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On nano πg^*s-closed sets in nano topological spaces

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Abstract

In this paper, a new class of set called nano π generalized star semi-closed sets in nano topological spaces is introduced and some of their basic properties are investigated. We shows that a new class of sets lies between the class of Nπg-closed sets and the class of Nπgs-closed sets. Further the notion of Nπg^*s-open sets, Nπg^*s-neighbourhoods, Nπg^*s-interior and Nπg^*s-closure are discussed. Several examples are also provided to illustrate the behaviour of new sets and functions.

Keywords

Nπg^*s-closed sets, Nπg^*s-open sets, Nπg^*s-neighbourhoods, Nπg^*s-interior, Nπg^*s-closure.

AMS Subject Classification

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Contents

1. Introduction

Levine [4] Introduced the class of g-closed sets, a super class of closed sets in 1970. This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology. Recently Lellis Thivagar introduced nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximation of X. The elements of nano topological space are called nano open sets. He has also defined nano closed sets, nano interior and nano closure of a set. He also introduced the weak forms of nano open sets. Bhuvaneswari [2] introduced nano g-closed sets and obtained some of the basic interesting results. Sathishmohan [9] et.al., studied the concept of θg^*-closed sets in topological spaces and investigate the composition of the functions between θg^*-continuous functions and continuous functions. In this paper, we define a new class of sets called nano π generalized star semi-closed and its open sets in nano topological spaces and study the relationships with other nano sets.

2. Preliminaries

Definition 2.1. *[5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X⊆U*

- *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = U_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(X) denotes the equivalence class determined by $X \in U$.*

- *The upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = U_{x\in U}\{R(x) : R(x) \cap X \neq \varphi\}$*

- *The boundary of the region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$*
  That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. *[5] If (U,R) is an approximation space and X,Y ⊆U, then*

- $L_R(X) \subseteq X \subseteq U_R(X)$.
- $L_R(\phi) = U_R(\phi) = \phi$
- $L_R(U) = U_R(U) = U$.
- $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- $U_R(X \cup Y) \supseteq U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) = U_R(X) \cap U_R(Y)$
- $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$
- $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$
- $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
- $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3. *[5] Let U be the Universe and R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X⊆U. $\tau_R(X)$ satisfies the following axioms:*

- *U and $\phi \in \tau_R(X)$.*
- *The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*
- *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

We call (U, $\tau_R(X)$) is a nano topological space. The elements of $\tau_R(X)$ are called a nano open sets and the complement of a nano open set is called nano closed sets.
Throughout this paper (U, $\tau_R(X)$) is a nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R.

Remark 2.4. *[5] If $\tau_R(X)$ is the nano topology on U with respect to X. The set*
B=$\{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5. *[5] If (U, $\tau_R(X)$) is a nano topological space with respect to X. Where $X \subseteq G$ and if $A \subseteq G$, then*

- *The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint(A), Nint(A) is the largest nano open subset of A.*
- *The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the smallest nano closed set containing A.*

Definition 2.6. *[10] A nano-subset of a nano topological spaces U is called nano-dense if Ncl(A)=U.*

Definition 2.7. *[12] A nano topological space (U, $\tau_R(X)$) is said to be nano extremally disconnected, if the nano-closure of each nano-open set is nano-open.*

Definition 2.8. *[10] A space U is called nano-submaximal if each nano-dense subset of U is nano-open.*

Definition 2.9. *Let (U, $\tau_R(X)$) be a nano topological space and $A \subseteq G$. Then A is said to be*

- *Nr-closed [5] if A = Ncl(Nint(A).*
- *Nα-closed [5] if Nint(Ncl(Nint(A))) $\subseteq$ A.*
- *Np-closed [5] if Nint(Ncl(A)) $\subseteq$ A.*
- *Ns-closed [2] if Ncl(Nint(A) $\subseteq$ A.*
- *Nβ-closed [2] If Ncl(Nint(Ncl(A))) $\subseteq$ A.*
- *Nπ-closed [1] if intersection of nano r-closed.*
- *Ng-closed [2] if Ncl(A)$\subset$G whenever A$\subset$G and G is nano-open in U.*
- *Ng^*-closed [5] if Ncl(A)$\subset$G whenever A$\subset$G and G is nano g-open in U.*
- *Ng^*s-closed [4] if Nscl(A)$\subset$G whenever A$\subset$G and G is nano g-open in U.*

- *$N\pi g$-closed [6] if Ncl(A)⊆G, whenever A⊆G and G is $N\pi$-open.*
- *$N\alpha g$-closed [11] if $N\alpha cl$(A)⊆G, whenever A⊆G and G is nano-open.*
- *Nsg-closed [2] if Nscl(A)⊆G, whenever A⊆G and G is Ns-open.*
- *$N\pi g\beta$-closed [8] if $N\beta cl$(A)⊆G, whenever A⊆G and G is $N\pi$-open.*
- *$N\pi gs$-closed [7] if Nscl(A)⊆G, whenever A⊆G and G is $N\pi$-open.*
- *Ngs-closed [3] if Nscl(A)⊆G, whenever A⊆G and G is nano-open.*

3. $N\pi g^*$s-closed sets

Definition 3.1. *A subset A of a nano topological space (U, $\tau_R(X)$) is called nano π generalized star semi-closed (briefly $N\pi g^*$s-closed) set if Nscl(A)⊆G whenever A⊆G and G is $N\pi g$-open in (U,$\tau_R(X)$).*

Theorem 3.2. *Every Nr-closed set is $N\pi g^*$s-closed.*
Proof: *Let A be a Nr-closed set in U. Let G be a $N\pi g$-open set such that A⊆G. Since A is Nr-closed we have Nrcl(A)=A⊆G. But, Nscl(A)⊆ $Nrcl(A)$ ⊆G. Therefore Nscl(A)⊆G. Hence A is a $N\pi g^*$s-closed set in X.*

Remark 3.3. *The converse of the above theorem need not be true as seen in the following example.*

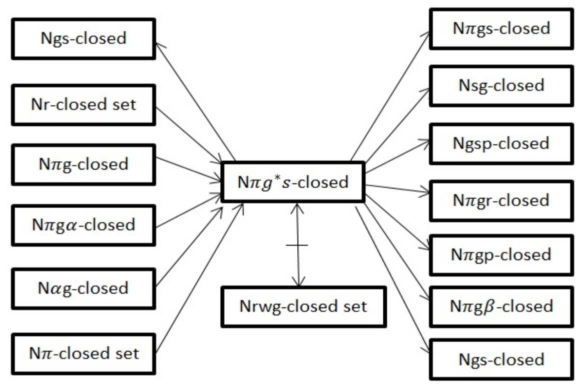
Example 3.4. *Let U=$\{a,b,c,d\}$ with U/R=$\{\{a\},\{b,c,d\}\}$. Let X=$\{a,c\}$⊆U and τ_R(X) = $\{\phi$, U,$\{a\}$,$\{b,c,d\}\}$, nano r-closed set = $\{\phi,U,\{b,c,d\},\{a\}\}$ and nano πg^*s-closed set = $\{\phi,U,\{a\},\{b,c\},\{b,d\},\{c,d\},\{b,c,d\}\}$. Let A= $\{b,d\}$. Thus the subset A is $N\pi g^*$s-closed but not Nr-closed set.*

Theorem 3.5. *Every $N\pi g^*$s-closed set is $N\pi gs$-closed.*
Proof: *Let A be any $N\pi g^*$s-closed set in U. Let A⊆G and G be $N\pi g$-open in U. Every nano open is $N\pi$-open and $N\pi g$-open and since A is $N\pi g^*$s-closed, we have Nscl(A)⊆G. Therefore A is $N\pi gs$-closed.*

Remark 3.6. *The converse of the above theorem need not be true as seen in the following example.*

Example 3.7. *Let U=$\{a,b,c,d\}$ with U/R=$\{\{a,c\},\{b\},\{d\}\}$. Let X=$\{a,b\}$⊆U and τ_R(X)=$\{\phi$,U,$\{b\}$,$\{a,b,c\}$,$\{a,c\}\}$, nano πgs-closed set = $\{\phi,U,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ and nano πg^*s-closed set = $\{\phi,U,\{a\},\{d\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Let A= $\{b\}$. Thus the subset A is $N\pi gs$-closed but not a $N\pi g^*$s-closed set.*

Remark 3.8. *The following diagram shows the relationship of $N\pi g^*$s-closed set with other known exixting sets.*



A⟶ B represents A implies B but not conversely.

Example 3.9. *Let U=$\{a,b,c,d\}$ with U/R = $\{\{a,b\},\{c\},\{d\}\}$. Let X = $\{a,d\}$⊆U and $\tau_R(X)$ = $\{\phi,U,\{d\},\{a,b,d\},\{a,b\}\}$. Then $N\pi g^*$s-closed=$\{\phi,U,\{c\},\{d\},\{a,b\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$, Nrwg-closed=$\{\phi,U,\{a,d\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$, $N\pi$-closed=$\{\phi,U,\{c\}\}$, $N\alpha g$-closed, $N\pi g$-closed and $N\pi g\alpha$-closed set=$\{\phi,U,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$. Let A=$\{d\}$. Then the subset A is $N\pi g^*$s-closed set but not $N\pi$-closed, $N\alpha g$-closed, $N\pi g$-closed, $N\pi g\alpha$-closed set*

Example 3.10. *Let U=$\{a,b,c,d\}$ with U/R=$\{\{c\},\{a,b,d\}\}$. Let X=$\{c,d\}$⊆U and $\tau_R(X)$ = $\{\phi,U,\{c\},\{a,b,d\}\}$. Nsg-closed, Ngsp-closed, $N\pi gr$-closed, $N\pi gp$-closed, $N\pi g\beta$-closed and Ngs-closed sets = $\{\phi,U,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ and Nano πg^*s-closed set = $\{\phi,U,\{c\},\{a,b,d\},\}$. Let A= $\{b\}$. Thus the subset A is Nsg-closed, Ngp-closed, Ngsp-closed, $N\pi gr$-closed, $N\pi gp$-closed, $N\pi g\beta$-closed, Ngs-closed sets but not a $N\pi g^*$s-closed set.*

Example 3.11. *Let U=$\{a,b,c,d\}$ with U/R=$\{\{a\},\{b,c,d\}\}$. Let X=$\{b,c\}$⊆U and τ_R(X)=$\{\phi$,U,$\{b,c,d\}\}$. $N\pi g^*$s-closed set=$\{\phi$,U,$\{a\}$,$\{b\}\}$ and Nrwg-closed set=$\{\phi$,U,$\{b,c,d\}\}$. Let A=$\{a\}$ and B=$\{b,c,d\}$. The subset A is $N\pi g^*$s-closed set but not in Nrwg-closed set. The subset B is in Nrwg-closed set but not in $N\pi g^*$s-closed set. Therefore Nrwg-closed sets and $N\pi g^*$s-closed sets are independent of each other.*

Theorem 3.12. *Union of any two $N\pi g^*$s-closed subset is $N\pi g^*$s-closed.*
Proof: *Let A and B be any two $N\pi g^*$s-closed sets in U, such that A⊆G and B⊆G where G is $N\pi g$-open in U and so A∪B ⊆G. Since A and B are $N\pi g^*$s-closed, we have A⊆Nscl(A)*

and B⊆Nscl(A) and hence A∪B ⊆ $Nscl(A) \cup Nscl(B) \subseteq Nscl(A \cup B)$. Thus, A∪B is $N\pi g^*$s-closed set in (U,$\tau_R(X)$).

Example 3.13. *Let U={a,b,c,d} with U/R = {{a},{b},{c,d}}. Let X = {a,c}⊆U and $\tau_R(X)$ = {ϕ,U,{a},{a,c,d},{c,d}}. Then $N\pi g^*$s-closed={ϕ,U,{a},{b},{a,b},{b,c,d}}. Let A={a} is πg^*s-closed set and B={b} is πg^*s-closed set, then A∪B ={a}∪{b}={a,b} is also a $N\pi g^*$s-closed set.*

Theorem 3.14. *Intersection of any two $N\pi g^*$s-closed subset is $N\pi g^*$s-closed.*
Proof: *Let A and B be any two $N\pi g^*$s-closed sets in U, such that A⊆G and B⊆G where G is Nπg-open in U and so A∩B ⊆G. Since A and B are $N\pi g^*$s-closed, we have A⊆Nscl(A) and B⊆Nscl(A) and hence A∩B ⊆ $Nscl(A) \cap Nscl(B) \subseteq Nscl(A \cap B)$. Thus, A∩B is a $N\pi g^*$s-closed set in (U,$\tau_R(X)$).*

Example 3.15. *Let U={a,b,c,d} with U/R = {{a},{b},{c,d}}. Let X = {a,c}⊆U and $\tau_R(X)$ = {ϕ,U,{a},{a,c,d},{c,d}}. Then $N\pi g^*$s-closed={ϕ,U,{a},{b},{a,b},{b,c,d}}. Let A={a,b} is a πg^*s-closed set and B={b,c,d} is a πg^*s-closed set, then A∩B ={a,b}∩{b,c,d}*
={b} is also a $N\pi g^*$s-closed set.

Theorem 3.16. *A subset A of U is $N\pi g^*$s-closed iff Nscl(A)-A contains no non-empty nano closed set.*
Proof: *Let A be a $N\pi g^*$s-closed set. Suppose F is a non-empty nano closed set such that F⊆Nscl(A)-A. Then F⊆Nscl(A) ∩ A^c, since Nscl(A)-A=Nscl(A)∩A^c. Therefore F⊆scl(A) and F⊆ A^c. Since F^c is nano open, it is Nπg-open. Now, by the definition of $N\pi g^*$s-closed set, Nscl(A)⊆ F^c, That is F⊆ $[Nscl(A)]^c$. Hence F⊆ $Nscl(A) \cap [Nscl(A)]^c = \phi$. That is F=ϕ, which is a contradiction. Thus, Nscl(A)-A contains no non-empty nano closed set in U. Conversely, assume that Nscl(A)-A contains no non-empty nano closed set. Let A⊆U, where G is πg-open. Suppose that Nscl(A) is not contained in U, then Nscl(A)∩U^c is a non-empty nano closed subset of Nscl(A)-A, which is a contradiction. Therefore Nscl(A)⊆U and hence A is $N\pi g^*$s-closed.*

Theorem 3.17. *If A is a $N\pi g^*$s-closed subset of U such that A⊆ B ⊆Nscl(A), then B is a $N\pi g^*$s-closed set in U.*
Proof: *Let A be a $N\pi g^*$s-closed set of U such that A⊆ B ⊆Nscl(A). Let G be a Nπg-open set of U such that B⊆G, then A⊆G. Since A is $N\pi g^*$s-closed. We have Nscl(A)⊆G. Now Nscl(B)⊆ $Nscl(Nscl(A)) = Nscl(A)$ ⊆G. Therefore B is a $N\pi g^*$s-closed set in U.*

Theorem 3.18. *For each {a} ∈ U, either {a} is nano closed set (or) $\{a\}^c$ is nano generalized closed in $\tau_R(X)$.*
Proof: *Suppose {a} is not nano closed in U. Then $\{a\}^c$ is not nano open and the only nano open set containing $\{a\}^c$ is G ⊆ U. That is $\{a\}^c$ ⊆ U. Therefore Nscl($\{a\}^c$)⊆U which implies $\{a^c\}$ is nano πg^*s-closed in $\tau_R(X)$.*

Corollary 3.19. *Let A be a $N\pi g^*$s-closed set and suppose that F is a nano closed set. Then A∪F is a $N\pi g^*$s-closed set which is given in the following example.*

Example 3.20. *Let U={a,b,c,d} with U/R = {{a,c},{b},{d}}. Let X = {a,b}⊆U and $\tau_R(X)^c$ = {ϕ,U,{a,c,d},{d},{b,d}}. Then $N\pi g^*$s-closed={ϕ,U,{b},{d},{a,b},{a,c},{a,d}, {b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}. Let A={a,c,d} and F= {b,c,d}. Then A∩F ={c,d} is a $N\pi g^*$s-closed set.*

Definition 3.21. *A subset A of a nano topological space (U, $\tau_R(X)$) is called nano π generalized star semi-open (briefly nano πg^*s-open) if A^c is nano πg^*s-closed.*

Theorem 3.22. *A subset A ⊆ U is $N\pi g^*$s-open iff F ⊆ Nint(A) whenever F is a Nπg-closed set and F ⊆ A.*
Proof: *Let A be nano πg^*s-open suppose F⊆int(A) where F is Nπg-closed and F⊆A. Let A^c⊆G where G=F^c is Nπg-open. Then G^c⊆A and G^c⊆Nint(A). Then we have A^c is $N\pi g^*$s-closed. Hence A is $N\pi g^*$s-open.*
Conversely, If A is $N\pi g^*$s-open, F⊆A and F is Nπg-closed. Then F^c is Nπg-open and A^c⊆F. Therefore Ncl(A^c)⊆(F^c). But Ncl(A^c)=$(NInt(A))^c$. Hence F⊆NInt(A).

Theorem 3.23. *If NInt(A) ⊆ B ⊆ A and if A is nano πg^*s-open, then B is nano πg^*s-open.*
Proof: *Let NInt(A) ⊆ B ⊆ A, then A^c is nano πg^*s-closed and hence B^c is also nano πg^*s-closed by the above Theorem 3.22. Therefore B is nano πg^*s-open.*

4. Nano πg^*s-interior and πg^*s-closure

Definition 4.1. *Let (U, $\tau_R(X)$) be a nano toplogical space and A ⊆ U then nano πg^*s-interior is defined as*
$N\pi g^*$s-int(A) = ∪ {B:B is nano πg^*s-open, B ⊂ A}
Clearly $N\pi g^*$s-int(A) is the largest nano πg^*s-open set over U which is contained in A.

Definition 4.2. *Let (U, $\tau_R(X)$) be a nano toplogical space and A ⊆ U then nano πg^*s-closure is defined as*
$N\pi g^*$s cl(A) = ∩ {F:F is nano πg^*s-closed, A ⊂ F}
Clearly $N\pi g^*$s-cl(A) is the smallest nano πg^*s-closed set over U which contains A.

Lemma 4.3. *Let A and B be any two subsets of U in a nano topological spaces (U, $\tau_R(X)$) and the following are true*
(i) $N\pi g^*$s-int(A)⊂A
(ii) A⊂B ⇒ $N\pi g^*$s-int(B) ⊂ $N\pi g^*$s-int(B)
(iii) $N\pi g^*$s-int(A) ∪ $N\pi g^*$s-int(B) ⊂ $N\pi g^*$s-int(A∪B)
(iv) $N\pi g^*$s-int(A)∩$N\pi g^*$s-int(B)⊂$N\pi g^*$s-int(A∩B)

Lemma 4.4. *For a subset A of U.*
(i)$N\pi g^*$s-cl(A)⊂Ncl(A)
(ii)N-int(A)⊂$N\pi g^*$s-cl(A)

Lemma 4.5. *A subset A of U is nano π generalized star semi-closed if and only if A = $N\pi g^*$s-cl(A).*

Theorem 4.6. *Let A and B be two subsets of nano topological space (U, $\tau_R(X)$). Then*
(i) $N\pi g^*$s-int(U) = U and $N\pi g^*$s-int(ϕ) = ϕ
(ii) $N\pi g^*$s-int(A) ⊂ A
(iii) If B is any $N\pi g^*$s-open set contained in A, then B ⊂

$N\pi g^*s$-int(A)
(iv) If $A \subset B$, then $N\pi g^*s$-int(A) $\subset$ $N\pi g^*s$-int(B)
(v) $N\pi g^*s$-int($N\pi g^*s$-int(A)) = $N\pi g^*s$-int(A).
Proof: *(i) Let U and ϕ are $N\pi g^*s$-open sets $N\pi g^*s$-int(U) = $\cup$ {B:B is a $N\pi g^*s$-open, $B\subset U$} = U $\cup$ all $N\pi g^*s$-open sets = U.*
(ie) int(U) = U. Since ϕ is the only $N\pi g^*s$-open set contained in ϕ, $N\pi g^*s$-int(ϕ) = ϕ.
(ii) Let $x \in N\pi g^*s$-int(A) $\Rightarrow$ x is a interior point of A.
$\Rightarrow$ A is a nbhd of x.
$\Rightarrow$ $x\in A$.
Thus, $x\in N\pi g^*s$-int(A)$\Rightarrow$ $x\in A$.
Hence $N\pi g^*s$-int(A) $\subset$ A.
(iii) Let B be any $N\pi g^*s$-open sets such that $B \subset A$. Let $x \in B$. Since B is a $N\pi g^*s$-open set contained in A. x is a $N\pi g^*s$-interior point of A.
(ie) $x \in N\pi g^*s$-int(A). Hence $B \subset N\pi g^*s$-int(A).
(iv) Let A and B be subsets of (U, $\tau_R(X)$) such that $A \subset B$. Let $x \in N\pi g^*s$-int(A). Then x is a $N\pi g^*s$-interior point of A and so A is a $N\pi g^*s$-nbhd of x. Since $B \supset A$, B is also $N\pi g^*s$-nbhd of x.
$\Rightarrow$ $x \in N\pi g^*s$-int(B). thus we have shown that $x\in N\pi g^*s$-int(B).

Theorem 4.7. *If a subset A of space $(U,\tau_R(X))$ is $N\pi g^*s$-open, then $N\pi g^*s$-int(A)=A.*
Proof: *Let A be $N\pi g^*s$-open subset of (U, $\tau_R(X)$). We know that $N\pi g^*s$-int(A) $\subset$ A. Also, A is $N\pi g^*s$-open set contained in A. From above Theorem 4.6(iii) $A\subset N\pi g^*s$-int(A) = A.*
The converse of the above theorem need not be true, as seen from the following example.

Example 4.8. *Let U= {a,b,c,d} with U/R={{a,b},{c},{d}}. Let X = {a,c} $\subseteq$ U. Then $\tau_R(X)$= {U, ϕ, {c}, {a,b,c}, {a,b}}. Then nano πg^*s-open subsets are {ϕ, U, {c},{a,b}, {a,c},{b,c},{c,d},{a,b,d},{a,c,d},{b,c,d}} $N\pi g^*s$-int({a,b,c}) = {c}$\cup${a,b}$\cup${b,c}$\cup${ϕ} = {a,b,c}. But {a,b,c} is not $N\pi g^*s$-open set in U.*

Theorem 4.9. *If A and B are subsets of (U, $\tau_R(X)$), then $N\pi g^*s$-int($A\cap B$) = $N\pi g^*s$-int(A)$\cap N\pi g^*s$-int(B).*
Proof: *We know that $A\cap B \subset B$. we have $N\pi g^*s$-int($A\cap B$) $\subset N\pi g^*s$-int(A) and $N\pi g^*s$-int($A\cap B$) $\subset$ $N\pi g^*s$-int(B). This implies that $N\pi g^*s$-int($A\cap B$) $\subset$ $N\pi g^*s$-int(A)$\cap N\pi g^*s$-int(B). Again let $x\in N\pi g^*s$-int(B). Then $x\in N\pi g^*s$-int(A) and $x\in N\pi g^*s$-int(B). Hence x is a $N\pi g^*s$-int point of each of sets A and B. It follows that A and B is $N\pi g^*s$-nbhd of x, so that their intersection $A\cap B$ is also a $N\pi g^*s$-nbhds of x. Hence $x\in N\pi g^*s$-int($A\cap B$). Thus $x\in N\pi g^*s$-int(A). $N\pi g^*s$-int(B) implies that $x\in N\pi g^*s$-int($A\cap B$). Therefore $N\pi g^*s$-int(A)$\cap N\pi g^*s$-int(B) $\subset$ $N\pi g^*s$-int($A\cap B$). From the above, We get $N\pi g^*s$-int($A\cap B$) = $N\pi g^*s$-int(A)$\cap N\pi g^*s$-int(B).*

Theorem 4.10. *If A is a subset of U, then Nint(A) $\subset$ $N\pi g^*s$-int(A).*
Proof: *Let $x\in$Nint(A) $\Rightarrow$ $x\cup${B:B is nano open, $B\subset A$}.*
$\Rightarrow$there exists an nano open set B such that $x\in B\subset A$.
$\Rightarrow$ there exists a $N\pi g^*s$-open set B such that $x\in B\subset A$, as every nano open set is a $N\pi g^*s$-open set in U.
$\Rightarrow x\in \cup${B:B is $N\pi g^*s$-open $B\subset A$}
$\Rightarrow x\in N\pi g^*s$-int(A).
Thus $x\in$Nint(A)$\Rightarrow x\in N\pi g^*s$-int(A).
Hence Nint(A)$\subset N\pi g^*s$-int(A).

Example 4.11. *Let U = {a,b,c,d} with U/R = {ϕ, U, {a,d},{b}, {c}}. Let X = {a,b}$\subseteq$U. Then $\tau_R(X)$ = {ϕ, U, {b}, {a,b,d}, {a,d}}. nano πg^*s-open subsets are {ϕ,U,{b},{a,d},{b,c}, {a,b,d},{a,c,d}} nano π-open subsets are {ϕ,U,{a,d}}.*
Let A = {a,c,d} $N\pi g^*s$-open = {a,c,d} and $N\pi$-open = {a,d}
It follows $N\pi$-int(A)$\subset N\pi g^*s$-int(A) and $N\pi$-int(A)$\neq N\pi g^*s$-int(A).

Theorem 4.12. *Let A be a subset of a nano topological space (U, $\tau_R(X)$). Then*
(i) $(N\pi g^*s\text{-cl}(A))^c$ = $N\pi g^*s$-int(A^c)
(ii) $(N\pi g^*s\text{-int}(A))^c$ = $N\pi g^*s$-cl(A^c)
Proof: *(i) $(N\pi g^*s\ \text{cl}(A))^c$ = $(\cap$ {F:F is $N\pi g^*s$ closed, $A\subset F\})^c$*
= $\cup$ {F^c:F^c is $N\pi g^*s$ open, $F^c \subset A^c$}
= $N\pi g^*s$ int(A^c)
(ii) $(N\pi g^*s\text{-int}(A))^c$ = $(\cup$ {B:B is $N\pi g^*s$-open, $B\subset A\})^c$
= $\cap$ {B^c:B^c is $N\pi g^*s$-closed, $A^c\subset B^c$}
= $N\pi g^*s$-cl(A^c).

Theorem 4.13. *If A is a subset of a space (U, $\tau_R(X)$), then $N\pi g^*s$-cl(A)$\subset N\pi g$-cl(A), where $N\pi g$-cl(A) is given by $N\pi g$-cl(A) = $\cap${F $\subset$U : A$\subset$F and F is a $N\pi g$-closed set in U}.*
Proof: *Let A be a subset of U. By definition of $N\pi g$-cl(A) = $\cap${F$\subset$U : A$\subset$F and F is a $N\pi g$-closed set in U}. If A$\subset$F and F is $N\pi g$-closed subset of x, then A$\subset$F$\in N\pi g$-cl(A), because every $N\pi g$-closed subset in U. That is $N\pi g^*s$-cl(A) $\subset$F. Therefore $N\pi g^*s$-cl(A)$\subset$ $\cap${F$\subset$U : A$\subset$F and F is a $N\pi g$-closed set in U} = $N\pi g$-cl(A).*
Hence $N\pi g^*s$-cl(A)$\subset N\pi g$-cl(A).

Theorem 4.14. *If A is a subset of a space (U, $\tau_R(X)$), then $N\pi g^*s$-cl(A) $\subset$Ncl(A).*
Proof: *Let A be a subset of a space (U, $\tau_R(X)$). By the definition of nano closure, Ncl(A) = $\cap$ {F : U$\subset$F$\in$C(U)}.*
If A$\subset$F$\in$NC(A). Then A$\subset$F$\in N\pi g^*s$(U), because every nano closed set is $N\pi g^*s$-closed. That is $N\pi g^*s$-cl(A)$\subset$F. Therefore $N\pi g^*s$-cl(A)$\subset$ $\cap${F $\subset$ X:F$\in N\pi g^*s$C(U)}=Ncl(A). Hence $N\pi g^*s$-cl(A)$\subset$Ncl(A).

Remark 4.15. *Containment relation in the above theorem may be proper as seen from the following example.*

Example 4.16. *Let U = {a,b,c,d} with U/R = {{a,b},{c},{d}}. Let X = {a,c}$\subseteq$U. Then $\tau_R(X)$ = {ϕ,U,{c},{a,b,c},{a,b}}, nano πg^*s-closed subsets are {ϕ,U,{a},{b},{c},{a,b},{a,d}, {b,d},{c,d},{a,b,d}} and nano πg-closed subsets are {ϕ,U,{a}, {b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c}, {a,c,d},{b,c,d}}.*
Let A = {a,d}, $N\pi g$-cl(A) = {a,b,d} and $N\pi g^*s$-cl(A) = {a,d}.
It follows $N\pi g^*s$-cl(A)$\subset$N πg-cl(A) and $N\pi g^*s$-cl(A) $\neq N\pi g$-cl(A).

5. Nano πg^*s-neighbourhoods

Definition 5.1. *A subset $M_x \subset U$ is called a nano πg^*s-neighbourhood (Nπg^*s-nghd) of a point $x \in U$ if and only if there exists a $A \in N\pi g^*sO(U,X)$ such that $x \in A \subset M_x$ and a point x is called Nπg^*s-nghd point of the set A.*

Definition 5.2. *The family of all Nπg^*s-nghd of the point $x \in U$ is called Nπg^*s-nghd system of U and is denoted by Nπg^*s-nghd x.*

Example 5.3. *Let U=$\{a,b,c,d\}$ with U/R=$\{\{a,c\},\{b\},\{d\}\}$. Let X=$\{a,b\} \subseteq U$ and $\tau_R(X)$= $\{\phi,U,\{b\},\{a,b,c\},\{a,c\}\}$, $N\pi g^*sO(U,X)=\{\phi,U,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\}\}$. Then $N\pi g^*$s-nghds(a)=$\{\phi,U,\{a\},\{a,b\},\{a,c\},\{a,b,c\},\{a,c,d\}\}$, $N\pi g^*$s-nghds(b)=$\{\phi,U,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\}\}$, $N\pi g^*$s-nghds(c)=$\{\phi,U,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\}\}$, $N\pi g^*$s-nghds(d)=$\{\phi,U,\{b,d\},\{c,d\},\{a,c,d\}\}$.*

Lemma 5.4. *An arbitrary union of Nπg^*s-nghds of a point x is again a Nπg^*s-nghd of x.*

Proof: *Let $\{A_\lambda\}_{\lambda\in I}$ be an arbitrary collection of Nπg^*s-nghds of a point $x \in U$. We have to prove that $\cup A_\lambda$ for $\lambda \in I$ (where I denote index set) also a Nπg^*s-nghd of x. For all $\lambda \in I$ there exists Nπg^*s-open set M_x such that $x \in M_x \subset A_\lambda \subset \cup A_\lambda$ i.e $x \cup M_x \subset \cup A_\lambda$ therefore $\cup A_\lambda$ for $\lambda \in I$ is a Nπg^*s-nghd of x. That is an arbitrary union of Nπg^*s-nghds of x is again a Nπg^*s-nghds of x.*

But intersection of Nπg^*s-nghds of a point is not a Nπg^*s-nghds of that point in general.

Example 5.5. *Let U=$\{a,b,c,d\}$ with U/R=$\{\{a,b\},\{c\},\{d\}\}$. Let X=$\{a,c\} \subseteq U$ and $\tau_R(X)$=$\{\phi,U,\{c\},\{a,b,c\},\{a,b\}\}$ be a nano topology on U. Now $N\pi g^*sO(U,X)$ = $\{\phi,U,\{c\},\{a,b\},\{a,c\},\{b,c\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Clearly $\{a,b\}$ and $\{b,c\}$ are Nπg^*s-nghd of $b \in U$ but $\{a,b\} \cap \{b,c\}$ = $\{b\}$ is not a Nπg^*s-nghd of b.*

Theorem 5.6. *The Nπg^*s-nghd system Nπg^*s-nghd(x) of a point $x \in U$ satisfies the following properties*
(a)if $N \in N\pi g^*s$-N(x) then $x \in N$
(b)if $N \in N\pi g^*s$-N(x) and $N \subset M$ then $M \in N\pi g^*s$-N(x)
(c)if $N \in N\pi g^*s$-N(x) then there exists a $G \in N\pi g^*s$-N(x) such that $G \subset N$ and $G \in N\pi g^*s$-N(y), for all $y \in G$.

Proof: *(a)Let $N \in N\pi g^*s$-N(x) implies N is the Nπg^*s-nghd of x. therefore $x \in N$*
(b)Let $N \in N\pi g^*s$-N(x) and $N \subset M$. Therefore there exists $G \in N\pi g^*sO(U,X)$ such that $x \in G \subset N \subset M$ implies M is a Nπg^*s-nghd of x and hence $M \in N\pi g^*s$-N(x)
(c)Let $N \in N\pi g^*s$-N(x) implies $G \in N\pi g^*sO(X)$ such that $x \in G \subset N$ G is Nπg^*s-nghd of each of its points implies for all $y \in G$, G is the Nπg^*s-nghd of y and hence $G \in N\pi g^*s$-N(y) for all $y \in G$.

Theorem 5.7. *If A be a subset of $(U,\tau_R(X))$. Then Nπg^*s-int(A) = $\cup\{B$:B is a Nπg^*s-open, $B \subset A\}$.*

Proof: *let A be a subset of $(U,\tau_R(X))$.*
$x \in N\pi g^*$s-int(A)
$\Leftrightarrow$ x is a Nπg^*s-interior point of A.
$\Leftrightarrow$A is a Nπg^*s-nbhd of point x.
$\Leftrightarrow$ there exists Nπg^*s-open set B such that $x \in B \subset A$
$\Leftrightarrow x \in \cup\{B$:B is a N$\pi g^*$s-open, $B \subset A\}$
Hence Nπg^*s-int(A) = $\cup\{B$:B is a Nπg^*s-open, $B \subset A\}$.

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