S = \{a\sqrt{3} | a \in \mathbb{Z}^-\}$ where $\mathbb{Z}^-$ is the set of negative odd integers. Then S is a ternary semigroup under usual multiplication.*

Definition 2.3. *The non-empty subset B of ternary semigroup S is called ternary subsemigroup if $xyz \in B$ for all $x,y,z \in B$.*

Definition 2.4. *A ternary subsemigroup B of S is called ternary bi -ideal if $(xwy)vz \in S$ for all $x,w,y,v,z \in S$.*

Definition 2.5. *Let $(L,\leq,\wedge,\vee)$ be a Complete Brouwerian Lattice with least element 0 and greatest element 1. Let S be a non empty set. By a L-set μ of S, we mean a mapping $\mu : S \to L$.*

Remark 2.6. *"1" is a L-set on S defined as $1(x) = 1$ for all $x \in S$.*

Definition 2.7. *The mapping $T : L\times L \to L$ is called a triangular norm[T-norm] on L which satisfies the following conditions:*

(i) $T(x,1) = T(1,x) = x$ (boundary condition)
(ii) $T(x,y) = T(y,x)$ (commutativity)
(iii)$T(x,T(y,z)) = T(T(x,y),z)$ (associativity)
(iv) If $x^* \leq x$ and $y^* \leq y$ then $T(x^*,y^*) \leq T(x,y)$ (monotonicity) for all $x,y,z \in L$.

Remark 2.8. *1. The T-norms on $(L,\leq,\wedge,\vee)$ are defined as follows:*
$T(x,y) = x \wedge y$,

2. Drastic product T-norm:

$$T_D(x,y) = \begin{cases} x \wedge y & \text{if } x = 1 \text{ or } y = 1 \\ 0 & \text{otherwise,} \end{cases}$$

Various T-norms on $L = [0,1]$ are defined as follows:

3. Product T-norm: $T_P(x,y) = x \cdot y$,

4. Lukasiewicz T-norm:
$T_L(x,y) = max\{x+y-1,0\}$,

5. and Hamacher class$T$-norms:
for any $\lambda \in [0,\infty)$

$$(T^H_\lambda)(x,y) = \begin{cases} T_D(x,y) & \text{if } \lambda = \infty \\ 0 & \text{if } \lambda = x = y = 0 \\ \frac{xy}{\lambda+(1-\lambda)(x+y-xy)} & \text{otherwise.} \end{cases}$$

3. Redefined TL-bi-ideals

Throughout this paper, S denotes a ternary semigroup L denotes complete brouwerian lattice with least element 0 and greatest element 1 and T denotes a triangular norm on L unless otherwise specified,

Definition 3.1. *Let μ be a L-set and T be a T norm on L. The L-set μ is said to be TL-ternary subsemigroup of S if $\mu(xyz) \geq T(\mu(x),T(\mu(y),\mu(z)))$ for all $x,y,z \in S$.*

Remark 3.2. *1. By taking $T(x,y) = x \wedge y$ in Definition 3.1, TL-ternary subsemigroup becomes in L-ternary subsemigroup.*

2. By taking $L = [0,1]$in Definition 3.1, then TL- ternary subsemigroup coincides with a T-fuzzy ternary subsemigroup.

3. Taking $L = [0,1]$ and $T(x,y) = min\{x,y\}$ is a Definition 3.1, TL-ternary subsemigroup is a fuzzy ternary subsemigroup.

Definition 3.3. *The TL-ternary subsemigroup μ of S is said to be a TL-ternary bi-ideal of S if $\mu(xwyvz) \geq T(\mu(x),T(\mu(y),\mu(z)))$ for all $x,y,z,w,v \in S$.*

Remark 3.4. *1. By taking $T(x,y) = x \wedge y$ in Definition 3.3, TL- ternary bi-ideal is the L-ternary bi-ideal.*

2. By taking $L = [0,1]$ in Definition 3.3, then TL- fuzzy ternary bi-ideal coincides with a T-fuzzy ternary bi-ideal.

3. By taking $L = [0,1]$ and $T(x,y) = min\{x,y\}$ inDefinition3.3, TL-ternary bi-ideal becomes is a fuzzy ternary bi-ideal.

Definition 3.5. *Let λ,μ and σ be the L- sets of a ternary semigroup S. Then ternary T-product on L-set λ,μ and σ is defined as follows:*

$$(\lambda \cdot_T \mu \cdot_T \sigma)(x) = \begin{cases} \bigvee_{x=abc} T(\lambda(a),T(\mu(b),\sigma(c))) & \text{if } x = abc \\ 0 & \text{otherwise} \end{cases}$$

Remark 3.6. *1. By taking $T(a,b) = a \wedge b$ in Definition 3.5, the ternary T-product is the ternary L product.*

2. By taking $L = [0,1]$ in Definition 3.5, the ternary T-product are referred to as T-fuzzy ternary product $\lambda \cdot \mu \cdot \sigma$ of λ, μ and σ respectively.

3. By taking $L = [0,1]$ in Definition 3.5, the ternary T-product coincides with a fuzzy ternary T- product.

Theorem 3.7. *The L-set of μ is a TL- ternary subsemigroup if and only if*
$\mu \cdot_T \mu \cdot_T \mu \subseteq \mu$.

Proof. Let μ be a TL-ternary subsemigroup of ternary semigroup . If x can not be expressible as $x = abc$,then $(\mu \cdot_T \mu \cdot_T \mu)(x) = 0 \leq \mu(x)$.
If$x = abc$, then
$\mu(x) = \mu(abc) \geq T(\mu(a),T(\mu(b),\mu(c)))$
Thus $\mu(x) \geq \bigvee_{x=abc} T(\mu(a),T(\mu(b),\mu(c)))$
Hence $\mu \cdot_T \mu \cdot_T \mu \subseteq \mu$
Conversely,

$$\begin{aligned} \mu(abc) &\geq (\mu \cdot_T \mu \cdot_T \mu)(abc) \\ &\geq T(\mu(a),T(\mu(b),\mu(c))) \end{aligned}$$

Hence μ is a TL-ternary subsemigroup of S.

Corollary 3.8. *The L-set μ is a L-ternary subsemigroup if and only if $\mu \cdot \mu \cdot \mu \subseteq \mu$.*

Proof. By taking $T(a,b) = a \wedge b$ in Theorem 3.7, we get the result.

Corollary 3.9. *The fuzzy set μ is a T-fuzzy ternary subsemigroup if and only if $\mu \cdot_T \mu \cdot_T \mu \subseteq \mu$.*

Proof. The proof follows by taking $L = [0,1]$ in Corollary 3.8.

Corollary 3.10. *The fuzzy set μ is a fuzzy ternary subsemigroup if and only if $\mu \cdot \mu \cdot \mu \subseteq \mu$.*

Proof. By taking $L = [0,1]$ and $T(a,b) = min\{a,b\}$ in Theorem 3.7, we get the result.

Theorem 3.11. *The L-set of μ is a TL-bi-ideal of S if and only if (i)$\mu \cdot_T \mu \cdot_T \mu \subseteq \mu$.*
(ii)$\mu \cdot_T 1 \cdot_T \mu \cdot_T 1 \cdot_T \mu \subseteq \mu$.

Proof. Let μ is a TL-bi-ideal of ternary subsemigroups of S. By Theorem 3.7,
$\mu \cdot_T \mu \cdot_T \mu \subseteq \mu$. If x cannot be expressible as $x = awbyc$.
Then $(\mu \cdot_T 1 \cdot_T \mu \cdot_T 1 \cdot_T \mu)(x) = 0 \leq \mu(x)$.
Then $\mu \cdot_T 1 \cdot_T \mu \cdot_T 1 \cdot_T \mu \subseteq \mu$. Now,

$$\begin{aligned}((\mu \cdot_T 1 \cdot_T \mu) \cdot_T 1 \cdot_T \mu)(x) &= \bigvee_{x=abc} T((\mu \cdot_T 1 \cdot_T \mu) \cdot_T 1 \cdot_T \mu) \\ &= \bigvee_{x=abc} T((\mu \cdot_T 1 \cdot_T \mu)(a), \mu(c)) \\ &= \bigvee_{x=abc} T(\bigvee_{a=stu} T(\mu(s), T(1(t), \mu(u))), \\ &\qquad \mu(c)) \\ &= \bigvee_{x=abc} T(\bigvee_{a=stu} T(\mu(s), \mu(u)), \mu(c)) \qquad (3.1)\end{aligned}$$

Now, $x = abc$, and $a = stu$ imply $x = (stu)bc$
Then, $\mu(x) \geq T(T(\mu(s), \mu(u)), \mu(c))$
Thus, $\mu(x) \geq \bigvee_{x=abc} T(\bigvee_{a=stu} T(\mu(s), \mu(u)), \mu(c))$
By Equation 3.1, $\mu(x) \geq (\mu \cdot_T 1 \cdot_T \mu \cdot_T 1 \cdot_T \mu)(x)$.
Therefore $\mu \cdot_T 1 \cdot_T \mu \cdot_T 1 \cdot_T \mu \subseteq \mu$.
Conversely, by Theorem 3.7, μ is a TL-ternary subsemigroup of S. Then,

$$\begin{aligned}\mu(xwyvz) &\geq ((\mu \cdot_T 1 \cdot_T \mu) \cdot_T 1 \cdot_T \mu)(xwyvz) \\ &= T((\mu \cdot_T 1 \cdot_T \mu)(xwy), T(1(v), \mu(z))) \\ &= T((\mu \cdot_T 1 \cdot_T \mu)(xwy), \mu(z)) \\ &\geq T(T(\mu(x), T(1(w), \mu(y))), \mu(z)) \\ &= T(T(\mu(x), \mu(y)), \mu(z)) \\ &= T(\mu(x), T(\mu(y), \mu(z)))\end{aligned}$$

Hence μ is a TL-bi-ideal of S.

Corollary 3.12. *The L-set μ is a L-bi-ideal if and only if (i)$\mu \cdot \mu \cdot \mu \subseteq \mu$. (ii)$\mu \cdot 1 \cdot \mu \cdot 1 \cdot \mu \subseteq \mu$.*

Proof. By taking $T(a,b) = a \wedge b$ in Theorem 3.11, we get the result.

Corollary 3.13. *The fuzzy set μ is a T-fuzzy bi-ideal if and only if (i)$\mu \cdot_T \mu \cdot_T \mu \subseteq \mu$.*
(ii)$\mu \cdot_T 1 \cdot_T \mu \cdot_T 1 \cdot_T \mu \subseteq \mu$.

Proof. The proof follows by taking $L = [0,1]$ in Corollary 3.12.

Corollary 3.14. *The fuzzy set μ is a fuzzy bi-ideal if and only if (i) $\mu \cdot \mu \cdot \mu \subseteq \mu$. (ii)$\mu \cdot 1 \cdot \mu \cdot 1 \cdot \mu \subseteq \mu$.*

Proof. By taking $L = [0,1]$ and
$T(a,b) = min\{a,b\}$ in Theorem 3.11, we get the result.

4. Homomorphism and TL-bi-ideals

Definition 4.1. *The mapping $f : S \to S'$ where S and S' are ternary semigroups is called a homomorphism of S into S' if $f(abc) = f(a)f(b)f(c)$, for all $a,b,c \in S$.*

Definition 4.2. *The image of μ under the mapping $f : S \to S'$ denoted by $f(\mu)$ is the L-set on S' that is defined as follows:*

$$(f(\mu))(y) = \begin{cases} \bigvee\{\mu(x) | x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

for all $y \in S'$

Definition 4.3. *The pre-image of λ under the mapping $f : S \to S'$ denoted by $f^{-1}(\lambda)$ is a L-set on S that is defined as follows:*

$$(f^{-1}(\lambda))(x) = \lambda(f(x))$$

for all $x \in S$.

Theorem 4.4. *If f:$S \to S'$ is a homomorphism, and if μ is a TL- bi-ideal of S', then $f^{-1}(\mu)$ is a TL-bi-ideal of S.*

Proof. Let μ be a TL- bi-ideal of S'. Let $x,y,z \in S$. Now,

$$\begin{aligned}(f^{-1}(\mu))(xyz) &= \mu(f(xyz)) \\ &= \mu(f(x)f(y)f(z)) \\ &\geq T(\mu(f(x)), T(\mu(f(y)), \mu(f(z)))) \\ &= T((f^{-1}(\mu))(x), T((f^{-1}(\mu))(y), \\ &\qquad (f^{-1}(\mu))(z)))\end{aligned}$$

Thus
$(f^{-1}(\mu))(xyz) \geq T((f^{-1}(\mu))(x), T((f^{-1}(\mu))(y), (f^{-1}(\mu))(z)))$
, for all $x,y,z \in S$. Now,

$$\begin{aligned}(f^{-1}(\mu))(xwyvz) &= \mu(f(xwyvz)) \\ &= \mu(f(x)f(w)f(y)f(v)f(z)) \\ &\geq T(\mu(f(x)), T(\mu(f(y)), \mu(f(z)))) \\ &= T((f^{-1}(\mu))(x), T((f^{-1}(\mu))(y), \\ &\qquad (f^{-1}(\mu))(z)))\end{aligned}$$

Therefore
$(f^{-1}(\mu))(xwyvz) \geq T((f^{-1}(\mu)(x), T(f^{-1}(\mu), f^{-1}(\mu)(x)))$
forall$x,y,z,w,v \in S$. Therefore $f^{-1}(\mu)$ is a TL-bi-ideal of S.

Theorem 4.5. *If f is a homomorphism from S onto S', then μ is a TL-bi-ideal of S' if and only if $f^{-1}(\mu)$ is a TL-bi-ideal of S.*

Proof. Let μ be a TL-bi-ideal of S'. Then by Theroem 4.4 , $f^{-1}(\mu)$ is a TL-bi-ideal of S.
Conversely, let $x', y', z' \in S'$. Then there exist $x,y,z \in S$ such that
$f(x) = x', f(y) = y', f(z) = z'$. Now,

$$\begin{aligned}\mu(x'y'z') &= \mu(f(x)f(y)f(z)) \\ &= \mu(f(xyz)) \\ &= (f^{-1}(\mu))(xyz) \\ &\geq T((f^{-1}(\mu))(x), T((f^{-1}(\mu))(y), \\ &\qquad (f^{-1}(\mu))(z))) \\ &= T(\mu(f(x)), T(\mu(f(y)), \mu(f(z)))) \\ &= T(\mu(x'), T(\mu(y'), \mu(z')))\end{aligned}$$

Therefore $\mu(x^{'}y^{'}z^{'}) \geq T(\mu(x^{'}),T(\mu(y^{'}),\mu(z^{'})))$, for all $x^{'},y^{'},z^{'} \in S^{'}$.

$$\begin{aligned}\mu(x^{'}w^{'}y^{'}v^{'}z^{'}) &= \mu(f(x)f(w)f(y)f(v)f(z)) \\ &= \mu(f(xwyvz)) \\ &= (f^{-1}(\mu))(xwyvz) \\ &\geq T((f^{-1}(\mu))(x),T((f^{-1}(\mu))(y), \\ &\qquad (f^{-1}(\mu))(z))) \\ &= T(\mu(f(x)),T(\mu(f(y)),\mu(f(z)))) \\ &= T(\mu(x^{'}),T(\mu(y^{'}),\mu(z^{'})))\end{aligned}$$

Therefore $\mu(x^{'}w^{'}y^{'}v^{'}z^{'}) \geq T(\mu(x^{'}),T(\mu(y^{'}),\mu(z^{'})))$,for all $x^{'},w^{'},y^{'},v^{'},z^{'} \in S^{'}$. Hence μ is a TL- bi-ideal of $S^{'}$.

Theorem 4.6. *If f is a homomorphism from S onto $S^{'}$ and μ is a TL- bi-ideal of S, then $f(\mu)$ is a TL-bi-ideal of $S^{'}$.*

Proof. Let μ be a TL-bi-ideal of S. For $x^{'},y^{'},z^{'} \in S^{'}$, there exist $x,y,z \in S$
such that $f(x) = x^{'}, f(y) = y^{'}, f(z) = z^{'}$. Now,

$$\begin{aligned}(f(\mu))(x^{'}y^{'}z^{'}) &= \bigvee\{\mu(xyz)|f(xyz) = x^{'}y^{'}z^{'}\} \\ &= \bigvee\{\mu(xyz)|f(x)f(y)f(z) = x^{'}y^{'}z^{'}\} \\ &\geq \bigvee\{T(\mu(x),T(\mu(y),\mu(z)))|f(x) = x^{'}, \\ &\qquad f(y) = y^{'}, f(z) = z^{'}\} \\ &= T(\bigvee\{\mu(x)|f(x) = x^{'}\},T(\bigvee\{\mu(y)|f(y) = y^{'},\} \\ &\qquad \bigvee\{\mu(z)|f(z) = z^{'}\})) \\ &= T(f(\mu)(x^{'}),T(f(\mu)(y^{'}),f(\mu)(z^{'})))\end{aligned}$$

Thus$(f(\mu))(x^{'}y^{'}z^{'}) \geq$
$T((f(\mu))(x^{'}),T((f(\mu))(y^{'}),(f(\mu))(z^{'})))$,
for all $x^{'},y^{'},z^{'} \in S^{'}$. Now,

$$\begin{aligned}(f(\mu))(x^{'}w^{'}y^{'}v^{'}z^{'}) &= \bigvee\{\mu(xwyvz)|f(xwyvz) = x^{'}w^{'}y^{'}v^{'}z^{'}\} \\ &= \bigvee\{\mu(xwyvz)|f(x)f(w) \\ &\qquad f(y)f(v)f(z) = x^{'}w^{'}y^{'}v^{'}z^{'}\} \\ &\geq \bigvee\{T(\mu(x),T(\mu(y),\mu(z)))|f(x) = x^{'}, \\ &\qquad f(y) = y^{'}, f(z) = z^{'}\} \\ &= T(\bigvee\{\mu(x)|f(x) = x^{'}\},T(\bigvee\{\mu(y)| \\ &\qquad f(y) = y^{'}\},\bigvee\{\mu(z)|f(z) = z^{'}\})) \\ &= T((f(\mu))(x^{'}),T((f(\mu))(y^{'}),(f(\mu))(z^{'})))\end{aligned}$$

Therefore$(f(\mu))(x^{'}w^{'}y^{'}v^{'}z^{'}) \geq$
$T((f(\mu))(x^{'}),T((f(\mu))(y^{'}),(f(\mu))(z^{'})))$, for all $x^{'}w^{'}y^{'}v^{'}z^{'} \in S$. Hence $f(\mu)$ is a TL-bi-ideal of $S^{'}$.

5. Example

Remark 5.1. *Converse of the above theorem need not be true by the following example.*

Example 5.2. *Let $\mathbb{Z}^{-}$ be the ternary semigroup of negative integers and $\mathbb{Z}_6$ be a ternary semigroup of integer modulo 6 under multiplication. The mapping $f : \mathbb{Z} \to \mathbb{Z}_6$, defined by $f(x) = x(mod 6)$. Clearly f is a homomorphism. By taking $L = [0,1]$, the L-sets μ on $\mathbb{Z}^{-}$ is defined as follows:*

$$\mu(x) = \begin{cases} 0.8 & if\ x = -12 \\ 0.3 & if\ x = -3 \\ 0.2 & otherwise. \end{cases}$$

Then ,

$$(f(\mu))(x) = \begin{cases} 0.8 & if\ x = 0 \\ 0.3 & if\ x = 3 \\ 0.2 & otherwise. \end{cases}$$

By taking T-norm as minimum norm , $f(\mu)$ is a TL-bi-ideal of $\mathbb{Z}_6$.

$$\begin{aligned}\mu(-27) &= \mu((-3).(-3).(-3)) = 0.2 \\ T(\mu(-3),T(\mu(-3),\mu(-3))) &= min\{0.3,0.3,0.3\} = 0.3 \\ \mu((-3).(-3).(-3)) = 0.2 &\ngeq 0.3 \\ &= T(\mu(-3),T(\mu(-3),\mu(-3)))\end{aligned}$$

Then μ is not a TL-bi-ideal $\mathbb{Z}^{-}$,
however $f(\mu)$ is a TL-bi-ideal on $\mathbb{Z}_6$.

T-intersection of TL-bi-ideals

Definition 5.3. *If μ and λ are two L- sets of S. Then T-intersection of μ and λ denoted by $T(\mu,\lambda)$ is defined as follows:*
$T(\mu,\lambda)(x) = T(\mu(x),\lambda(x))$ for all $x \in S$.

Theorem 5.4. *If μ and λ are TL-ternary subsemigroups of S, then $T(\mu,\lambda)$ is a TL-ternary subsemigroup of S.*

Proof. Let μ and λ be the TL-ternary subsemigroups of S .

$$\begin{aligned}T(\mu,\lambda)(xyz) &= T(\mu(xyz),\lambda(xyz)) \\ &\geq T(T[\mu(x),T(\mu(y),\mu(z))],T[\lambda(x),T(\lambda(y),\lambda(z))]) \\ &= T(\mu(x),T[T(\mu(y),\mu(z)),T[\lambda(x),T(\lambda(y),\lambda(z))]]) \\ &= T(\mu(x),T[T[\lambda(x),T(\lambda(y),\lambda(z))],T(\mu(y),\mu(z))]) \\ &= T(T[T(\mu(x),\lambda(x)),T(\lambda(y),\lambda(z))],T(\mu(y),\mu(z))) \\ &= T(T(\mu(x),\lambda(x),T[T(\lambda(y),\lambda(z)),T(\mu(y),\mu(z))]) \\ &= T(T(\mu(x),\lambda(x),T[T(\mu(y),\mu(z)),T(\lambda(y),\lambda(z))]) \\ &= T(T(\mu(x),\lambda(x),T[\mu(y),T(\mu(z),T(\lambda(y),\lambda(z)))]) \\ &= T(T(\mu(x),\lambda(x),T[\mu(y),T(T(\lambda(y),\lambda(z)),\mu(z))]) \\ &= T(T(\mu(x),\lambda(x),T[\mu(y),T(\lambda(y),T(\lambda(z),\mu(z)))]) \\ &= T(T(\mu(x),\lambda(x),T[T(\mu(y),\lambda(y)),T(\mu(z),\lambda(z))]) \\ &= T(T(\mu(x),\lambda(x),T[T(\mu(y),\lambda(y),T(\mu(z),\lambda(z)])\end{aligned}$$

Thus$T(\mu,\lambda)(xyz) \geq$
$T(T(\mu,\lambda)(x),T(T(\mu,\lambda)(y),T(\mu,\lambda)(z)))$,
for all $x,y,z \in S$. Hence $T(\mu,\lambda)$ is a TL-ternary subsemigroup of S.

Corollary 5.5. *If μ and λ are L-ternary subsemigroups of S, then $\mu \wedge \lambda$ is a L-ternary subsemigroup.*

Proof. By taking $T(a,b) = a \wedge b$ in Theorem 5.4, we get the result.

Corollary 5.6. *If μ and λ are T-fuzzy ternary subsemigroups of S, then $\mu \cap \lambda$ is a T-fuzzy ternary subsemigroup.*

Proof. The proof follows by taking $L = [0,1]$ in Theorem 5.4.

Corollary 5.7. *If μ and λ are fuzzy ternary subsemigroups of S, then $\mu \cap \lambda$ is a fuzzy ternary subsemigroup.*

Proof. By taking $L = [0,1]$ and $T(a,b) = min\{a,b\}$ in Theorem 5.4, we get result.

Theorem 5.8. *If μ and λ are TL-bi-ideals of S, then $T(\mu,\lambda)$ is the TL-bi-ideal of S.*

Proof. Let μ and λ be TL-bi-ideals of S.

$$\begin{aligned} T(\mu,\lambda)(xwyvz) &= T(\mu(xwyvz),\lambda(xwyvz)) \\ &\geq T(T[\mu(x),T(\mu(y),\mu(z))],T[\lambda(x),T(\lambda(y),\lambda(z))]) \\ &= T(\mu(x),T[T(\mu(y),\mu(z)),T[\lambda(x),T(\lambda(y),\lambda(z))]]) \\ &= T(\mu(x),T[T[\lambda(x),T(\lambda(y),\lambda(z)),T(\mu(y),\mu(z))]]) \\ &= T(T[T(\mu(x),\lambda(x)),T(\lambda(y),\lambda(z))],T(\mu(y),\mu(z))) \\ &= T(\mu(x),\lambda(x),T[T(\lambda(y),\lambda(z)),T(\mu(y),\mu(z))]) \\ &= T(\mu(x),\lambda(x),T[T(\mu(y),\mu(z)),T(\lambda(y),\lambda(z))]) \\ &= T(\mu(x),\lambda(x),T[\mu(y),T(\mu(z),T(\lambda(y),\lambda(z)))]) \\ &= T(\mu(x),\lambda(x),T[\mu(y),T(T(\lambda(y),\lambda(z)),\mu(z))]) \\ &= T(\mu(x),\lambda(x),T[\mu(y),T(\lambda(y),T(\lambda(z),\mu(z)))]) \\ &= T(\mu(x),\lambda(x),T[T(\mu(y),\lambda(y)),T(\mu(z),\lambda(z))]) \end{aligned}$$

Therefore

$$T(\mu,\lambda)(xwyvz) \geq T(T(\mu,\lambda)(x),T(T(\mu,\lambda)(y),T(\mu,\lambda)(z))),$$

for all $x,w,y,v,z \in S$. Hence $T(\mu,\lambda)$ is a TL-ternary bi-ideal of S.

Corollary 5.9. *If μ and λ are L-ternary bi-ideals of S, then $\mu \wedge \lambda$ is a L-ternary bi-ideal.*

Proof. By taking $T(a,b) = a \wedge b$ in Theorem 5.8, we get the result

Corollary 5.10. *If μ and λ are T-fuzzy bi-ideals of S, then $\mu \cap \lambda$ is a T-fuzzy bi-ideal.*

Proof. The proof follows by taking $L = [0,1]$ in Theorem 5.8.

Corollary 5.11. *If μ and λ are fuzzy bi-ideals of S, then $\mu \cap \lambda$ is a fuzzy bi-ideal.*

Proof. By taking $L = [0,1]$ and $T(a,b) = min\{a,b\}$ in Theorem 5.8, we get result.

Definition 5.12. *For a L-set λ of S and $r,s,t \in L$, we define TL-level set of λ, denoted by $T(\lambda : (r,(s,t))$ is defined as follows:*

$$T(\lambda : (r,(s,t)) = \{x \in S | \lambda(x) \geq T(r,T(s,t))\}$$

.

Theorem 5.13. *If μ is a L-set of S and $T(\mu : (r,(s,t)))$ is a bi-ideal of S, for all $r,s,t \in Im\mu$, then μ is a TL-bi-ideal of S.*

Proof. If there exist $x,y,z \in S$ such that $\mu(xyz) < T(\mu(x),T(\mu(y),\mu(z)))$, then choose $r = \mu(x), s = \mu(y)$ and $t = \mu(z)$. Thus $xyz \notin T(\mu : (r,(s,t)))$. Now,

$$\begin{aligned} \mu(x) &= r \\ &= T(r,1) \\ &= T(r,T(1,1)) \\ &\geq T(r,T(s,t)) \end{aligned}$$

Thus $x \in T(\mu : (r,(s,t)))$. Similarly $y,z \in T(\mu : (r,(s,t)))$. Then $x,y,z \in T(\mu : (r,(s,t)))$,but $xyz \notin T(\mu : (r,(s,t)))$, which is a contradiction. If there exist $x,w,y,v,z \in S$ such that $\mu(xwyvz) < T(\mu(x),T(\mu(y),\mu(z)))$ then take $r = \mu(x), s = \mu(y)$ and $t = \mu(z)$.But $xwyvz \notin T(\mu : (r,(s,t)))$, then $x,y,z \in T(\mu : (r,(s,t)))$, which is a contradiction . Thus μ is a TL-bi- ideal of S.

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