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Bounds on the covering radius of repetition code in $\mathbb{Z}_2\mathbb{Z}_6$

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Abstract. In this paper, the covering radius of codes over $R = \mathbb{Z}_2\mathbb{Z}_6$ with different weight are discussed. The block repetition codes over R is defined and the covering radius for block repetition codes R are obtained.

AMS Subject Classifications: 16P10,11T71, 94B05, 11H71, 94B65.

Keywords: Finite ring, Additive codes, Covering radius, Different weight.

Contents

1. Introduction and Background

Codes over finite commutative rings have been studied for almost 50 years. The main motivation of studying codes over rings is that they can be associated with codes over finite fields through the Gray map. Recently, coding theory over finite commutative non-chain rings is a hot research topic. Recently, there has been substantial interest in the class of additive codes. In [11, 12], Delsarte contributes to the algebraic theory of association scheme where the main idea is to characterize the subgroups of the underlying abelian group in a given association scheme.

The covering radius is an important geometric parameter of codes. It not only indicates the maximum error correcting capability of codes, but also relates to some practical problems such as the data compression and transmission. Studying of the covering radius of codes has attracted many coding scientists for almost 30 years. The covering radius of linear codes over binary finite fields was studied in [9].

Additive codes over $\mathbb{Z}_2\mathbb{Z}_4$ have been extensively studied in [2, 4–6]. In [7], the authors, in particular, gave lower and upper bounds on the covering radius of codes over the ring $\mathbb{Z}_6$ with respect to different distance. Using above results motivate us to work in this Paper.

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2. Preliminaries

In $\mathbb{Z}_2$ and $\mathbb{Z}_6$ be the rings of integers modulo 2 and 6. Let $\mathbb{Z}_2^n$ and $\mathbb{Z}_6^n$ denote the space of n-tuples over these rings. A ring $R = \mathbb{Z}_2\mathbb{Z}_6 = \{00, 01, 02, 03, 04, 05, 10, 11, 12, 13, 14, 15\}$, with integer modulo is 2 and 6. If C be a non-empty subset of $\mathbb{Z}_2^n$ is called a *code* and if that subcode is a linear space, then C is said to be *linear code*. Similarly, any non-empty subset C of $\mathbb{Z}_6^n$ is called a *linear senary code*.

In this section, some preliminary results are given [4, 6, 15]. A non-empty set C is a R-additive code if it is a subgroup of $\mathbb{Z}_2^\gamma \times {\mathbb{Z}_6}^\delta$. In this case, C is also isomorphic to an abelian structure $\mathbb{Z}_2^\lambda \times \mathbb{Z}_6^\mu$ for some λ and μ and type of C is a $2^\lambda 6^\mu$ as a group. It pursue that it has $\mid C \mid = 2^{\lambda+2\mu}$ codewords and the number of order for two codewords in C is $\mid C \mid = 2^{\lambda+\mu}$.

A linear code C of length n over $\mathbb{Z}_6$ is an additive subgroup of $\mathbb{Z}_6^n$. An element of C is called a *codeword* of C and a generator matrix of C is a matrix whose rows generate C. The Hamming weight $w_H(x)$ of a vector $x \in \mathbb{Z}_6^n$ is the number of non-zero components. The Lee weight $w_L(x)$ of a vector $x = (x_1, x_2, \cdots, x_n)$ is

$$w_L(x_i) = \begin{cases} 0 & \text{if } x = 0, \\ 1 & \text{if } x = 1, 5, \\ 2 & \text{if } x = 2, 4, \\ 3 & \text{if } x = 3. \end{cases}$$

The Euclidean weight $w_E(x)$ of a vector x is

$$w_E(x_i) = \begin{cases} 0 & \text{if } x = 0, \\ 1 & \text{if } x = 1, 5, \\ 4 & \text{if } x = 2, 4, \\ 9 & \text{if } x = 3. \end{cases}$$

The Chinese Euclidean weight $w_E(x)$ of a vector x is

$$w_{CE}(x_i) = \begin{cases} 0 & \text{if } x = 0, \\ 1 & \text{if } x = 1, 5, \\ 3 & \text{if } x = 2, 4, \\ 4 & \text{if } x = 3. \end{cases}$$

The Hamming, Lee, Euclidean, Chinese Euclidean distances $d_H(x, y), d_L(x, y)$, $d_E(x, y)$ and $d_{CE}(x, y)$ between two vectors x and y are $w_H(x-y), w_L(x-y), w_E(x-y)$ and $w_{CE}(x-y)$ respectively. The minimum Hamming, Lee, Euclidean and Chinese Euclidean weights, d_H, d_L, d_E and d_{CE} of C are the smallest Hamming, Lee, Euclidean and Chinese Euclidean weights among all non-zero codewords of C respectively.

The Gray map: $\mu : \mathbb{Z}_6 \to \mathbb{Z}_2\mathbb{Z}_3$ is defined as $\mu(0) = (00), \mu(1) = (11), \mu(2) = (02), \mu(3) = (10), \mu(4) = (01), \mu(5) = (12)$ and the extension of the Gray map
$\rho : \mathbb{Z}_2^\gamma \times \mathbb{Z}_6^\delta \to \mathbb{Z}_2^n\mathbb{Z}_3$, where $n = \gamma + \delta$ is given by

$$\rho(u, w) = (u, \mu(w_1), \cdots, \mu(w_\delta)), \forall u \in \mathbb{Z}_2^\gamma \text{ and } (w_1, \cdots, w_\delta) \in \mathbb{Z}_6^\delta.$$

Then the binary image of a R-additive code under the extended Gray map is called a R-*linear code* of length $n = \gamma + \delta$.

The Hamming weight of u denoted by $w_H(u)$ and $w_L(w), w_E(w), w_{CE}(w)$ be the Lee, Euclidean and Chinese Euclidean weights of w respectively, where $u \in \mathbb{Z}_2^{\gamma}$ and $w \in \mathbb{Z}_6^{\delta}$.

In Lee, Euclidean and Chinese Euclidean weights are x is defined as $w_D(x) = w_H(u) + w_D(w)$, where $D = \{$Lee(L), Euclidean(E), Chinese Euclidean(CE)$\}$, and $x = (u, w) \in \mathbb{Z}_2^{\gamma} \times \mathbb{Z}_6^{\delta}$, and $u = (u_1, \cdots, u_{\gamma}) \in \mathbb{Z}_2^{\gamma}$ and $w = (w_1, \cdots, w_{\delta}) \in \mathbb{Z}_6^{\delta}$. The Gray map defined above is an isometry which transforms the Lee distance defined over $\mathbb{Z}_2^{\gamma} \times \mathbb{Z}_6^{\delta}$ to the Hamming distance defined over $\mathbb{Z}_2^{n}$, with $n = \gamma + \delta$.

3. The covering radius of code and the block repetition codes over R

The covering radius of a code C is the smallest number r such that the spheres of radius r around the codewords cover $\mathbb{Z}_2^{\gamma} \times \mathbb{Z}_6^{\delta} = R$ and thus the covering radius of a code C over R with respect to the different distance, such as(Lee, Euclidean, Chinese Euclidean) is given $r_d(C) = \max\limits_{u \in R}\{\min\limits_{c \in C} d(u, c)\}$.

In $F_q = \{0, 1, \beta_2, \cdots, \beta_{q-1}\}$ is a finite field. Let C be a q-ary *repetition code* C over F_q. That is $C = \{\bar{\beta} = (\beta\beta\cdots\beta) | \beta \in F_q\}$ and the repetition code C is an $[n, 1, n]$ code. Therefore, the covering radius of the code C is $\lceil \frac{n(q-1)}{q} \rceil$ this true for binary repetition code. In [7], the authors studied for different classes of repetition codes over $\mathbb{Z}_6$ and their covering radius has been obtained. Now, generalize those results for codes over R.

Consider the repetition codes over R. For a fixed $1 \leq i \leq 11$. For all $1 \leq j \neq i \leq 11, n_j = 0$, then the code $C^n = C^{n_i}$ is denoted by C_i. Therefore, the eleven basic repetition codes are the following table,

Generator Matrix	Code	Parameters $[n, k, d_*], (n, M, d_*)$
$G_1 = [\overbrace{01(\mathbf{05}) \cdots 01(\mathbf{05})}^{n}] = G_{\mathbf{5}}$	C_1, C_5	$[n, 1, n, n, n]$
$G_2 = [\overbrace{02(\mathbf{04}) \cdots 02(\mathbf{04})}^{n}] = G_{\mathbf{4}}$	C_2, C_4	$(n, 3, 2n, 4n, 3n)$
$G_3 = [\overbrace{03 \cdots 03}^{n}]$	C_3	$(n, 2, 3n, 9n, 4n)$
$G_6 = [\overbrace{10 \cdots 10}^{n}]$	C_6	$(n, 2, n, n, n)$
$G_7 = [\overbrace{11(\mathbf{15}) \cdots 11(\mathbf{15})}^{n}] = G_{\mathbf{11}}$	C_7, C_{11}	$[n, 1, n, n, n]$
$G_8 = [\overbrace{12(\mathbf{14}) \cdots 12(\mathbf{14})}^{n}] = G_{\mathbf{10}}$	C_8, C_{10}	$(n, 6, n, n, n)$
$G_9 = [\overbrace{13 \cdots 13}^{n}]$	C_9	$(n, 4, n, n, n)$

here,$* = L(E)(CE)$
$C_1 = \{c_0, c_1, c_2, c_3, c_4, c_5\} = C_5$,
$C_2 = \{c_0, c_2, c_4\} = C_4$,
$C_3 = \{c_0, c_3\}$,
$C_6 = \{c_0, c_6\}$
$C_7 = \{c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}\} = C_{11}$,
$C_8 = \{c_0, c_2, c_4, c_6, c_8\} = C_{10}$,
$C_9 = \{c_0, c_3, c_6, c_9\}$ and $\{c_0 = 00 \cdots 00, c_1 = 01 \cdots 01, c_2 = 02 \cdots 02, c_3 = 03 \cdots 03, c_4 = 04 \cdots 04,$
$c_5 = 05 \cdots 05, c_6 = 10 \cdots 10, c_7 = 11 \cdots 11, c_8 = 12 \cdots 12, c_9 = 13 \cdots 13, c_{10} = 14 \cdots 14, c_{11} = 15 \cdots 15\}$.

Theorem 3.1. *Let $C_{j, 1 \leq j \leq 11}$, be a code in R. Then,*

1. $\frac{3n}{4} \leq r_L(C_1) = r_L(C_5) \leq \frac{7n}{3}$,
2. $\frac{2n}{3} \leq r_L(C_2) = r_L(C_4) \leq \frac{7n}{3}$,
3. $\frac{3n}{4} \leq r_L(C_3) \leq 2n$,

4. $\frac{n}{4} \leq r_L(C_6) \leq 3n$,

5. $n \leq r_L(C_7) = r_L(C_{11}) \leq 2n$,

6. $\frac{11n}{12} \leq r_L(C_8) = r_L(C_{10}) \leq 2n$ *and*

7. $n \leq r_L(C_9) \leq 2n$, *where* $r_L(C_j)$ *is a covering radius of* C_j, $_{1\leq j\leq 11}$ *with Lee distance.*

Proof. For $c \in C_{j,1\leq j\leq 11}$ be a codeword of code C_j in R. Let $t_i(c)_{,0\leq i\leq 11}$ is the number of occurrences of symbol i in the codeword c.

Let $x \in R^n$ by $(t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_8, t_9, t_{10}, t_{11})$, where $\sum_{j=0}^{11} t_j = n$, then

$$
\begin{aligned}
d_L(x, \overline{00}) &= n - t_0 + t_2 + 2t_3 + t_4 + t_7 + 2t_8 + 3t_9 + 2t_{10} + t_{11},\\
d_L(x, \overline{01}) &= n - t_1 + t_3 + 2t_4 + t_5 + t_6 + t_8 + 2t_9 + 3t_{10} + 2t_{11},\\
d_L(x, \overline{02}) &= n - t_2 + t_0 + t_4 + 2t_5 + 2t_6 + t_7 + t_9 + 2t_{10} + 3t_{11},\\
d_L(x, \overline{03}) &= n - t_3 + t_1 + t_5 + 3t_6 + 2t_7 + t_8 + t_{10} + 2t_{11} + 2t_0,\\
d_L(x, \overline{04}) &= n - t_4 + t_0 + 2t_1 + t_2 + 2t_6 + 3t_7 + 2t_8 + t_9 + t_{11},\\
d_L(x, \overline{05}) &= n - t_5 + t_1 + 2t_2 + t_3 + t_6 + 2t_7 + 3t_8 + 2t_9 + t_{10},\\
d_L(x, \overline{10}) &= n - t_6 + t_1 + 2t_2 + 3t_3 + 2t_4 + t_5 + t_8 + 2t_9 + t_{10},\\
d_L(x, \overline{11}) &= n - t_7 + t_0 + t_2 + 2t_3 + 3t_4 + 2t_5 + t_9 + 2t_{10} + t_{11},\\
d_L(x, \overline{12}) &= n - t_8 + 2t_0 + t_1 + t_3 + 2t_4 + 3t_5 + t_6 + t_{10} + 2t_{11},\\
d_L(x, \overline{13}) &= n - t_9 + 3t_0 + 2t_1 + t_2 + t_4 + 2t_5 + 2t_6 + t_7 + t_{11},\\
d_L(x, \overline{14}) &= n - t_{10} + 2t_0 + 3t_1 + 2t_2 + t_3 + t_5 + t_6 + 2t_7 + t_8,\\
d_L(x, \overline{15}) &= n - t_{11} + t_0 + 2t_1 + 3t_2 + 2t_3 + t_4 + t_7 + 2t_8 + t_9,
\end{aligned}
$$

In code $C_7 = C_{11} \in R$, then $d_L(x, C_7) = d_L(x, C_{11}) = \min\{d_L(x, \overline{01}), d_L(x, \overline{01}),$ $d_L(x, \overline{02}), d_L(x, \overline{03}), d_L(x, \overline{04}), d_L(x, \overline{05}), d_L(x, \overline{10}), d_L(x, \overline{11}) d_L(x, \overline{12}), d_L(x, \overline{13}),$ $d_L(x, \overline{14}), d_L(x, \overline{15})\} \leq 2n$ and hence

$$r_L(C_7) = r_L(C_{11}) \leq 2n.$$

If $x = (\overbrace{00\cdots 00}^{\frac{n}{12}}\overbrace{01\cdots 01}^{\frac{n}{12}}\overbrace{02\cdots 02}^{\frac{n}{12}}\overbrace{03\cdots 03}^{\frac{n}{12}}\overbrace{04\cdots 04}^{\frac{n}{12}}\overbrace{05\cdots 05}^{\frac{n}{12}}\overbrace{10\cdots 10}^{\frac{n}{12}}\overbrace{11\cdots 11}^{\frac{n}{12}}$ $\overbrace{12\cdots 12}^{\frac{n}{12}}\overbrace{13\cdots 13}^{\frac{n}{12}}\overbrace{14\cdots 14}^{\frac{n}{12}}\overbrace{15\cdots 15}^{\frac{n}{12}}) \in R^n$. Then $d_L(x, \overline{00}) = d_L(x, \overline{01}) = d_L(x, \overline{02}) = d_L(x, \overline{03}) =$ $d_L(x, \overline{04}) = d_L(x, \overline{05}) = d_L(x, \overline{10}) = d_L(x, \overline{11}) = d_L(x, \overline{12}) = d_L(x, \overline{13}) = d_L(x, \overline{14}) = d_L(x, \overline{15}) =$ $\frac{n}{24} + 2(\frac{n}{24}) + 3(\frac{n}{24}) + 2(\frac{n}{24}) + \frac{n}{24} + \frac{n}{24} + 2(\frac{n}{24}) + 3(\frac{n}{24}) + 4(\frac{n}{24}) + 3(\frac{n}{24}) + 2(\frac{n}{24}) = n$. Thus $r_L(C_7) = r_L(C_{11}) \geq n$ and hence, $n \leq r_L(C_7) = r_L(C_{11}) \leq 2n$.

In Code, $C_3 \in R$, $d_L(x, C_3) = \min\{d_L(x, \overline{00}), d_L(x, \overline{03})\} \leq \frac{2n-n+3n}{2} = 2n$. Then $r_L(C_3) \leq 2n$.

If $x = (\overbrace{00\cdots 00}^{\frac{n}{2}}\overbrace{03\cdots 03}^{\frac{n}{2}}) \in R^n$, then $d_L(x, \overline{00}) = d_L(x, \overline{03}) = 3(\frac{n}{4}) = \frac{3n}{4}$. Thus $r_L(C_2) \geq \frac{3n}{4}$ and so $\frac{3n}{4} \leq r_L(C_3) \leq 2n$.

The remaining part of proof is follows from the above computation with respect to code. ■

Theorem 3.2. *In Euclidean weight for the code* $C_{j,1\leq j\leq 7}$, *prove the following*

1. $\frac{19n}{12} \leq r_E(C_1) = r_{CE}(C_5) \leq 4n$,

2. $\frac{4n}{3} \leq r_E(C_2) = r_{CE}(C_4) \leq \frac{13n}{3}$,

3. $\frac{9n}{4} \leq r_E(C_3) \leq 5n$,

4. $\frac{n}{4} \le r_E(C_6) \le 7n$,
5. $\frac{11n}{6} \le r_E(C_7) = r_{CE}(C_{11}) \le \frac{11n}{3}$,
6. $\frac{19n}{12} \le r_E(C_8) = r_{CE}(C_{10}) \le 4n$ *and*
7. $\frac{5n}{2} \le r_E(C_9) \le 5n$.

Proof. Use to theorem 3.1 and in Code $C_i,\ _{i=1 \text{ to } 11}$ with Euclidean weight. ■

Theorem 3.3. *In Chinese Euclidean weight of code of $C_{j, 1\le j\le 11,}$ to find*

1. $\frac{5n}{6} \le r_{CE}(C_1) = r_{CE}(C_5) \le \frac{17n}{6}$,
2. $n \le r_{CE}(C_2) = r_{CE}(C_4) \le \frac{8n}{3}$,
3. $n \le r_{CE}(C_3) \le \frac{5n}{2}$,
4. $\frac{n}{4} \le r_{CE}(C_6) \le 4n$,
5. $r_{CE}(C_7) = r_{CE}(C_{11}) \le \frac{5n}{2}$,
6. $\frac{5n}{4} \le r_{CE}(C_8) = r_{CE}(C_{10}) \le \frac{7n}{3}$ *and*
7. $\frac{5n}{4} \le r_{CE}(C_9) \le \frac{5n}{2}$.

Proof. In Code $C_i,\ _{i=1 \text{ to } 11}$ with Chinese Euclidean weight is apply to theorem 3.1. ■

Block repetition code in R

The block repetition code C^n over R is a R-additive code.

Let $G = [\overbrace{0101\cdots01}^{n_1}\overbrace{0202\cdots02}^{n_2}\overbrace{0303\cdots03}^{n_3}\overbrace{0404\cdots04}^{n_4}\overbrace{0505\cdots05}^{n_5}\overbrace{1010\cdots10}^{n_6}$ $\overbrace{1111\cdots11}^{n_7}\overbrace{1212\cdots12}^{n_8}\overbrace{1313\cdots13}^{n_9}\overbrace{1414\cdots14}^{n_{10}}\overbrace{1515\cdots15}^{n_{11}}]$ be a generator matrix with the parameters of C^n : $[n = \sum_{j=1}^{11} n_j, 12, d_L = \min\{\sum_{j=6}^{11} n_j, \sum_{j=1,2,4,5,}^{7,8,10,11} 2n_j\}, d_E = \min\{\sum_{j=6}^{11} n_j, \sum_{j=1,2,4,5,}^{7,8,10,11} 4n_j\}, d_{CE} = \min\{\sum_{j=6}^{11} n_j, \sum_{j=1,2,4,5,}^{7,8,10,11} 3n_j\}]$.

Theorem 3.4. *Let C^n be the block repetition code in R with length is n. Then the covering radius of block repetition code is*

1. $\frac{9(n_1+n_3+n_5)+8(n_2+n_4)+3n_6+12(n_7+n_9+n_{11})+11(n_8+n_{10})}{12} \le r_L(C^n) \le \frac{30(n_1+n_3+n_5)+31n_2+24n_4+36n_6+26(n_7+n_8)+24(n_9+n_{11})+25n_{10}}{12}$,
2. $\frac{19(n_1+n_5+n_8+n_{10})+16(n_2+n_4)+27n_3+3n_6+22(n_7+n_{11})+30n_9}{12} \le r_E(C^n) \le \frac{52n_1+56(n_2+n_4)+66n_3+50(n_5+n_8+n_{10})+60(n_6+n_9)+45n_7+44n_{11}}{12}$ *and*
3. $\frac{10(n_1+n_5)+12(n_2+n_3)+3n_6+30(n_7+n_{11})+15(n_8+n_9+n_{10})}{12} \le r_{CE}(C^n) \le \frac{36(n_1+n_2+n_3+n_4+n_5)+48n_6+32n_7+31(n_8+n_{10})+30(n_9+n_{11})}{12}$.

Proof. Using [9], Theorem 3.1, 3.2 and 3.3, thus

- $\frac{9(n_1+n_3+n_5)+8(n_2+n_4)+3n_6+12(n_7+n_9+n_{11})+11(n_8+n_{10})}{12} \le r_L(C^n)$,

- $\frac{19(n_1+n_5+n_8+n_{10})+16(n_2+n_4)+27n_3+3n_6+22(n_7+n_{11})+30n_9}{12} \leq r_E(C^n)$ and
- $\frac{10(n_1+n_5)+12(n_2+n_3)+3n_6+30(n_7+n_{11})+15(n_8+n_9+n_{10})}{12} \leq r_{CE}(C^n)$.

Let $x = x_1x_2x_3x_4x_5x_6x_7x_8x_9x_{10}x_{11} \in R^n$ with $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9,$ x_{10}, x_{11} is $(a_i), (b_i), (c_i), (d_i), (e_i), (f_i), (g_i), (h_i), (k_i), (l_i), (m_i),$ $_{i=0,1,2,3,4,5,6,7,8,9,10,11}$ respectively such that $n_1 = \sum_{j=0}^{11} a_j$, $n_2 = \sum_{j=0}^{11} b_j$, $n_3 = \sum_{j=0}^{11} c_j$, $n_4 = \sum_{j=0}^{11} d_j$, $n_5 = \sum_{j=0}^{11} e_j$, $n_6 = \sum_{j=0}^{11} f_j$, $n_7 = \sum_{j=0}^{11} g_j$, $n_7 = \sum_{j=0}^{11} g_j$, $n_8 = \sum_{j=0}^{11} h_j$, $n_9 = \sum_{j=0}^{11} k_j$, $n_{10} = \sum_{j=0}^{11} l_j$, $n_{11} = \sum_{j=0}^{11} m_j$.

Thus, $r_L(C^n) \leq \frac{30(n_1+n_3+n_5)+31n_2+24n_4+36n_6+26(n_7+n_8)+24(n_9+n_{11})+25n_{10}}{12}$, $r_E(C^n) \leq \frac{52n_1+56(n_2+n_4)+66n_3+50(n_5+n_8+n_{10})+60(n_6+n_9)+45n_7+44n_{11}}{12}$ and r$_{CE}(C^n) \leq \frac{36(n_1+n_2+n_3+n_4+n_5)+48n_6+32n_7+31(n_8+n_{10})+30(n_9+n_{11})}{12}$. ■

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