



Kamal decomposition method and its application in solving coupled system of nonlinear PDE's

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Abstract

In this paper we are solving coupled system of non linear partial differential equations by a new method called Kamal decomposition method (KDM). The new method is coupling of the Kamal transform and the Adomian decomposition method. The generalized solution has been proved .Kamal decomposition method (KDM) is very successful tool for finding the exact solution of linear and non linear partial differential equations. The existence and uniqueness of solution is based on KDM.

Keywords

Kamal decomposition method (KDM), coupled system of nonlinear PDE's.

AMS Subject Classification

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1. Introduction

Multiple problems in and mathematics carved by non-linear partial differential equation. Various researchers are putting and efforts to go through these problems finding the exact or approximate solutions using diverse methods. Many researchers were keen in solving differential equations as well as paid immersion in going through the solution of nonlinear partial differential equations by various methods. In the past few years, a number of integral transforms has been introduced .which helps us in solving ODEs and PDEs. we will apply Kamal decomposition method to find the exact solution to for solving coupled system of nonlinear partial

differential equations (CSNLPDEs).A new Kamal Transform are introduced by [6] Abdelilah Kamal(2016). Dualities between Kamal integral transforms and some integral transforms has been found [8].The utility of Kamal integral transform method [7, 9] is exist in the literature to solve partial differential equations, ordinary differential equations, fractional ordinary differential equation and integral equations. We can see that several problem in the field of physics and engineering to show the accuracy of the KDM. For more details, see for instance [5,10-17].

2. Preliminaries & Definitions of Kamal transform

2.1 Kamal transform

The Kamal transform is denoted by operator $K(\cdot)$ and Kamal transform of $f(t)$ is defined by the integral equation:

$$K(f(t)) = G(v) = \int_0^{\infty} f(t)e^{\frac{-t}{v}} dt, \quad t \geq 0,$$

$$\text{and } k_1 \leq v \leq k_2, \quad (2.1)$$

in a set A the function is defined in the form

$$A = \{f(t) : \exists M, k_1, k_2 > 0. |f(t)|$$

$$< Me^{\frac{|t|}{k_j}}, \text{ if } t \in (-1)^j \times [0, \infty)\}, \quad (2.2)$$

where k_1 and k_2 may be finite or infinite and the constant M must be finite number. For existence of Kamal transform is that $f(t)$ for $t \geq 0$ is piece wise continuous and of exponential order, else it will not be exist.

Remark 2.1. *The reader can be read more about the Kamal transform in [6].*

2.2 Derivative of Kamal transform

Let function $f(t)$ then derivative of $f(t)$ with respect to t and the n^{th} order derivative of the same with respect to t are respectively. Then Kamal transform of derivative given by:

$$K[f^n(t)] = \frac{1}{v^n} G(v) - \sum_{k=0}^{n-1} v^{k-n+1} f^k(0). \tag{2.3}$$

If we put $n = 1, 2, 3, \dots$ in equation (2.3), then we get Kamal transform of first and second derivative of $f(t)$ with respect to t .

$$K[f'(t)] = \frac{1}{v} G(v) - f(0)$$

$$K[f''(t)] = \frac{1}{v^2} G(v) - \frac{1}{v} f(0) - f'(0)$$

2.3 Adomian decomposition method

Adomian decomposition method is a semi analytical method for solving varied types of differential and integral equation, both linear and non-linear, and including partial differential equations. This method was developed from the 1970s to the 1990s by George Adomian [1–3]. The main advantage of this method is that it reduces the size of computation work and maintains the high accuracy of the analytical solution in terms of a rapidly convergence series [4]. In Adomian decomposition method, a solution can be decomposed into an infinite series that converges rapidly into the exact solution. The linear and non-linear portion of the equation can be separated by Adomian decomposition method. The inversion of linear operator can be represented by the linear operator any given condition is taken into consideration. The decomposition of a series is obtained by non linear portion which is called Adomian polynomials. By the using Adomian polynomials we can find a solution in the form of a series which can be determined by the recursive relationship.

3. Analysis of Kamal dcomposition method (KDM)

In this section we explain the Kamal decomposition method (KDM) for non linear non- homogeneous system of PDEs of the form :

$$G_t u + G_y w + L_1(u, w) = s_1(y, t) \tag{3.1}$$

$$G_t w + G_y u + L_2(u, w) = s_2(y, t),$$

with subject to initial conditions

$$u(y, 0) = h_1(y) \tag{3.2}$$

$$w(y, 0) = h_2(y).$$

where $s_1(y, t), s_2(y, t)$ are the non- homogeneous terms (source term). G_t and G_y are first differential operators. $L_1(u, w)$ and $L_2(u, w)$ are the non linear operators.

we apply the Kamal transform to Eq.(3.1) and Eq.(3.2) to get :

$$\frac{1}{v} u(y, v) - u(y, 0) + K[w_y] + K[L_1(u, w)] = K[s_1(y, t)] \tag{3.3}$$

$$\frac{1}{v} w(y, v) - w(y, 0) + K[u_y] + K[L_2(u, w)] = K[s_2(y, t)].$$

By substituting the given initial condition in Eq.(3.2) in to Eq.(3.3), we obtain

$$u(y, v) = v h_1(y) + v K[s_1(y, t)] - v K[w_y + L_1(u, w)] \tag{3.4}$$

$$u(y, v) = v h_2(y) + v K[s_2(y, t)] - v K[w_y + L_2(u, w)]$$

taking the inverse Kamal transform of Eq.(4.4) we get:

$$u(y, t) = S_1(y, t) - K^{-1}[v K[w_y + L_1(u, w)]] \tag{3.5}$$

$$w(y, t) = S_2(y, t) - K^{-1}[v K[u_y + L_2(u, w)]]$$

where the terms $S_1(y, t)$ and $S_2(y, t)$ coming from the source terms.

we have function $u(y, t)$ and $w(y, t)$ which is unknown functions, for these functions we adopt infinite series solution of the form:

$$u(y, t) = \sum_{n=0} u_n(y, t) \tag{3.6}$$

$$w(y, t) = \sum_{n=0} w_n(y, t)$$

Now, we can easily decompose the non linear terms $L_1(u, w)$ and $L_2(u, w)$ are can be written as:

$$L_1(u, w) = \sum_{n=0} A_n(y, t) \tag{3.7}$$

$$L_2(u, w) = \sum_{n=0} B_n(y, t)$$

where A_n and B_n are Adomian polynomials which is given by :

$$A_n = \frac{1}{n!} \frac{d^n}{dy^n} \left[F \left(\sum_{i=0}^n \mu^i u_i \right) \right]_{\mu=0} \tag{3.8}$$

$$B_n = \frac{1}{n!} \frac{d^n}{dy^n} \left[F \left(\sum_{i=0}^n \mu^i w_i \right) \right]_{\mu=0}$$

where $n = 0, 1, 2, 3, \dots$

Using Eq. (3.8) and Eq.(3.7),we get :

$$\sum_{n=0} u_n(y, t) = S_1(y, t) - K^{-1} \left[v K \left[\sum_{n=0} w_{ny} + \sum_{n=0} A_n \right] \right] \tag{3.9}$$

$$\sum_{n=0} w_n(y, t) = S_2(y, t) - K^{-1} \left[v K \left[\sum_{n=0} u_{ny} + \sum_{n=0} B_n \right] \right]$$



Therefore, from Eq.(3.9). we can find the recursive relation .which is given by

$$\begin{aligned}
 u_0(y,t) &= S_1(y,t) \\
 u_1(y,t) &= -K^{-1} \left[vK \left[\sum_{n=0} w_{0y} + \sum_{n=0} A_0 \right] \right] \\
 u_2(y,t) &= -K^{-1} \left[vK \left[\sum_{n=0} w_{1y} + \sum_{n=0} A_1 \right] \right]
 \end{aligned}
 \tag{3.10}$$

thus,

$$u_{n+1}(y,t) = -K^{-1} [vK [w_{ny} + A_0]] n \geq 0.
 \tag{3.11}$$

Similarly,

$$\begin{aligned}
 w_0(y,t) &= S_2(y,t) \\
 w_1(y,t) &= -K^{-1} \left[vK \left[\sum_{n=0} u_{0y} + \sum_{n=0} B_0 \right] \right] \\
 w_2(y,t) &= -K^{-1} \left[vK \left[\sum_{n=0} u_{1y} + \sum_{n=0} B_1 \right] \right]
 \end{aligned}
 \tag{3.12}$$

Similarly, we arrive at

$$w_{n+1}(y,t) = -K^{-1} [vK [u_{ny} + B_0]] n \geq 0.$$

The exact solutions of the non linear system are given by :

$$\begin{aligned}
 u(y,t) &= \sum_{n=0} u_n(y,t) \\
 w(y,t) &= \sum_{n=0} w_n(y,t)
 \end{aligned}$$

Thus, the Adomian decomposition method gives a convergent series solution which is absolute and uniformly convergent.

4. Relevance of Kamal decomposition method

In this section, we apply the Kamal decomposition method (KDM) for two coupled systems and then compare our solutions to existing exact solutions.

Example 4.1. Consider the coupled system of nonlinear PDEs of the form:

$$\begin{aligned}
 u_t - u_{yy} - 2uu_y + (uw)_y &= 0 \\
 w_t - w_{yy} - 2ww_y + (uw)_y &= 0,
 \end{aligned}
 \tag{4.1}$$

with subject to initial conditions,

$$\begin{aligned}
 u(y,0) &= \sin y \\
 w(y,0) &= \sin y.
 \end{aligned}
 \tag{4.2}$$

By taking Kamal transform of derivatives on both sides of Eq.(4.1), we get

$$\begin{aligned}
 \frac{1}{v} u(y,v) - u(y,0) - K[u_{yy}] - 2K[uu_y] + K[(uw)_y] &= 0 \\
 \frac{1}{v} w(y,v) - w(y,0) - K[w_{yy}] - 2K[ww_y] + K[(uw)_y] &= 0.
 \end{aligned}
 \tag{4.3}$$

now using initial conditions, we arrive

$$\begin{aligned}
 u(y,v) &= v \sin y + vK[u_{yy} + 2uu_y - (uw)_y] \\
 w(y,v) &= v \sin y + vK[w_{yy} + 2ww_y - (uw)_y]
 \end{aligned}
 \tag{4.4}$$

taking the inverse Kamal transform of Eq.(4.5), we obtain

$$\begin{aligned}
 u(y,t) &= \sin y + K^{-1} [vK[u_{yy} + 2uu_y - (uw)_y]] \\
 w(y,t) &= \sin y + K^{-1} [vK[w_{yy} + 2ww_y - (uw)_y]]
 \end{aligned}
 \tag{4.5}$$

now, we assume a series solution for the unknown function $u(y,t)$ and $w(y,t)$ of the form:

$$\begin{aligned}
 u(y,t) &= \sum_{n=0} u_n(y,t) \\
 w(y,t) &= \sum_{n=0} w_n(y,t)
 \end{aligned}
 \tag{4.6}$$

then Eq.(4.5) becomes

$$\begin{aligned}
 u(y,t) &= \sin y + K^{-1} \left[vK \left[\sum_{n=0} u_{nyy} + 2A_n(u) \right. \right. \\
 &\quad \left. \left. - \left(\sum_{n=0} B_n(u,w) \right)_y \right] \right] \\
 w(y,t) &= \sin y \\
 &\quad + K^{-1} \left[vK \left[\sum_{n=0} w_{nyy} + 2C_n(w) \right. \right. \\
 &\quad \left. \left. - \left(\sum_{n=0} B_n(u,w) \right)_y \right] \right],
 \end{aligned}
 \tag{4.7}$$

where A_n, B_n and C_n are Adomian polynomials. which characterized the non linear terms $uu_y, (uw)_y$ and ww_y relatively. Now using of aforementioned techniques, we reach the following recursive relation as follows:

$$\begin{aligned}
 u_0(y,t) &= \sin y \\
 u_1(y,t) &= K^{-1} [vK [u_{0yy} + 2A_0(u) - (B_0(u,w))_y]] \\
 u_2(y,t) &= K^{-1} [vK [u_{1yy} + 2A_1(u) - (B_1(u,w))_y]] \\
 w_0(y,t) &= \sin y \\
 w_1(y,t) &= K^{-1} [vK [w_{0yy} + 2C_0(w) - (B_0(u,w))_y]] \\
 w_2(y,t) &= K^{-1} [vK [w_{1yy} + 2C_1(w) - (B_1(u,w))_y]].
 \end{aligned}
 \tag{4.9}$$

thus,

similarly,

$$\begin{aligned}
 u_{n+1}(y,t) &= K^{-1} [vK [u_{nyy} + 2A_n(u) - (B_n(u,w))_y]], \quad n \geq 0, \\
 w_{n+1}(y,t) &= K^{-1} [vK [w_{nyy} + 2C_n(w) - (B_n(u,w))_y]], \quad n \geq 0.
 \end{aligned}
 \tag{4.10}$$

Finally,

$$\begin{aligned}
 u_{n+1}(y,t) &= K^{-1} [vK [u_{nyy} + 2A_n(u) - (B_n(u,w))_y]], \quad n \geq 0. \\
 w_{n+1}(y,t) &= K^{-1} [vK [w_{nyy} + 2C_n(w) - (B_n(u,w))_y]], \quad n \geq 0.
 \end{aligned}
 \tag{4.11}$$



Consequently, from the recurrence relation in Eq.(4.9) and Eq.(4.11).we can determine remaining components of the solution as follows:

$$\begin{aligned}
 u_1(y,t) &= K^{-1} [vK [u_{0yy} + 2A_0(u) - (B_0(u,w))_y]] \\
 &= K^{-1} [vK [u_{0yy} + 2u_0u_{0y} - (u_0w_0)_y]] \\
 &= K^{-1} [vK [-\sin y + 2\sin y \cos y - 2\sin y \cos y]] \\
 &= K^{-1} [vK [\sin y]] \\
 &= -\sin y K^{-1} [vK [1]] \\
 &= -t \sin y. \\
 w_1(y,t) &= K^{-1} [vK [w_{0yy} + 2C_0(u) - (B_0(u,w))_y]] \\
 &= K^{-1} [vK [w_{0yy} + 2w_0w_{0y} - (u_0w_0)_y]] \\
 &= K^{-1} [vK [-\sin y + 2\sin y \cos y - 2\sin y \cos y]] \\
 &= K^{-1} [vK [\sin y]] \\
 &= -\sin y K^{-1} [vK [1]] \\
 &= -t \sin y.
 \end{aligned}
 \tag{4.12}$$

and

$$\begin{aligned}
 u_2(y,t) &= K^{-1} [vK [u_{1yy} + 2A_1(u) - (B_1(u,w))_y]] \\
 &= K^{-1} [vK [u_{1yy} + 2(u_1u_{0y} + u_0u_{1y}) \\
 &\quad - (u_1w_0 + u_0w_{1y} + w_1u_{0y} + w_0u_{1y})]] \\
 &= K^{-1} [vK [t \sin y - 4t \sin y \cos y + 4t \sin y \cos y]] \\
 &= \sin y K^{-1} [vK [t]] \\
 &= \sin y K^{-1} [v^3] \\
 &= \frac{t^2}{2!} \sin y. \\
 w_2(y,t) &= K^{-1} [vK [w_{1yy} + 2C_1(u) - (B_1(u,w))_y]] \\
 &= K^{-1} [vK [w_{1yy} + 2(w_1w_{0y} + w_0w_{1y}) \\
 &\quad - (u_1w_0 + u_0w_{1y} + w_1u_{0y} + w_0u_{1y})]] \\
 &= K^{-1} [vK [t \sin y - 4t \sin y \cos y + 4t \sin y \cos y]] \\
 &= \sin y K^{-1} [vK [t]] \\
 &= \sin y K^{-1} [v^3] \\
 &= \frac{t^2}{2!} \sin y.
 \end{aligned}
 \tag{4.15}$$

Finally, the approximate solution of the known functions $u(y,t)$

and $w(y,t)$ are given by:

$$\begin{aligned}
 u(y,t) &= \sum_{n=0} u_n(y,t) \\
 &= u_0(y,t) + u_1(y,t) + u_2(y,t) + \dots \\
 &= \sin y - t \sin y + \frac{t^2}{2!} \sin y + \dots \\
 &= \sin y \left(1 - t + \frac{t^2}{2!} + \dots \right) \\
 &= e^{-t} \sin y.
 \end{aligned}
 \tag{4.16}$$

and

$$\begin{aligned}
 w(y,t) &= \sum_{n=0} w_n(y,t) \\
 &= w_0(y,t) + w_1(y,t) + w_2(y,t) + \dots \\
 &= \sin y - t \sin y + \frac{t^2}{2!} \sin y + \dots \\
 &= \sin y \left(1 - t + \frac{t^2}{2!} + \dots \right) \\
 &= e^{-t} \sin y.
 \end{aligned}
 \tag{4.17}$$

Thus, we get exact solution of the given non linear coupled system are given by

$$\begin{aligned}
 u(y,t) &= e^{-t} \sin y \\
 w(y,t) &= e^{-t} \sin y.
 \end{aligned}
 \tag{4.18}$$

Example 4.2. We consider the coupled system of nonlinear PDE of the form:

$$\begin{aligned}
 p_t + u_x \phi_y - u_y \phi_x &= -p \\
 u_t + w_x p_y + p_x \phi_y &= u \\
 \phi_t + p_x u_y + p_y u_x &= \phi,
 \end{aligned}
 \tag{4.19}$$

subject to the initial conditions

$$\begin{aligned}
 p(x,y,0) &= e^{x+y} \\
 u(x,y,0) &= e^{x-y} \\
 \phi(x,y,0) &= e^{y-x}.
 \end{aligned}
 \tag{4.20}$$

Taking the Kamal transform of derivatives both sides of Eq.(4.19). we get,

$$\begin{aligned}
 \frac{1}{v} p(x,y,v) - p(x,y,0) + K[u_x \phi_y] - K[u_y \phi_x] &= K[-p] \\
 \frac{1}{v} u(x,y,v) - u(x,y,0) + K[\phi_x p_y] + K[p_x \phi_x] &= K[u] \\
 \frac{1}{v} \phi(x,y,v) - \phi(x,y,0) + K[p_x u_y] + K[p_y u_x] &= K[\phi].
 \end{aligned}
 \tag{4.21}$$

Then using the initial conditions of Eq.(4.20) into Eq.(4.21).we arrived

$$\begin{aligned}
 p(x,y,v) &= v e^{x+y} + vK [u_y \phi_x - u_x \phi_y - p] \\
 u(x,y,v) &= v e^{x-y} + vK [u - \phi_x p_y - p_x \phi_y] \\
 w(x,y,v) &= v e^{y-x} + vK [\phi - p_x u_y - p_y u_x]
 \end{aligned}
 \tag{4.22}$$



by taking inverse Kamal transform of Eq. (4.22), we obtain

$$\begin{aligned} p(x, y, t) &= e^{x+y} + K^{-1} [vK[u_y \phi_x - u_x \phi_y - p]] \quad (4.23) \\ u(x, y, t) &= e^{x-y} + K^{-1} [vK[u - \phi_x p_y - p_x \phi_y]] \\ w(x, y, v) &= e^{y-x} + K^{-1} [vK[\phi - p_x u_y - p_y u_x]] \end{aligned}$$

we have functions $p(x, y, t), u(x, y, t)$ and $\phi(x, y, t)$. Which is unknown functions, for these functions we adopt infinite series solutions of the form :

$$\begin{aligned} p(x, y, t) &= \sum_{n=0} p_n(x, y, t) \\ u(x, y, t) &= \sum_{n=0} u_n(x, y, t) \\ w(x, y, t) &= \sum_{n=0} \phi_n(x, y, t). \end{aligned}$$

From Eq. (4.23) can be re-written in the form:

$$\begin{aligned} p(x, y, t) &= e^{x+y} + K^{-1} \left[vK \left[\sum_{n=0} A_n(u, \phi) \right. \right. \\ &\quad \left. \left. - \sum_{n=0} B_n(u, \phi) - \sum_{n=0} p_n \right] \right] \quad (4.25) \\ u(x, y, t) &= e^{x-y} + K^{-1} \left[vK \left[\sum_{n=0} u_n \right. \right. \\ &\quad \left. \left. - \sum_{n=0} C_n(\phi, p) - \sum_{n=0} D_n(\phi, p) \right] \right] \\ \phi(x, y, t) &= e^{y-x} + K^{-1} \left[vK \left[\sum_{n=0} \phi_n \right. \right. \\ &\quad \left. \left. - \sum_{n=0} E_n(p, u) - \sum_{n=0} F_n(p, u) \right] \right] \end{aligned}$$

where A_n, B_n, C_n, D_n, E_n and F_n are Adomian polynomials. Which characterized the nonlinear terms $u_y \phi_x, u_x \phi_y, \phi_x p_y, p_x \phi_y, p_x u_y$ and $p_y u_x$ appropriately. Now we can obtain the recursive relation by Eq. (4.25) as follows:

$$\begin{aligned} p_0(x, y, t) &= e^{x+y} \quad (4.26) \\ p_1(x, y, t) &= K^{-1} [vK[A_0(u, \phi) - B_0(u, \phi) - p_0]] \\ p_2(x, y, t) &= K^{-1} [vK[A_1(u, \phi) - B_1(u, \phi) - p_1]] \end{aligned}$$

similarly, we can obtain $p_{n+1}(x, y, t)$. Which is given by,

$$\begin{aligned} p_{n+1}(x, y, t) &= K^{-1} [vK[A_n(u, \phi) - B_n(u, \phi) - p_n]] \quad n \geq 0. \quad (4.27) \end{aligned}$$

Again, we continue in the same manner for term $u_{n+1}(x, y, t)$ and $\phi_{n+1}(x, y, t)$ can be obtain by Eq.(4.25) easily. we will eventually have

$$\begin{aligned} u_{n+1}(x, y, t) &= K^{-1} [vK[u_n - C_n(\phi, p) - D_n(\phi, p)]], \quad n \geq 0. \quad (4.28) \end{aligned}$$

$$\begin{aligned} \phi_{n+1}(x, y, t) &= K^{-1} [vK[\phi_n - E_n(p, u) - F_n(p, u)]], \quad n \geq 0. \quad (4.29) \end{aligned}$$

Hence using the Eq. (4.27), Eq. (4.28) and Eq. (4.29). we can obtain the remaining components of the functions $p(x, y, t), u(x, y, t)$ and $\phi(x, y, t)$. Which is unknown functions as follows:

$$\begin{aligned} p_1(x, y, t) &= K^{-1} [vK[A_0(u, \phi) - B_0(u, \phi) - p_0]] \quad (4.30) \\ &= K^{-1} [vK[u_{0y} \phi_{0x} - u_{0x} \phi_{0y} - p_0]] \\ &= K^{-1} [vK[e^{x-y} e^{y-x} - e^{x-y} e^{y-x} - e^{x+y}]] \\ &= -e^{x+y} K^{-1} [vK[1]] \\ &= -e^{x+y} t. \end{aligned}$$

$$\begin{aligned} u_1(x, y, t) &= K^{-1} [vK[u_0 - C_0(\phi, p) - D_0(\phi, p)]] \quad (4.31) \\ &= K^{-1} [vK[u_0 - \phi_{0x} p_{0y} - p_{0x} \phi_{0y}]] \\ &= K^{-1} [vK[e^{x-y} - e^{y-x} e^{x+y} - e^{x+y} e^{y-x}]] \\ &= -e^{x-y} K^{-1} [vK[1]] \\ &= t e^{x-y}. \end{aligned}$$

$$\begin{aligned} \phi_1(x, y, t) &= K^{-1} [vK[\phi_0 - E_0(p, u) - F_0(p, u)]] \quad (4.32) \\ &= K^{-1} [vK[\phi_0 - p_{0x} u_{0y} - p_{0y} u_{0x}]] \\ &= K^{-1} [vK[e^{-x+y} - e^{x+y} e^{x-y} - e^{x+y} e^{x-y}]] \\ &= e^{-x+y} K^{-1} [vK[1]] \\ &= t e^{y-x}, \end{aligned}$$

and

$$\begin{aligned} p_2(x, y, t) &= K^{-1} [vK[A_1(u, \phi) - B_1(u, \phi) - p_1]] \quad (4.33) \\ &= K^{-1} [vK[(u_{1y} \phi_{0x} - u_{0y} \phi_{1x}) \\ &\quad - (u_{1x} \phi_{0x} + u_{0x} \phi_{1y}) - p_1]] \\ &= K^{-1} [vK[(t e^{x-y} e^{y-x} + e^{x-y} t e^{y-x}) \\ &\quad - (t e^{x-y} e^{-x+y} + e^{x-y} t e^{-x+y}) + t e^{x+y}]] \\ &= e^{x+y} K^{-1} [vK[t]] \\ &= e^{x+y} K^{-1} [v^3] \\ &= \frac{t^2 e^{x+y}}{2!}. \end{aligned}$$

$$\begin{aligned} u_2(x, y, t) &= K^{-1} [vK[u_1 - C_1(\phi, p) - D_1(\phi, p)]] \quad (4.34) \\ &= K^{-1} [vK[u_1 - (\phi_{1x} p_{0y} + \phi_{0x} p_{1y}) \\ &\quad - (p_{1x} \phi_{0y} + p_{0x} \phi_{1y})]] \\ &= K^{-1} [vK[(t e^{x-y} - t e^{y-x} e^{x+y} + e^{-x+y} t e^{x+y}) \\ &\quad - (t e^{x+y} e^{y-x} + e^{x+y} t e^{-x+y})]] \\ &= e^{x-y} K^{-1} [vK[t]] \\ &= e^{x-y} K^{-1} [v^3] \\ &= \frac{t^2 e^{x-y}}{2!}. \end{aligned}$$



$$\begin{aligned}
 \phi_2(x, y, t) &= K^{-1}[vK[\phi_1 - E_1(p, u) - F_1(p, u)]] \quad (4.35) \\
 &= K^{-1}[vK[\phi_1 - (p_{1x}u_{0y} + p_{0x}u_{1y}) \\
 &\quad - (p_{1y}u_{0x} + p_{0y}u_{1x})]] \\
 &= K^{-1}[vK[te^{-x+y} - (te^{x+y}e^{x-y} - e^{x+y}te^{x-y}) \\
 &\quad - (-te^{x+y}e^{x-y} + e^{x+y}te^{x-y})]] \\
 &= e^{-x+y}K^{-1}[vK[t]] \\
 &= e^{y-x}, K^{-1}[v^3] \\
 &= \frac{t^2e^{y-x}}{2!}.
 \end{aligned}$$

Finally, the approximate solution of the unknown function $p(x, y, t)$, $u(x, y, t)$ and $\phi(x, y, t)$ is given by:

$$\begin{aligned}
 p(x, y, t) &= \sum_{n=0} p_n(x, y, t) \quad (4.36) \\
 &= p_0(x, y, t) + p_1(x, y, t) + p_2(x, y, t) + \dots \\
 &= e^{x+y} - te^{x+y} + \frac{t^2e^{x+y}}{2!} + \dots \\
 &= e^{x+y} \left[1 - t + \frac{t^2}{2!} + \dots \right] \\
 &= e^{x+y-t},
 \end{aligned}$$

and

$$\begin{aligned}
 u(x, y, t) &= \sum_{n=0} u_n(x, y, t) \quad (4.37) \\
 &= u_0(x, y, t) + u_1(x, y, t) + u_2(x, y, t) + \dots \\
 &= e^{x-y} - te^{x-y} + \frac{t^2e^{x-y}}{2!} + \dots \\
 &= e^{x-y} \left[1 - t + \frac{t^2}{2!} + \dots \right] \\
 &= e^{x-y+t}.
 \end{aligned}$$

Subsequently,

$$\begin{aligned}
 \phi(x, y, t) &= \sum_{n=0} \phi_n(x, y, t) \quad (4.36) \\
 &= \phi_0(x, y, t) + \phi_1(x, y, t) + \phi_2(x, y, t) + \dots \\
 &= e^{y-x} - te^{y-x} + \frac{t^2e^{y-x}}{2!} + \dots \\
 &= e^{y-x} \left[1 - t + \frac{t^2}{2!} + \dots \right] \\
 &= e^{y-x+t},
 \end{aligned}$$

Hence the exact solution of the unknown functions which is given by:

$$\begin{aligned}
 p(x, y, t) &= e^{x+y-t} \quad (4.39) \\
 u(x, y, t) &= e^{x-y+t} \\
 \phi(x, y, t) &= e^{y-x+t}.
 \end{aligned}$$

This is same results obtained by NDM [5].

5. Conclusion

In this paper the Kamal decomposition method (KDM) was used for solving system of non linear coupled partial differential equation with initial conditions. We found KDM is powerful and easy-to-use analytic tool for PDE's and thus, the present study highlights the efficiency of the method. Also, we get exact solution as well as compare the result with NDM [5]. This clearly shows that Kamal decomposition method technique play important role in future for solving nonlinear PDE's.

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