Malaya Journal of Matematik, Vol. 6, No. 3, 626-631, 2018

https://doi.org/10.26637/MJM0603/0025

Doubly connected geodetic number of graphs

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Abstract

In this paper, we investigate the results on doubly connected geodetic number of a simple graph G, shadow distance graph and corona of two graphs. Further we verify how doubly connected geodetic number is affected by adding a pendant vertex.

Keywords

Corona, Geodetic number, shadow distance graphs, Vertex covering number.

AMS Subject Classification

10C75, 10C10, 10C12.

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Article History: Received **22** April **2018**; Accepted **05** August **2018**

Contents

1. Introduction

Let $I(u,v)$ be the set of all vertices lying on some $u-v$ geodesic of G and for a nonempty subset S of $V(G)$, $I[S] = \cup_{u,v \in S} I(u,v)$. The set S of vertices of G is called a geodetic set in G if $I[S] = V(G)$ and a geodetic set of minimum cardinality is a minimum geodetic set in G is called the geodetic number $g(G)$ was studied in [2].

Nonsplit geodetic number of a graph was studied by Tejaswini, Venkanagouda M Goudar in [6]. Venkanagouda M Goudar, et al., [7] introduced the concept of strong nonsplit geodetic number of a graph. The connected geodetic number was introduced by Santhakumaran, et al., in [5].

A set $S \subseteq V$ in a graph G is a doubly connected geodetic set $[DCGS]$ if S is a geodetic set and induced subgraphs $< S >$ and $< V - S >$ are both connected. The minimum cardinality of a doubly connected geodetic set and it is denoted by $g_{dc}(G)$ is called doubly connected geodetic number of a G. A doubly connected geodetic set of cardinality $g_{dc}(G)$ is called $g_{dc}(G)$-set and it was introduced by Bhavyavenu and Venkanagouda M Goudar[1]. To illustrate this concepts, consider the following examples:

Example 1.1. We depicted a graph G given in Figure 1, $S_1 = \{v_1, v_2, v_3, v_4, v_5\}$ is a g_{dc}-set so that $g_{dc}(G) = 5$. Also $S_2 = \{v_3, v_4, v_5, v_6, v_7\}$ is another g_{dc}-set of G is as shown in Figure 1.

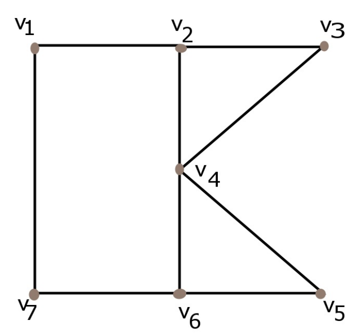

Figure 1. G

For any undefined term in this paper, see [2,3,4,10].

The following theorems are used in the sequel.

Theorem 1.1. *[2] For any cycle C_n of order $n \geq 3$,*

$$g(C_n) = \begin{cases} 2 & \text{if } n \text{ is even,} \\ 3 & \text{if } n \text{ is odd.} \end{cases}$$

Theorem 1.2. *[2] Every geodetic set of a graph contains its extreme vertices.*

Theorem 1.3. *[2] For integers $r,s \geq 2$, $g(K_{r,s}) = min\{r,s,4\}$.*

Theorem 1.4. *[3] For any cycle C_n of order $n \geq 3$,*

$$\alpha_o(C_n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even,} \\ \frac{n+1}{2} & \text{if } n \text{ is odd.} \end{cases}$$

Theorem 1.5. *[3] For any cycle C_n of order $n \geq 3$,*

$$\alpha_1(C_n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even,} \\ \frac{n+1}{2} & \text{if } n \text{ is odd.} \end{cases}$$

Theorem 1.6. *[6] For any cycle C_n of order $n \geq 3$,*

$$g_{ns}(C_n) = \begin{cases} \frac{n}{2}+1 & \text{if } n \text{ is even,} \\ \lfloor \frac{n}{2} \rfloor + 2 & \text{if } n \text{ is odd.} \end{cases}$$

2. Main results

Theorem 2.1. *For the wheel $W_n = K_1 + C_{n-1}$, $n \geq 5$, $g_{dc}(W_n) = n-2$.*

Proof. Let $W_n = K_1 + C_{n-1}$ and let $V(W_n) = \{x, v_1, ...v_{n-1}\}$, where $deg(x) = n-1 > 3$ and $deg(v_i) = 3$ for all $i \in \{1,2,...n-1\}$. Let $P = \{x, v_{n-1}\}$ be the set and $S_1 = V(W_n) - P$ be the doubly connected geodetic set such that the induced subgraphs $< S_1 >$ and $< V - S_1 >$ are connected. Hence $|S_1| = n-2$. Therefore $g_{dc}(W_n) = n-2$. □

Corollary 2.2. *For any wheel W_n $n \geq 4$, $g_{dc}(G) = n-d$, where d is the diameter of wheel.*

Proof. Since the diameter of wheel is 2, hence it follows by the Theorem 2.1, that $g_{dc}(G) = n-d$. □

Theorem 2.3. *Let $K_{r,s}$ be the complete bipartite graph, such that $r \geq 2$ and $s \geq 3$. Then $g_{dc}(K_{r,s}) = 4$.*

Proof. Let $G = K_{r,s}$ such that $X = \{x_1, x_2, ...x_r\}$ and $Y = \{y_1, y_2, ...y_r\}$ are the partite set of G and $V = X \cup Y$. Let $S = \{x_i, x_j, y_k, y_l\}$ for some $1 \leq i,j \leq r$, $1 \leq k,l \leq s$, be the doubly connected geodetic set such that both induced subgraphs $< S >$ and $< V - S >$ are connected. Hence S is a doubly connected geodetic set. Therefore $g_{dc}(K_{r,s}) = 4$. □

Corollary 2.4. *For any complete bipartite graph $K_{r,s}$, $r,s \geq 4$, $g_{dc}(K_{r,s}) = d+2$, where d is a diameter.*

Proof. For any complete bipartite graph, the diameter is 2. Hence the proof follows from the Theorem 2.3. □

Corollary 2.5. *For any complete tripartite graph $K_{r,s,t}$, $r,s,t \geq 3$,*
$g_{dc}(K_{r,s,t}) = 4$.

Corollary 2.6. *For any connected graph G, $g(G) \leq g_c(G) \leq g_{dc}(G)$.*

Theorem 2.7. *For any cycle C_n with $n \geq 4$, $g_{dc}(C_n) = g_c(C_n)$.*

Proof. Let C_n be a cycle. We consider the two cases.

Case(i) Suppose n is even, $n = 2p$. Let $S = \{v_1, v_2, ...v_{p+1}\}$ be a connected geodetic set of C_n such that $< S >$ is connected and observed that $< V(C_n) - S >$ is also connected. Hence $g_c(C_n) = g_{dc}(C_n)$.

Case(ii) Suppose n is odd, $n = 2p+1$. Let $S = \{v_1, v_2, ...v_{p+1}, v_{p+2}\}$ be a connected geodetic set of C_n. So that $< S >$ is connected and also observed that $< V(C_n) - S >$ is also connected. Hence $g_c(C_n) = g_{dc}(C_n)$. Clearly $g_c(C_n) = g_{dc}(C_n)$. □

Theorem 2.8. *For any cycle C_n, $g_{ns}(C_n) = g_{dc}(C_n)$.*

Proof. Let C_n be a cycle. We discuss the two cases.

Case(i) Suppose n is even, $n = 2p$. Let $S = \{v_1, v_2, ...v_{p+1}\}$ be a nonsplit geodetic set of C_n such that $< V(C_n) - S >$ is connected and observed that $< S >$ is also connected, which is a doubly connected geodetic set . Hence $g_{ns}(C_n) = g_{dc}(C_n)$.

Case(ii) Suppose n is odd, $n = 2p+1$. Let $S = \{v_1, v_2, ...v_{p+1}, v_{p+2}\}$ be a nonsplit geodetic set of C_n so that $< V(C_n) - S >$ is connected and observed that $< S >$ is also connected. So that the set S itself forms the minimum doubly connected geodetic set. Hence $g_{ns}(C_n) = g_{dc}(C_n)$. Clearly $g_{ns}(C_n) = g_{dc}(C_n)$. □

Corollary 2.9. *For cycle C_n, $n \geq 4$, $g_{dc}(C_n) = \alpha_0(C_n) + 1$.*

In the following results we show that how the doubly connected geodetic number is affected by removing the edges of K_n.

Theorem 2.10. *If $G = K_n - e$ is a graph obtained from K_n by removing an edge e with $e = \{v_i, v_j\}$, $n \geq 5$, then $g_{dc}(G) = 3$.*

Proof. Let $G = K_n - e$ be a graph with $e = \{v_i, v_j\}$. Clearly $S = \{v_i, v_j\}$ forms a strong non-split geodetic set. But $< S >$ is not connected. Consider $S_1 = S \cup \{v_k\}$, where $\{v_k\}$ is the any vertex of G such that induced subgraphs $< S_1 >$ and $< V - S_1 >$ both are connected. Clearly S_1 is a doubly connected geodetic set. Thus $g_{dc}(K_n - e) = |S_1| = |S \cup \{v_k\}| = |S| + |\{v_k\}| = 3$. □

Theorem 2.11. *If $G = K_n - \{e_1, e_2\}$ of order $n \geq 5$, then*

$$g_{dc}(G) = \begin{cases} 4 & \text{when } e_1, e_2 \text{ are adjacent,} \\ 3 & \text{when } e_1, e_2 \text{ are not adjacent.} \end{cases}$$

Proof. Let $G = K_n - \{e_1, e_2\}$. We discuss the following cases.

Case(i) Suppose e_1 and e_2 are adjacent. Let $e_1 = xy$ and $e_2 = xz$ for some $x, y, z \in V(G)$ and they are extreme vertices. Clearly no two vertex set forms a geodetic set. Let $S = \{x, y, z\}$ be the nonsplit geodetic set with minimum cardinality. But the induced subgraph $< S >$ is not connected. Consider $S_1 = S \cup \{u\}$, where $u \in V(G)$, be the doubly connected geodetic set of G such that induced subgraphs $< S_1 >$ and $< V - S_1 >$ are connected. Hence $|S_1| = |S \cup \{u\}| = |S| + |\{u\}| = 4$. Therefore $g_{dc}(K_n - \{e_1, e_2\}) = 4$.

Case(ii) Suppose e_1 and e_2 are not adjacent. Let $e_1 = uv$ and $e_2 = xy$ for some $\{u, v, x, y\} \in V(G)$. Then $S = \{x, y\} = \{u, v\}$ is the nonsplit geodetic set with minimum cardinality. But $< S >$ is not connected. Consider $S_1 = S \cup \{w\}$, where $w \in V(G)$, be the doubly connected geodetic set of G. Hence $|S_1| = |S \cup \{w\}| = |S| + |\{w\}| = 3$. Therefore $g_{dc}(K_n - \{e_1, e_2\}) = 3$. □

Theorem 2.12. *For a graph G, there is no doubly connected geodetic set if and only if G is one of the graph P_n or K_n or $K_{1,s}$. Where P_n, K_n and $K_{1,s}$ are the path, complete bipartite and star.*

Proof. Let $V(P_n) = \{v_1, v_2, ...v_n\}$ and $S = \{v_1, v_n\}$ be the nonsplit geodetic set but $\langle S \rangle$ is not connected. To make S connected, we required all the internal vertices of P_n. Clearly $g_{dc}(P_n) = n$ but $V - S = \{\emptyset\}$, which is contradiction to our assumption. Therefore for path P_n there is no doubly connected geodetic set.

Let $V(K_n) = \{v_1, v_2, ...v_n\}$ be the vertex set of K_n and $d(v_i, v_j) = 1$ in K_n. Therefore $S = V(K_n)$. Clearly the set S contains all the vertices of K_n for the geodetic set, then $V - S = \{\emptyset\}$. Therefore there is no doubly connected geodetic set for K_n.

Let $V(K_{1,s}) = \{x, v_1, v_2, ...v_s\}$ be the vertex set of $K_{1,s}$. consider the geodetic set $S = \{v_1, v_2, ...v_s\}$, but $\langle S \rangle$ is not connected. To make S connected, we required internal vertex $\{x\}$. Consider $S_1 = \{x\}$ and $A = S \cup S_1 = S \cup \{x\}$ which contains all the vertices of $K_{1,s}$ that is $V - S = \{\emptyset\}$, which is contradiction to our assumption. Therefore for star $K_{1,s}$ there is no doubly connected geodetic set. □

3. Results on adding a pendant vertex

In the next theorem we show that how doubly connected geodetic number is affected by adding pendent vertices.

Theorem 3.1. *Let $G = C_n$ be a cycle of order $n \geq 4$ and G' be the graph obtained by adding a pendant edge (v_i, u) with $v_i \in G$, $u \notin G$, then*

$$g_{dc}(G') = \begin{cases} \frac{n}{2} + 2 & \text{if } n \text{ is even,} \\ \frac{n-1}{2} + 3 & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let $V(G) = \{v_1, v_2, v_3, ...v_n, v_1\}$ be the cycle with n vertices. If G' be the graph obtained from $G = C_n$ by adding a pendant edge (v_i, u) such that $u \notin G$ and for any $v_i \in G$. We have two cases.

Case(i) Suppose n is even. Then $S = \{u, v_j\}$ be a nonsplit geodetic set of G', where v_j is the antipodal vertex of v_i. But the induced subgraph $< S >$ is not connected. Consider $S' = \{v_i, v_{i+1}, ...v_{j-1}\}$ and let $S_1 = S \cup S'$ is a doubly connected geodetic set. Clearly induced subgraphs $< S_1 >$ and $< V - S_1 >$ are connected. Hence $|S_1| = |S| + |S'| = 2 + \frac{n}{2}$. Therefore $g_{dc}(G') = \frac{n}{2} + 2$.

Case(ii) Suppose n is odd. Then $S = \{u, v_j, v_{j+1}\}$ be a nonsplit geodetic set with minimum cardinality of G', where v_j, v_{j+1} are the antipodal vertices of v_i. But $< S >$ is not connected. Consider $S' = \{v_i, v_{i+1}, ...v_{j-1}\}$ and let $S_1 = S \cup S'$ is a doubly connected geodetic set. Clearly induced sub graphs $< S_1 >$ and $< V - S_1 >$ are connected. Hence $|S_1| = |S| + |S'| = 3 + \frac{n-1}{2}$. Therefore $g_{dc}(G') = \frac{n-1}{2} + 3$. □

Theorem 3.2. *Let $G = C_n$ be a cycle of order $n \geq 4$ and G' be the graph obtained by adding k pendant edges $\{(v_i, w_1), (v_i, w_2), (v_i, w_3), ...(v_i, w_k)\}$ to the cycle C_n with $v_i \in G$ and $\{w_1, w_2, w_3, ...w_k\} \notin G$ then,*

$$g_{dc}(G') = \begin{cases} \frac{n}{2} + 1 + k & \text{if } n \text{ is even,} \\ \frac{n-1}{2} + 2 + k & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Consider $V(G) = \{v_1, v_2, v_3, ...v_n, v_1\}$ be the cycle with n vertices. If G' be the graph obtained from $G = C_n$ by adding pendant edges $\{(v_i, w_1), (v_i, w_2), (v_i, w_3), ...(v_i, w_k)\}$ such that $\{w_1, w_2, w_3, ...w_k\} \notin G$ and v_i is a single vertex of G. We discuss the following cases.

Case(i) Let n is even. Then $S = \{w_1, w_2, w_3, ...w_k \cup v_j\}$ be a geodetic set of G', where $\{w_1, w_2, w_3, ...w_k\}$ are the k pendant vertices and v_j is the antipodal vertex of v_i. But the induced subgraph $< S >$ is not connected. Consider $S' = \{v_i, v_{i+1}, ...v_{j-1}\}$ and let $S_1 = S \cup S'$ is a doubly connected geodetic set. Clearly induced subgraphs $< S_1 >$ and $< V - S_1 >$ are connected. Hence

$|S_1| = |S| + |S'|$
$= |\{w_1, w_2, w_3, ...w_k \cup v_j\}| + |S'|$
$= |\{w_1, w_2, w_3, ...w_k\}| + |v_j| + |S'|$
$= k + 1 + \frac{n}{2}$. Therefore $g_{dc}(G') = \frac{n}{2} + k + 1$.

Case(ii) Let n is odd. Then $S=\{w_1,w_2,w_3,...w_k,v_j,v_{j+1}\}$ be a geodetic set with minimum cardinality of $G^{'}$, where $\{w_1,w_2,w_3,...w_k\}$ are the k pendant vertices and v_j,v_{j+1} are the antipodal vertices of v_i. But the induced subgraph $<S>$ is not connected. Consider $S^{'}=\{v_i,v_{i+1},...v_{j-1}\}$ and let $S_1=S\cup S^{'}$ is a doubly connected geodetic set. Clearly induced subgraphs $<S_1>$ and $<V-S_1>$ are connected. Hence
$|S_1|=|S|+|S^{'}|$
$=|\{w_1,w_2,w_3,...w_k\cup v_j\cup v_{j+1}\}|+|S^{'}|$
$=|\{w_1,w_2,w_3,...w_k\}+v_j+v_k|+|S^{'}|$
$=k+1+1+\frac{n-1}{2}$
$=k+2+\frac{n-1}{2}$. Therefore $g_{dc}(G^{'})=\frac{n-1}{2}+k+2$. □

Theorem 3.3. *Let $G=C_n$ be a graph and $G^{'}$ obtained from G by adding k pendent veritces $\{u,v_1,v_2,v_3,...v_k\}$ with $v_1,v_2,...v_k\notin G$ and $u\in G$ then $g_{dc}(G^{'})=\alpha_o(C_n)+1+k$.*

Proof. Let C_n be the cycle of order $n\geq 4$. We have two cases.

Case(i) Let C_n be the cycle and it is even and $\alpha_o(c_n)$ be the vertex covering number. By Theorem 3.2, for even cycle $g_{dc}(G^{'})=\frac{n}{2}+k+1$. Also by Theorem 1.4, $\alpha_o(C_n)=\frac{n}{2}$. Therefore $g_{dc}(C_n)=\alpha_o(C_n)+k+1$.

Case(ii) Let C_n be the cycle and it is odd and $\alpha_o(c_n)$ be the vertex covering number. By theorem 3.2, for odd cycle $g_{dc}(G^{'})=\frac{n+1}{2}+k+1$. Also by theorem 1.4, $\alpha_o(C_n)=\frac{n+1}{2}$, hence $g_{dc}(C_n)=\alpha_o(C_n)+k+1$. □

In the next section, we obtain the doubly connected geodetic number on shadow distance graph.

4. Shadow distance graph

Definition 4.1.
Let D be the set of all distance between distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The distance graph G denoted by $D(G,D_s)$ is the graph having the same vertex as that of G and two vertices u and v are adjacent in $D(G,D_s)$ whenever $d(u,v)\in D_s$.
The shadow distance graph of G, denoted by $D_{sd}(G,D_s)$ is constructed from G with the following conditions:
i) Consider two copies of G say G itself and G'
ii)if $u\in V(G)$ (first copy) then we denoted the corresponding vertex as $u'\in V(G')$ (second copy)
iii) The vertex set of $D_{sd}(G,D_s)$ is $V(G)\cup V(G')$
iv) The edge set of $D_{sd}(G,D_s)$ is $E(G)\cup E(G')\cup E_{ds}$, where E_{ds} is the set of all edges between two distinct vertices $u\in V(G)$ and $v'\in V(G')$ that satisfy the condition $d(u,v)\in D_s$ in G.

The shadow distance graphs $D_{sd}(P_8,\{2\})$ and $D_{sd}(C_7,\{3\})$ are as shown in figure 2 and 3.

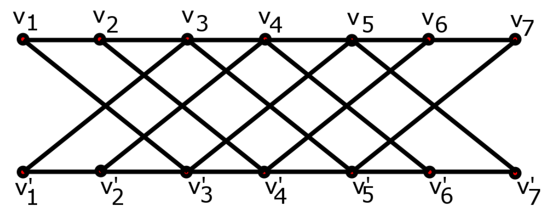

Figure 2. $D_{sd}(P_8,\{2\})$

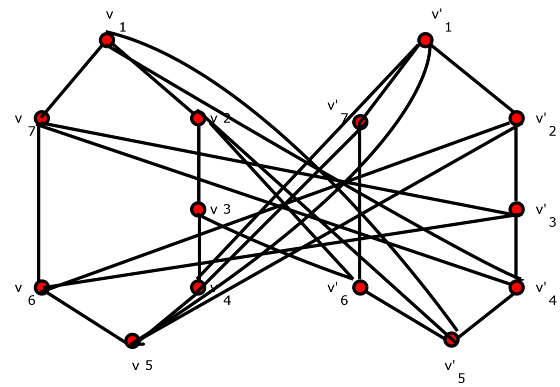

Figure 3. $D_{sd}(C_7,\{3\})$

Theorem 4.2. *For $n\geq 3$,*

$$g_{dc}(D_{sd}(P_n,\{2\}))=\begin{cases} n+1 & \text{for n=3,}\\ n+2 & \text{for } n>3. \end{cases}$$

Proof. Let $G=g_{dc}(D_{sd}(P_n,\{2\}))$ be the shadow distance graph. Let $V(G)=V_1\cup V_1'$, where $V_1=\{v_1,v_2,...v_i/1\leq i\leq n\}$ be the vertex set of P_n and $V_1'=\{v_1',v_2',...v_i'/1\leq i\leq n\}$ be the vertex set of P_n'. Then $|V(G)|=2n$.
Case(i) For $n=3$, $G=C_6$. By Theorem 3.7 it follows that $g_{dc}(G)=4$. Hence $g_{dc}(D_{sd}(P_3,\{2\}))=n+1$.
Case(ii) For $n>3$. Consider $S=\{v_1,v_n,v_1',v_n'\}$ be the geodetic set with minimum cordinality, but $\langle S\rangle$ is not connected. Consider $S_1=S\cup\{v_2,v_3,...v_{n-1}\}$ be the doubly connected geodetic set of G.
Suppose $v_i\in V(P_n)$ for $2\leq i\leq n-1$ and $v_i\notin S_1$, then $\langle S_1\rangle$ is disconnected. So that every vertices of P_n must exists in the set S_1. For $v_i'\in V'(P_n)$, where v_i' is the end vertices of $V'(P_n)$ and $v_i'\notin S_1$ for i=1 or n then $\langle S_1\rangle$ is not a geodetic set. So that end vertices of P_n' must exists in the set S_1. Therefore $g_{dc}(G)=|S_1|=|S\cup\{v_2,v_3,...v_{n-1}\}|=4+n-2=n+2$.
Hence $g_{dc}(D_{sd}(P_n,\{2\}))=n+2$. □

Corollary 4.3. *For $n\geq 4$, $g_{dc}(D_{sd}(P_n,\{3\}))=n+2$.*

Theorem 4.4. *For $n \geq 4$,*

$$g_{dc}(D_{sd}(C_n,\{2\})) = \begin{cases} 4 & \text{for n=4,5} \\ \frac{n}{2}+2 & \text{for n=6,8} \\ \frac{n+1}{2}+2 & \text{for n=7,9} \\ \frac{n+6}{2} & \text{for } n \geq 10 \text{ and n is even} \\ \frac{n+9}{2} & \text{for } n \geq 11 \text{ and n is odd.} \end{cases}$$

Proof. Consider two copies of C_n namely C_n itself and C'_n. Let $\{u_1,u_2,...u_n\}$ be the vertices of first copy of C_n and $\{u'_1,u'_2,...u'_n\}$ be the vertices of second copy of C'_n. Let $G = D_{sd}(C_n,\{2\})$ be a shadow distance graph with $|V(G)| = |V(C_n) \cup V(C'_n)| = 2n$ and $|E(G)| = 4n$. We discuss the following cases.

Case(i) For $n = 4,5$. The set $S = \{v_1,v_2,v_3,v'_1\}$ is a doubly connected geodetic set with minimum cardinality. Hence $g_{dc}(D_{sd}(C_n,\{2\})) = 4$.

Case(ii) For $n = 6,8$. The geodetic set $S = \{v_1,v_{\frac{n}{2}},v'_1,v'_{\frac{n}{2}}\}$, but $\langle S\rangle$ is not connected, then S is not a doubly connected geodetic set. Now consider $S_1 = S \cup \{v_2,...v_{\frac{n}{2}-1}\}$ forms a $g_{dc} - set$. Therefore $g_{dc}(D_{sd}(C_n,\{2\})) = |S_1| = \frac{n}{2}+2$.

Case(iii) For $n = 7,9$. The geodetic set $S = \{v_1,v_{\frac{n+1}{2}},v'_1,v'_{\frac{n+1}{2}}\}$, but $\langle S\rangle$ is not connected . Now consider $S_1 = S \cup \{v_2,...v_{\frac{n-1}{2}}\}$ forms a $g_{dc} - set$. Therefore $g_{dc}(D_{sd}(C_n,\{2\})) = |S_1| = \frac{n+1}{2} + 2$.

Case(iv) For $n \geq 10$ and is even. Consider the set $S = \{v_1,v_2,...v_i,v'_1,v'_i/1 \leq i \leq \frac{n}{2}+1\}$ is a $g_{dc} - set$ with minimum cardinality . Suppose for any vertex $v_j \in S$, $F = S - v_j$ is not a doubly connected geodetic set in G, because $\langle F\rangle$ or $\langle V - F\rangle$ are not connected. If v_j is an end vertex then it is not a geodetic set. Therefore $g_{dc}(D_{sd}(C_n,\{2\})) = |S| = \frac{n+6}{2}$.

Case(v) For $n \geq 11$ and is odd. Consider the set $S = \{v_1,v_{\frac{n+1}{2}}, v_{\frac{n+1}{2}+1},v'_1,v'_{\frac{n+1}{2}},v'_{\frac{n+1}{2}+1}\}$ is a nonsplit geodetic set. But $\langle S\rangle$ is not connected. Consider a set $S_1 = \{v_2,v_3,...v_{\frac{n+1}{2}-1}\}$ are the vertices between v_1 and $v_{\frac{n+1}{2}+1}$. Then doubly connected geodetic set $A = S \cup S_1$.Therefore $g_{dc}(D_{sd}(C_n,\{2\})) = |A| = |S \cup S_1| = |S| + |S_1| = 6 + \frac{n-3}{2} = \frac{n+9}{2}$. ☐

Corollary 4.5. *For $n \geq 8$,*

$$g_{dc}(D_{sd}(C_n,\{3\})) = \begin{cases} \frac{n+6}{2} & \text{for n is even} \\ \frac{n+9}{2} & \text{for n is odd.} \end{cases}$$

In the next section, we obtain the doubly connected geodetic number on corona of two graphs.

5. Corona of two graphs

Definition 5.1. *Let G and H be two graphs and let n be the order of G. The corona product $G \circ H$ is defined as the graph obtained from G and H by taking one copy of G and n copies of H and then joining by an edge, all the vertices form the i^{th} copy of H with the i^{th} vertex of G.*

Theorem 5.2. *For the cycle C_n of order $n \geq 4$, $g_{dc}(K_1 \circ C_n) = n-2$.*

Proof. Consider $H = K_1$ and $G = C_n$, $n > 3$. Let $u_1 \in V(K_1)$ and $v_i \in V(C_n)$, $1 \leq i \leq n$. For each v_i is the adjacent to the vertex u_1, then $K_1 \circ C_n$ form the wheel. By the Theorem 2.2, we have $g_{dc}(W_n) = n-2$. It follows that $g_{dc}(K_1 \circ C_n) = n-2$. ☐

Theorem 5.3. *Let G be a connected graph of order n, such that $\triangle(G) = n-1$. Then $g_{dc}(K_1 \circ G) = g(G)+1$, where $g(G)$ is the geodetic number of G.*

Proof. Let $H = K_1 = \{u\}$ and $V(G) = \{v_1,v_2,v_3,...v_n\}$ be the vertex set of H and G respectively, we have $\triangle(G) = n-1$. Let $S = \{v_1,v_2,...v_l/1 \leq l < n\}$ be the geodetic set of G. Consider $G_1 = H \circ G$ be the corona graph. It is easy to verify that S is the geodetic set of G_1, but the induced subgraph $< S >$ is not connected. Consider $S_1 = S \cup \{u\}$ be a doubly connected geodetic set so that induced subgraphs $< S >$ and $< V - S >$ are connected. Hence $|S_1| = |S \cup \{u\}| = |S| + 1$. Therefore $g_{dc}(G_1) = g(G)+1$. ☐

Theorem 5.4. *Let P_{n_1} and P_{n_2} be the paths of order $n_1 \geq 2$, $n_2 \geq 3$ then $g_{dc}(P_{n_1} \circ P_{n_2}) = n_1 + n_1n_2 - 1$.*

Proof. Let $P_{n_1} : v_1,v_2,...v_{n_1}$ and $P_{n_2} : u_1,u_2,...u_{n_2}$ are the vertices of P_{n_1} and P_{n_2} with $|V((P_{n_1} \circ P_{n_2}))| = n_1 + n_1n_2$.

Case(i) If n_2 is odd, then $S = \{\cup_{i=1}^{n_1} v_i \cup \{u_1,u_3,...u_{n_2}\}_{n_1 times}\}$ is a connected geodetic set but it is not a doubly connected geodetic set, because $\langle V(P_{n_1} \circ P_{n_2}) - S\rangle$ is not connected. To make $\langle V(P_{n_1} \circ P_{n_2}) - S\rangle$ connected, we consider $A = S \cup \{u_2,u_4,...u_{n_2-1}\}_{n_1 times} - \{u\}$ where $u \in V(P_{n_2})$ which forms a doubly connected geodetic set of $P_{n_1} \circ P_{n_2}$ with minimum cardinality. Thus we have,

$g_{dc}(P_{n_1} \circ P_{n_2}) = |A|$
$=|S \cup \{u_2,u_4,...u_{n_2-1}\} - \{u\}|$
$= |S| + |\{u_2,u_4,...u_{n_2-1}\} - \{u\}|$
$=|\{\cup_{i=1}^{n_1} v_i \cup \{u_1,u_3,...u_{n_2}\}_{n_1 times}\}| + |\{u_2,u_4,...u_{n_2-1}\}_{n_1 times} - \{u\}|$
$=n_1 + \frac{n_1n_2}{2} + \frac{n_1n_2}{2} - 1$
$g_{dc}(P_{n_1} \circ P_{n_2}) = n_1 + n_1n_2 - 1$.

Case(ii) If n_2 is even, then $S = \{\cup_{i=1}^{n_1} v_i \cup \{u_1,u_3,...,u_{n_2-1},u_{n_2}\}_{n_1 times}\}$ is a connected geodetic set but it is not a doubly connected geodetic set, because $\langle V(P_{n_1} \circ P_{n_2}) - S\rangle$ is not connected. To make $\langle V(P_{n_1} \circ P_{n_2}) - S\rangle$ connected we consider $A = S \cup \{u_2,u_4,...u_{n_2-2}\}_{n_1 times} -$

$\{u\}$ where $u \in V(P_{n_2})$ which forms a doubly connected geodetic set of $P_{n_1} \circ P_{n_2}$ with minimum cardinality. Thus we have,
$g_{dc}(P_{n_1} \circ P_{n_2}) = |A|$
$=|S \cup \{u_2, u_4, ...u_{n_2-2}\} - \{u\}|$
$= |S| + |\{u_2, u_4, ...u_{n_2-1}\} - \{u\}|$
$=|\{\cup_{i=1}^{n_1} v_i \cup \{u_1, u_3, ..., u_{n_2-1}, u_{n_2}\}_{n_1 times}\}|$
$+|\{u_2, u_4, ...u_{n_2-2}\}_{n_1 times} - \{u\}|$
$=n_1 + \frac{n_1n_2+1}{2} + \frac{n_1n_2-1}{2} - 1.$
Therefore $g_{dc}(P_{n_1} \circ P_{n_2}) = n_1 + n_1n_2 - 1.$ □

Corollary 5.5. *Let P_{n_1} and C_{n_2} are the path and cycle of order $n_1 \geq 2$, $n_2 \geq 4$, then $g_{dc}(P_{n_1} \circ C_{n_2}) = n_1 + n_1n_2 - 1$.*

Corollary 5.6. *Let C_{n_1} and C_{n_2} are the cycles of order $n_1 \geq 3$, $n_2 \geq 4$, then $g_{dc}(C_{n_1} \circ C_{n_2}) = n_1 + n_1n_2 - 1$.*

Corollary 5.7. *Let C_{n_1} and C_{n_2} are the cycles of order $n_1 \geq 3$, $n_2 \geq 4$, then $g_{dc}(C_{n_1} \circ C_{n_2}) = n_1 + n_1n_2 - 1$.*

6. Conclusion

In this paper, we discussed the doubly connected geodetic number of a graph. Also we obtained how doubly connected geodetic number is affected by adding a pendant vertex,corona and shadow distance graph.

Acknowledgment

We are very thankful to the referees for reading the paper and giving valuable suggestions.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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