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Some characterizations of stochastic orders

D. Rajan[1]* and D. Vijayabalan[2]

Abstract

In this paper, we define stochastic orders of fuzzy random variables with some properties. Also we define the hazard rate ordering of fuzzy random variables with related properties.

Keywords

Fuzzy random variables, fuzzy stochastic order, fuzzy hazard rate order.

AMS Subject Classification

60A86.

[1,2] *Department of Mathematics, T.B.M.L College, Porayar–609307, India.*

***Corresponding author**: [1] dan_rajan@rediffmail.com; [2]vijayabalantqb@gmail.com

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Contents

1. Introduction

The concept of fuzzy random variable was introduced by Kwakernaak [3] and Puri and Ralescu [7]. A fuzzy random variable is just a random variable that takes on values in space of fuzzy sets. The outcomes of Kwakernaak's fuzzy random variables are fuzzy real. Subsets and the extreme points of their α-cuts are classical random variables. Fuzzy random variables are Mathematical descriptions for fuzzy stochastic phenomena, but only one time descriptions.

Fuzzy random variables generalize random variables and random sets. Kwakernaak [4] introduced the concept of a fuzzy random variables as a function $X:\Omega\to F(R)$ where $(\Omega,\mathscr{A},P)$ is a probability triple and F(R) denotes the set of all canonical fuzzy numbers. Puri and Ralescu [7] defined the notion of fuzzy random variable as a function X: $\Omega\to$ F(R^n) where $(\Omega,\mathscr{A},P)$ is a probability space and $F(R^n)$ indicates all functions $U:R^n\to[0,1]$.

Stochastic ordering of fuzzy random variables is one of the most applicatble in statistics and probability. Various concept of statistic comparison between random variables have been defined and studied in the literature, because of their usefulness in modeling for reliability economics applications and as mathematical tools for proving important results in applied probability [8].

2. Preliminaries

Definition 2.1. *Let X be a universal set. Then a fuzzy set* $\tilde{A}=\{(x,\mu_A(x))/x\in X\}$ *of X is defined by its membership function*
$\mu_(R)\tilde{A}:X\to[0,1]$.

Definition 2.2. *For each* $0\leqslant\alpha\leqslant1$, *the* α*-cut of set of* $\tilde{A}$ *is denoted by its* $\tilde{A}_\alpha=\left\{x\in X:\mu_(R)\tilde{A}(x)\geqslant\alpha\right\}$.

Definition 2.3. *A fuzzy number is a fuzzy set of R such that the following conditions are satisfies:*

1. $\tilde{A}$ *is normal if there exists* $x\in X$ *such that* $\mu_(R)\tilde{A}(x)=1$
2. $\tilde{A}$ *is called convex if*
$\mu_(R)\tilde{A}\left(\lambda x_1+(1-\lambda x_2)\right)\geqslant\min\left(\mu_(R)\tilde{A}\left(x_1\right),\mu_(R)\tilde{A}\left(x_2\right)\right)$
3. $\tilde{A}$ *is called upper semi continuous with compact support; that is for every* $\varepsilon>0$, *there exists*
$\delta>0;\ |x-y|<\delta\Rightarrow\mu_(R)\tilde{A}(x)<\mu_(R)\tilde{A}(y)+\varepsilon$
4. *The* α*-cut of fuzzy number is closed interval denoted by* $A_\alpha=\left[A_\alpha^L,A_\alpha^U\right]$, *where* $A_\alpha^L=\inf\left\{x\in R;\mu_(R)\tilde{A}(x)\geqslant\alpha\right\}$ *and* $A_\alpha^U=\sup\left\{x\in R;\mu_(R)\tilde{A}(x)\geqslant\alpha\right\}$.
5. *If* $\tilde{A}$ *is closed and bounded fuzzy number with* A_α^L,A_α^U *and its membership function is strictly increasing on* $\left[A_\alpha^L,A_\alpha^U\right]$ *and strictly decreasing on* $\left[A_1^U,A_\alpha^U\right]$ *then* $\tilde{A}$ *is called canonical fuzzy number.*

Definition 2.4. *A fuzzy random variable is a fuzzy set consisting of a membership function and a basic set of underlying variables. A fuzzy random variable X is a map $X:\Omega \to F(R)$ satisfying the following conditions:*

1. *For each $\alpha \in (0,1]$ both X_α^L and X_α^U defined as $X_\alpha^L(\omega)(x) = \inf\{x \in R; X(\omega)(x) \geqslant \alpha\}$ and $X_\alpha^U(\omega)(x) = \sup\{x \in R; X(\omega)(x) \geqslant \alpha\}$ are finite real valued random variables defined on such (Ω, A, P) that the mathematical expectations EX_α^L and EX_α^U exist.*
2. *For each $\omega \in \Omega$ and $\alpha \in (0,1], X_\alpha^L(\omega)(x) \geqslant \alpha$ and $X_\alpha^U(\omega)(x) \geqslant \alpha$.*

Definition 2.5. *A fuzzy set valued mapping $X:\Omega \to F_0^m(R) = F_0(R) \times \cdots \times F_0(R)$ represented by $X(\omega) = \big(X(1,\omega) \times \cdots \times X(m,\omega)\big)$ is called the fuzzy random vector if for each K, $1 \leqslant K \leqslant m, X(K,\omega)$ is a fuzzy random variable.*

Definition 2.6. *$X(\omega)$ is a fuzzy random variable if and only if $X_\alpha(\omega) = \left[X_\alpha^L(\omega), X_\alpha^U(\omega)\right]$, where $X_\alpha^L(\omega)$ and $X_\alpha^U(\omega)$ are both random variables for each $\alpha \in (0,1]$ and $X(\omega) = \underset{\alpha\in(0,1]}{\cup} \alpha X_\alpha(\omega)$.*

Definition 2.7. *The α cut of the distribution function F of fuzzy random variable $\tilde{X}$ is defined by*

$$D_\alpha = \left[\min\left\{\min_{\alpha\leqslant\beta} F\left(\tilde{x}_\beta^L\right), \min_{\alpha\leqslant\beta} F\left(\tilde{x}_\beta^U\right)\right\},\right.$$
$$\left.\max\left\{\max_{\alpha\leqslant\beta} F\left(\tilde{x}_\beta^L\right), \max_{\alpha\leqslant\beta} F\left(\tilde{x}_\beta^U\right)\right\}\right].$$

Definition 2.8. *The membership function of the probability distribution function of $\tilde{x}$, denoted as $\tilde{F}(\tilde{x})$. It is defined as*

$$\mu_{\tilde{F}(x)}(r) = ^{\sup}_{\alpha\in[0,1]\bigcap Q} \alpha 1 D_\alpha(r).$$

Definition 2.9. *The α-cut of probability density functions $f(x)$ where $x \in \left[\tilde{x}_\alpha^L, \tilde{x}_\alpha^U\right]$ is given by*

$$[A_\alpha = \left[\min\left\{\min_{\alpha\leqslant\beta} f\left(\tilde{x}_\beta^L\right), f\left(\tilde{x}_\beta^U\right)\right\},\right.$$
$$\left.\max\left\{\max_{\alpha\leqslant\beta} f\left(\tilde{x}_\beta^L\right), \max_{\alpha\leqslant\beta} f\left(\tilde{x}_\beta^U\right)\right\}\right].$$

Definition 2.10. *The membership function of the probability distribution function of $\tilde{x}$, denoted as $\tilde{f}(\tilde{x})$ is given by*

$$\mu_{\tilde{f}(x)}(r) = \sup_{\alpha\in[0,1]\bigcap Q} \alpha 1 A_\alpha(r).$$

3. Stochastic Orders

Definition 3.1. *If $\tilde{X}$ and $\tilde{Y}$ be fuzzy random variables with fuzzy cumulative distribution function $\tilde{F}$ and $\tilde{G}$ respectively then $X \leqslant_{st} Y \Leftrightarrow \tilde{F}(t) \geqslant \tilde{G}(t) \forall t$*

Definition 3.2. *If X and Y are two fuzzy random variables then*

$$X \leqslant_{st} Y \Leftrightarrow \left\{P\left(X_\alpha^L \geqslant t\right) \vee P\left(X_\alpha^U \geqslant t\right)\right\}$$
$$\leqslant \left\{P\left(Y_\alpha^L \geqslant t\right) \vee P\left(Y_\alpha^U \geqslant t\right)\right\}.$$

Definition 3.3. *If X and Y are two fuzzy random variables then*

$$X \leqslant_{st} Y \Longleftrightarrow E\left[f\left(X_\alpha^L\right)\right] \vee E\left[f\left(X_\alpha^U\right)\right]$$
$$\leqslant E\left[f\left(Y_\alpha^L\right)\right] \vee E\left[f\left(Y_\alpha^U\right)\right],$$

for all increasing functions f.

Example 3.4. *If X and Y are two exponential fuzzy random variables with mean λ and μ respectively such that $\lambda < \mu$ then $X \leqslant_{st} Y$.*

3.1 Properties of stochastic orders

Proposition 3.5 (Ordering of Mean values)**.** *If $X \leqslant_{st} Y$ and $E(Y)$ is well defined then $E(X) \leqslant E(Y)$.*

Proof. Assume first that X and Y are non negative value fuzzy random variables. Then

$$E\left[f\left(X_\alpha^L\right)\right] \vee E\left[f\left(X_\alpha^U\right)\right]$$
$$= \int_0^\infty P\left(X_\alpha^L > a\right)da \vee \int_0^\infty P\left(X_\alpha^U > a\right)da$$
$$\leqslant \int_0^\infty P\left(Y_\alpha^L > a\right)da \vee \int_0^\infty P\left(Y_\alpha^U > a\right)da$$
$$= E\left[f\left(Y_\alpha^L\right)\right] \vee E\left[f\left(Y_\alpha^U\right)\right]$$

Generally one can express any fuzzy random variables Z as the difference of two non negative fuzzy random variables.

Let $Z = Z^+ - Z^-$. For each $x \in R$,

$$Z^+(\omega)(x) = \begin{cases} Z(\omega)(x); & Z(\omega)(x) \geqslant 0 \\ 0; & Z(\omega)(x) < 0 \end{cases}$$

$$Z^-(\omega)(x) = \begin{cases} 0; & Z(\omega)(x) < 0 \\ -Z(\omega)(x); & Z(\omega)(x) \geqslant 0 \end{cases}$$

$X \leqslant_{st} Y$ implies $P\left\{X_\alpha^L > a\right\} \vee P\left\{X_\alpha^U > a\right\}$
$\leqslant P\left\{Y_\alpha^L > a\right\} \vee P\left\{Y_\alpha^U > a\right\}$
Let $X(\omega)(x) \geqslant 0$ and $Y(\omega)(x) \geqslant 0$ then

$$P\left\{\left(X^+\right)_\alpha^L > a\right\} \vee P\left\{\left(X^+\right)_\alpha^U > a\right\}$$
$$\leqslant P\left\{\left(Y^+\right)_\alpha^L > a\right\} \vee P\left\{\left(Y^+\right)_\alpha^U > a\right\}$$

This implies $X^+ \leqslant_{st} Y^+$.
Similarly we can prove $X^- \leqslant_{st} Y^-$.

Therefore

$$\begin{aligned}
&E\left[\left(X_\alpha^L\right)\right] \vee E\left[\left(X_\alpha^U\right)\right] \\
&= E\left[\left(X^+\right)_\alpha^L\right] \vee E\left[\left(X^+\right)_\alpha^U\right] \\
&-\left\{E\left[\left(X^-\right)_\alpha^L\right] \vee E\left[\left(X^-\right)_\alpha^U\right]\right\} \\
&\leqslant E\left[\left(Y^+\right)_\alpha^L\right] \vee E\left[\left(Y^+\right)_\alpha^U\right] \\
&-\left\{E\left[\left(Y^-\right)_\alpha^L\right] \vee E\left[\left(Y^-\right)_\alpha^U\right]\right\} \\
&= E\left[\left(Y_\alpha^L\right)\right] \vee E\left[\left(Y_\alpha^U\right)\right]
\end{aligned}$$

The above inequality true for all $\alpha \in [0,1]$

$$\begin{aligned}
&[E\left[\bigcup_{\alpha\in[0,1]} \alpha X_\alpha^L\right] \vee E\left[\bigcup_{\alpha\in[0,1]} \alpha X_\alpha^U\right] \\
&\leqslant E\left[\bigcup_{\alpha\in[0,1]} \alpha Y_\alpha^L\right] \vee E\left[\bigcup_{\alpha\in[0,1]} \alpha Y_\alpha^U\right]
\end{aligned}$$

This shows that,

$$E\left[\bigcup_{\alpha\in[0,1]} \alpha\left[X_\alpha^L, X_\alpha^U\right]\right] \leqslant E\left[\bigcup_{\alpha\in[0,1]} \alpha\left[Y_\alpha^L, Y_\alpha^U\right]\right]$$

That is $E(X) \leqslant E(Y)$. □

Proposition 3.6 (Closed under transformations by increasing function)**.** *If X and Y are two fuzzy random variables and if $X \leqslant_{st} Y$ then $f(X) \leqslant_{st} f(Y)$, for any increasing function f.*

Proof. Suppose first that $X \leqslant_{st} Y$, and let f be an increasing function.
Let $f^{-1}(a) = \inf\{x; f(x) \geqslant a\}$ Then for $\alpha \in [0,1]$

$$\begin{aligned}
&P\left\{f\left(X_\alpha^L\right) > a\right\} \vee P\left\{f\left(X_\alpha^U\right) > a\right\} \\
&= P\left\{X_\alpha^L > f^{-1}(a)\right\} \vee P\left\{X_\alpha^U > f^{-1}(a)\right\} \\
&\leqslant P\left\{Y_\alpha^L > f^{-1}(a)\right\} \vee P\left\{Y_\alpha^U > f^{-1}(a)\right\} \\
&= P\left\{f\left(Y_\alpha^L\right) > a\right\} \vee P\left\{f\left(Y_\alpha^U\right) > a\right\}
\end{aligned}$$

This shows that $f(X) \geqslant_{st} f(Y)$.

Conversely suppose that $(f(X)) \leqslant (f(Y))$ for all increasing function f.

For any a Let f_a denote the increasing function $f_a(x) = \begin{cases} 1; & x > a \\ 0; & x \leqslant a \end{cases}$

Then for $\alpha \in [0,1]$,

$$\begin{aligned}
&\left[f_a\left(X_\alpha^L\right)\right] \vee \left[f_a\left(X_\alpha^U\right)\right] \\
&= P\left\{X_\alpha^L > a\right\} \vee P\left\{X_\alpha^U > a\right\} \\
&\leqslant P\left\{Y_\alpha^L > a\right\} \vee P\left\{Y_\alpha^U > a\right\}
\end{aligned}$$

This shows that $X \leqslant_{st} Y$. □

4. Hazard Rate Order

Definition 4.1. *Let X be a non negative fuzzy random variables with fuzzy distribution function F and fuzzy Probability density function f Then the α-level fuzzy Hazard rate of X at $t \geqslant 0$ is defined as*

$$H_\alpha = \left[\frac{\min\left\{\min\limits_{\alpha\leqslant\beta\leqslant 1} f\left(\tilde{x}_\beta^L\right), \min\limits_{\alpha\leqslant\beta\leqslant 1} f\left(\tilde{x}_\beta^U\right)\right\}}{\min\left\{\min\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^L\right), \min\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^U\right)\right\}}, \frac{\max\left\{\max\limits_{\alpha\leqslant\beta\leqslant 1} f\left(\tilde{x}_\beta^L\right), \max\limits_{\alpha\leqslant\beta\leqslant 1} f\left(\tilde{x}_\beta^U\right)\right\}}{\max\left\{\max\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^L\right), \max\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^U\right)\right\}}\right]$$

The fuzzy hazard rate of $\tilde{x}$ denoted as $\tilde{r}(\tilde{x})$ the membership function of $\tilde{r}(\tilde{x})$ is defined as $\mu_{\tilde{r}(\tilde{x})}^{(r)} = \sup\limits_{0\leqslant\alpha\leqslant 1} \alpha I H_\alpha(r)$.

Definition 4.2. *Let X and Y be two non negative fuzzy random variable with continuous distribution functions and with fuzzy hazard rate functions $\tilde{r}(\tilde{x})$ and $\tilde{q}(\tilde{x})$ respectively. X is smaller than Y in the hazard rate order*

Denoted as $X \geqslant_{hr} Y$ if,

$$\frac{\min\left\{\min\limits_{\alpha\leqslant\beta\leqslant 1} f\left(\tilde{x}_\beta^L\right), \min\limits_{\alpha\leqslant\beta\leqslant 1} f\left(\tilde{x}_\beta^U\right)\right\}}{\min\left\{\min\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^L\right), \min\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^U\right)\right\}} \geqslant \frac{\min\left\{\min\limits_{\alpha\leqslant\beta\leqslant 1} g\left(\tilde{y}_\beta^L\right), \min\limits_{\alpha\leqslant\beta\leqslant 1} g\left(\tilde{y}_\beta^U\right)\right\}}{\min\left\{\min\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{G}\left(\tilde{y}_\beta^L\right), \min\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{G}\left(\tilde{y}_\beta^U\right)\right\}}$$

and

$$\frac{\max\left\{\max\limits_{\alpha\leqslant\beta\leqslant 1} f\left(\tilde{x}_\beta^L\right), \max\limits_{\alpha\leqslant\beta\leqslant 1} f\left(\tilde{x}_\beta^U\right)\right\}}{\max\left\{\max\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^L\right), \max\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^U\right)\right\}} \geqslant \frac{\max\left\{\max\limits_{\alpha\leqslant\beta\leqslant 1} g\left(\tilde{x}_\beta^L\right), \max\limits_{\alpha\leqslant\beta\leqslant 1} g\left(\tilde{x}_\beta^U\right)\right\}}{\max\left\{\max\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{G}\left(\tilde{x}_\beta^L\right), \max\limits_{\alpha\leqslant\beta\leqslant 1} \tilde{G}\left(\tilde{x}_\beta^U\right)\right\}}$$

For each $\alpha, \beta \in (0,1] \bigcap Q$ where $\tilde{F}, f$ are the survival and density functions of X respectively and $\tilde{G}, g$ are the survival and density functions of Y respectively.

Proposition 4.3. *If X and Y are fuzzy random variables with continuous distribution functions and if $X \geqslant_{hr} Y$ then $\bar{F}(t) \leqslant \bar{G}$ for all t, where $F(t)$ is fuzzy probability distribution function of X and $G(t)$ is fuzzy probability distribution function of Y.*

Proposition 4.4. *If X and Y are two fuzzy random variables such that $X \geqslant_{hr} Y$ then $X \geqslant_{st} Y$*

Proof. If $X \geqslant_{hr} Y$ then the survival function of X is smaller than that of Y for each $\alpha \leqslant \beta \leqslant 1$ and $\alpha, \beta \in (0,1] \bigcap Q$. This Shows that

$$\min\left\{\min_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^U\right)\right\}$$
$$\leqslant \min\left\{\min_{\alpha\leqslant\beta\leqslant 1} \tilde{G}\left(\tilde{x}_\beta^L\right), \min_{\alpha\leqslant\beta\leqslant 1} \tilde{G}\left(\tilde{x}_\beta^U\right)\right\}$$

and

$$\max\left\{\max_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leqslant\beta\leqslant 1} \tilde{F}\left(\tilde{x}_\beta^U\right)\right\}$$
$$\leqslant \max\left\{\max_{\alpha\leqslant\beta\leqslant 1} \tilde{G}\left(\tilde{x}_\beta^L\right), \max_{\alpha\leqslant\beta\leqslant 1} \tilde{G}\left(\tilde{x}_\beta^U\right)\right\}.$$

This shows that

$$\min\left\{\min_{\alpha\leqslant\beta\leqslant 1} P\left(\tilde{x}_\beta^L > t\right), \min_{\alpha\leqslant\beta\leqslant 1} P\left(\tilde{x}_\beta^U > t\right)\right\}$$
$$\leqslant \min\left\{\min_{\alpha\leqslant\beta\leqslant 1} P\left(\tilde{y}_\beta^L > t\right), \min_{\alpha\leqslant\beta\leqslant 1} P\left(\tilde{y}_\beta^U > t\right)\right\},$$
$$\max\left\{\max_{\alpha\leqslant\beta\leqslant 1} P\left(\tilde{x}_\beta^L > t\right), \max_{\alpha\leqslant\beta\leqslant 1} P\left(\tilde{x}_\beta^U > t\right)\right\}$$
$$\leqslant \max\left\{\max_{\alpha\leqslant\beta\leqslant 1} P\left(\tilde{y}_\beta^L > t\right), \max_{\alpha\leqslant\beta\leqslant 1} P\left(\tilde{x}_\beta^U > t\right)\right\}.$$

The above establishes that $X \geqslant_{st} Y$ and the proof is complete. □

Proposition 4.5 (Closed under increasing function)**.** *If X and Y be two non negative fuzzy random variables with have the survival functions $\bar{F}g^{-1}$ and $\bar{G}g^{-1}$. If $X \geqslant_{hr} Y$ then $\bar{F}g^{-1}(t) \leqslant \bar{G}g^{-1}(t)$, where $\bar{G}g^{-1}(t) > 0$ and $\bar{G}g^{-1}(t) \leqslant \bar{F}g^{-1}(t)$, where $\bar{F}g^{-1}(t) > 0$.*

Proof. Given $X \geqslant_{hr} Y \Rightarrow \min\left\{\min \bar{F}g^{-1}\left(\tilde{x}_\beta^L\right), \min \bar{F}g^{-1}\left(\tilde{x}_\beta^U\right)\right\}$ $\leqslant \min\left\{\min \bar{G}g^{-1}\left(\tilde{y}_\beta^L\right), \min \bar{F}g^{-1}\left(\tilde{x}_\beta^U\right)\right\}$ and

$$\Rightarrow \max\left\{\max \bar{F}g^{-1}\left(\tilde{x}_\beta^L\right), \max \bar{F}g^{-1}\left(\tilde{x}_\beta^U\right)\right\}$$
$$\leqslant \max\left\{\max \bar{G}g^{-1}\left(\tilde{y}_\beta^L\right), \max \bar{F}g^{-1}\left(\tilde{x}_\beta^U\right)\right\}$$
$$\Rightarrow \min\left\{\min P\left(\tilde{x}_\beta^L > g^{-1}(t)\right), \min P\left(\tilde{x}_\beta^U\right) > g^{-1}(t)\right\}$$
$$\leqslant \min\left\{\min P\left(\tilde{y}_\beta^L > g^{-1}(t)\right), \min P\left(\tilde{x}_\beta^U > g^{-1}(t)\right)\right\}$$
$$\Rightarrow \max\left\{\max P\left(\tilde{x}_\beta^L > g^{-1}(t)\right), \max P\left(\tilde{x}_\beta^U\right) > g^{-1}(t)\right\}$$
$$\leqslant \max\left\{\max P\left(\tilde{y}_\beta^L > g^{-1}(t)\right), \max P\left(\tilde{x}_\beta^U > g^{-1}(t)\right)\right\}$$
$$\Rightarrow \min\left\{\min P\left(g\left(\tilde{x}_\beta^L\right) > t\right), \min P\left(g\left(\tilde{x}_\beta^U\right)\right) > (t)\right\}$$
$$\leqslant \min\left\{\min P\left(\tilde{y}_\beta^L > g^{-1}(t)\right), \min P\left(\tilde{x}_\beta^U > g^{-1}(t)\right)\right\}$$
$$\Rightarrow \max\left\{\max P\left(g\left(\tilde{x}_\beta^L\right) > t\right), \max P\left(g\left(\tilde{x}_\beta^U\right)\right) > (t)\right\}$$
$$\leqslant \max\left\{\max P\left(\tilde{y}_\beta^L > g^{-1}(t)\right), \max P\left(\tilde{x}_\beta^U > g^{-1}(t)\right)\right\}$$
$$\Rightarrow g(X) \leqslant g(Y).$$

□

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