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Edge geodetic parameters of snake graphs

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Abstract

In this paper, we investigate the different edge geodetic parameters of triangular snake graph, double triangular snake graph, alternate triangular snake graph, double alternate triangular snake graph, quadrilateral snake graph, double quadrilateral snake graph, alternate quadrilateral snake graph, double alternate quadrilateral snake graph.

Keywords

Edge geodetic set, Snake Graphs, Split edge geodetic Set.

AMS Subject Classification

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Contents

1. Introduction

An edge geodetic set of G is a set $S \subseteq V(G)$ such that every edge of G is contained in a geodesic joining some pair of vertices in S. The edge geodetic number $g_1(G)$ of G is the minimum order of its edge geodetic sets. This concept was introduced in [5]. The concept of split edge geodetic number (g_{1s}) was introduced in [7]. A. P. Santhakumaran et al. [6] introduced the concept of restrained edge geodetic number (eg_r). In [8] Venkanagouda M Goudar and Shobha introduced total edge geodetic number (g_{1t}). The concept of strong split geodetic number (g_{ss}) was introduced in [1]. Further the concept of nonsplit geodetic number (g_{ns}) was introduced in [10]. Let $P_n : v_1, v_2, ..., v_n$ be the path of length $n-1$.

In this paper, we investigated the edge geodetic number, split edge geodetic number, strong split geodetic number of different snake graphs in terms of blocks, regions, vertex covering number. For more details on this theory, we suggest the reader to refer [2,3,4,9].

2. Edge geodetic parameters of Snake graphs

Definition 2.1. *The triangular snake T_n is obtained from a path P_n by joining v_i and v_{i+1} to a new vertex u_i.*

Theorem 2.2. *Let $G = T_n$ be the triangular snake with $(n \geqslant 3)$ then $g_1(G) = n+1$.*

Proof. Let $G = T_n$. Let $| V(G) | = 2n-1$ and $| E(G) | = 3(n-1)$. Let $S = \{v_1, v_n\} \cup Q$ where $\{v_1, v_n\}$ are the end vertices of path P_n and $Q = \cup u_i$ are the new vertices joined to v_i and v_{i+1}. Since each edge of G lies on a geodesic joining any two vertices of S then S is an edge geodetic set of G. Hence

$$\begin{aligned} g_1(G) &= |S| \\ &= n+1. \end{aligned}$$

Corollary 2.3. *For the triangular snake $G = T_n$ $(n \geqslant 4)$, $eg_r(G) = n+1$.*

Corollary 2.4. *Let T_n $(n \geqslant 3)$ be the triangular snake then $g_{ns}(T_n) = n+1$.*

Corollary 2.5. *Let $G = T_n$ be the triangular snake with $(n \geqslant 3)$, $g_{1t}(G) = n+1$.*

Theorem 2.6. *The strong split geodetic number for a triangular snake graph T_n is*

$$g_{ss}(G)=\begin{cases}\frac{3n}{2} & \text{if } n \text{ is even },\\ \frac{3n-1}{2} & \text{if } n \text{ is odd }.\end{cases}$$

Proof. Let $V(T_n) = \{v_1,v_2,...v_n,u_1,u_2,...u_{n-1}\}$ where $\{v_1,v_2,...,v_n\} \in V(P_n)$ and $\{u_1,u_2,...u_{n-1}\}$ are the new vertices joined to v_i and v_{i+1} for $1 \leq i \leq n-1$ so that a triangle $C_3 = \{v_i,w_i,v_{i+1}\}$ is obtained. We consider the following cases:

Case 1 For n is even. Let $S = \{v_1,v_n,u_1,u_2,...u_{n-1}\}$ be the minimum geodetic set of T_n. But induced subgraph $\langle V-S\rangle$ is connected. Consider $S^{'} = S \cup \{v_3,v_5,...v_{n-1}\}$. Clearly $\langle V-S^{'}\rangle$ is totally disconnected. Therefore

$$\begin{aligned} g_{ss}(G) &= \mid S^{'} \mid, \\ &= \mid S \mid + n + 1, \\ &= \frac{3n}{2}. \end{aligned}$$

Case 2 For n is odd. Consider the geodetic set $S = \{v_1,v_n,u_1,u_2,...u_{n-1}\}$ and $I[S] = V(G)$. Let $S^{'} = S \cup \{v_3,v_5,...v_{n-2}\}$. Now induced subgraph $\langle V-S^{'}\rangle$ has isolated vertices. Thus S is the strong split geodetic set. Hence

$$\begin{aligned} g_{ss}(G) &= \mid S^{'} \mid, \\ &= \mid S \mid + n + 1, \\ &= \frac{3n-1}{2}. \end{aligned}$$

□

Theorem 2.7. *For the triangular snake $G = T_n$ with $(n \geqslant 6)$, $g_{1s}(G) = r+2$ where r is the number of regions in G.*

Proof. Let $V(G) = \{v_1,v_2,...v_n,u_1,u_2,...u_{n-1}\}$ where $v_i \in V(P_n)$ and $\{u_j/1 \leq j \leq n-1\}$ are the new vertices joined to v_i and v_{i+1}. Then $\mid V(G) \mid = 2n-1$ and $\mid E(G) \mid = 3(n-1)$. Let $R = \{r_1,r_2,r_3,...r_n\}$ be the region set of G where each region consists of $C_3 = \{v_i,u_i,v_{i+1}\}$ and $\mid R \mid = r$. Let $S = \{v_1,v_n\} \cup Q$ where $\{v_1,v_n\}$ are the end vertices of P_n and $Q = \cup u_j$. Clearly S is an edge geodetic set of G and $V-S$ is connected. Let $S^{'} = S \cup \{v_k\}$ where $\{v_k\}, 3 \leq k \leq n-2$ is any one internal vertex of P_n. Then $V-S^{'}$ is disconnected. Therefore

$$\begin{aligned} g_{1s}(G) &= \mid S^{'} \mid \\ &= \mid S \cup v_k \mid \\ &= n+2 \\ &= \mid R \mid + 2 \\ &= r+2. \end{aligned}$$

□

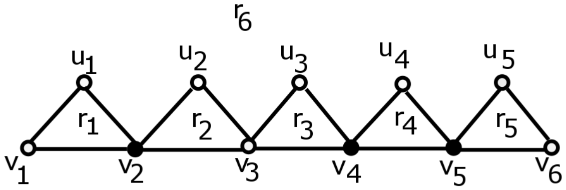

Figure 1. G

Example: For a triangular snake graph T_6 given in Figure 1. The empty color vertices is its split edge geodetic set.

$S = \{v_1,u_1,u_2,u_3,u_4,u_5,v_6\}$ is the edge geodetic set so that $g_1(T_6) = 7$ and $S^{'} = \{v_1,v_3,u_1,u_2,u_3,u_4,u_5,v_6\}$ is the split edge geodetic set so that $g_{1s}(T_6) = 8 = 6+2 = r+2$.

Definition 2.8. *The double triangular snake DT_n is obtained by path P_n by joining v_i and v_{i+1} to a new vertex u_i for $i = 1,2,...n-1$ and to a new vertex w_i for $i = 1,2,...n-1$. Let $V(G) = \{v_1,v_2,...v_n,u_1,u_2,...u_{n-1},w_1,w_2,...w_{n-1}\}$ where u_i adjacent to v_i and v_{i+1} in upward direction and w_i adjacent to v_i and v_{i+1} in downward direction.*

Theorem 2.9. *For the double triangular snake $G = DT_n$ $(n \geqslant 2)$, $g_1(G) = b+r-k$ where b be the number of blocks , r be the number of regions and k be the number of internal vertices.*

Proof. Let $G = DT_n$. By Definition 2.8, $\mid V(G) \mid = 2n-1$ and $\mid E(G) \mid = 5(n-1)$. Let $B = \{B_1,B_2,...B_{n-1}\}$ be the blocks such that $B_i = \{v_i,u_i,w_i,v_{i+1}\}$ for $1 \leq i \leq n-1$. Let $\{R_1,R_2,...,R_{2n-1}\}$ be the number of regions and $\mid \{R_1,R_2,...,R_{2n-1}\} \mid = r$. The set $S = \{u_1,u_2,...u_{n-1},w_1,w_2,...w_{n-1}\}$ forms the minimum geodetic set of G. But each edge of G does not lie on geodesic joining any two vertices of S. Let $S^{'} = S \cup \{v_1,v_n\}$. Clearly $S^{'}$ is edge geodetic set of G. Hence $g_1(G) = b+r-k$. □

Corollary 2.10. *Let $G = DT_n$ $(n \geqslant 4)$ be the double triangular snake, then $eg_r(G) = b+r-k$ where b be the number of blocks , r be the number of regions and k be the number of internal vertices.*

Corollary 2.11. *For the double triangular snake DT_n $(n \geqslant 2)$, $g_{ns}(DT_n) = b+r-k$ where b be the number of blocks , r be the number of regions and k be the number of internal vertices.*

Corollary 2.12. *Let $G = DT_n$ $(n \geqslant 2)$ be the double triangular snake, then $g_{1t}(G) = b+r-k$ where b be the number of blocks , r be the number of regions and k be the number of internal vertices.*

Theorem 2.13. *For the double triangular snake* DT_n $(n \geqslant 3)$,

$$g_{ss}(DT_n) = \begin{cases} \frac{5n-4}{2} & if\ n \equiv 0(mod2), \\ \frac{5(n-1)}{2} & otherwise\ . \end{cases}$$

.

Proof. Let $G = DT_n$. By Definition 2.8, let $\{u_i, w_i / 1 \leq i \leq n-1\}$ be the new vertices added to v_i and v_{i+1} in upward and downward direction. Let $\{B_1, B_2, ...B_{n-1}\}$ be the blocks of DT_n. Now, the geodetic set of G must have vertices of degree 2 from each block and hence $S = \{u_i, w_i\}$ is the geodetic set. Hence, to attain the minimum strong split geodetic set of DT_n, we construct a vertex set $X \subset V(DT_n)$ as follows:

$$X = \begin{cases} \{v_1, v_3, ..., v_{n-1}, u_i, w_i\} & \text{if } n \equiv 0(mod2), \\ \{v_2, v_4, ..., v_{n-1}, u_i, w_i\} & \text{otherwise} \end{cases}$$

where $1 \leq i \leq n-1$.
Then

$$|\, X \,| = \begin{cases} \frac{5n-4}{2} & \text{if } n \equiv 0(mod2), \\ \frac{5(n-1)}{2} & \text{otherwise .} \end{cases}$$

Since each vertex in $V(DT_n)$ is either in X or is adjacent to a vertex in X, it follows that X is the minimum strong split geodetic set as $< V - X >$ is totally disconnected. Thus,

$$g_{ss}(DT_n) = \begin{cases} \frac{5n-4}{2} & \text{if } n \equiv 0(mod2), \\ \frac{5(n-1)}{2} & \text{otherwise .} \end{cases}$$

☐

Theorem 2.14. *For the double triangular snake* $G = DT_n$ $(n \geqslant 6)$, $g_{1s}(G) = 2n+1$.

Proof. By Definition 2.8, let $V(G) = \{v_1, v_2, ...v_n, u_1, u_2, ...u_{n-1}, w_1, w_2, ...w_{n-1}\}$ such that $|\, V(G) \,| = 2n-1$ and $|\, E(G) \,| = 5(n-1)$. Let $S = S_1 \cup S_2 \cup S_3$ where

$$\begin{aligned} S_1 &= \{v_1, v_n\} \\ S_2 &= \cup\{w_i\} \\ S_3 &= \cup\{u_i\} \end{aligned}$$

and $1 \leq i \leq n-1$. Then S is an edge geodetic set. But $< V - S >$ is connected , consider $S' = S \cup \{v_j\}, 3 \leq j \leq n-2$ is any one internal vertex of P_n. Clearly $< V - S' >$ is disconnected. Therefore

$$\begin{aligned} g_{1s}(G) &= |\, S' \,| \\ &= 2n+1. \end{aligned}$$

☐

Definition 2.15. *An alternate triangular snake* AT_n *is obtained from a path* P_n *by joining* v_i *and* v_{i+1} *alternatively to a new vertex* u_i *where* $1 \leq i \leq n$ *for n even and* $1 \leq i \leq n-1$ *for n odd .*

Theorem 2.16. *Let* $G = AT_n$ $(n \geqslant 3)$ *be the alternate triangular snake, then* $g_1(G) = \lfloor \frac{n}{2} \rfloor + 2$.

Proof. Let $G = AT_n$. By Definition 2.15

$$V(G) = \begin{cases} \frac{3n}{2} & \text{if } n \equiv 0(mod2), \\ \frac{3(n-1)}{2} + 1 & \text{if } n \equiv 1(mod2)\ . \end{cases}$$

In order to obtain the geodetic set of G, the set must contain vertices of degree two in each cycle for n even while for n odd, it must contain the vertices of degree two and a pendent vertex. Hence, in order to attain the minimum cardinality of a vertex set of G, we can construct the vertex set of G as follows:

$$S = \begin{cases} \{v_1, v_n, u_1, u_2, ..., u_{\frac{n}{2}}\} & \text{if n is even,} \\ \{v_1, v_n, u_1, u_2, ..., u_{\frac{n-1}{2}}\} & \text{if n is odd .} \end{cases}$$

Clearly S is also an edge geodetic set of G. Then $g_1(G) = |\, S \,| = \lfloor \frac{n}{2} \rfloor + 2$ ☐

Corollary 2.17. *Let* $G = AT_n$ $(n \geqslant 4)$ *be the alternate triangular snake, then* $eg_r(G) = \lfloor \frac{n}{2} \rfloor + 2$.

Corollary 2.18. *Let* $G = AT_n$ $(n \geqslant 3)$ *be the alternate triangular snake, then* $g_{ns}(G) = \lfloor \frac{n}{2} \rfloor + 2$.

Theorem 2.19. *For the alternate triangular snake* AT_n $(n \geqslant 5)$,

$$g_{ss}(AT_n) = \begin{cases} n+1 & if\ n\ is\ even, \\ n & otherwise\ . \end{cases}$$

.

Proof. Let AT_n be the alternate triangular snake graph obtained by replacing every alternate edges of P_n by a triangle C_3. Let $U = \{v_1, v_2, ..., v_n\}$ be the vertices of path P_n and $W = \{u_1, u_2, ..., u_{\lfloor \frac{n}{2} \rfloor}\}$ be the new vertices which are joined alternatively to v_i and v_{i+1} such that $V(AT_n) = U \cup W$. We discuss the following cases:
Case 1 Let n be even. Let $S = S_1 \cup S_2$ where $S_1 = \{v_1, v_n\} \subseteq U$ and $S_2 = \{u_1, u_2, ..., u_{\frac{n}{2}}\} \subseteq W$ having $\frac{n}{2}$ vertices. Let S be the minimum set of vertices, such that $I[S] = V(AT_n)$ and the set of vertices of the induced subgarph $< V(AT_n) - S >$ is connected. Let $S' = S \cup \{v_2, v_4, ...v_{n-2}\}$. Clearly $< V(AT_n) - S >$ has isolated vertices. Therefore $g_{ss}(AT_n) = |\, S' \,| = n+1$.
Case 2 Let n be odd. Consider $S = \{v_1, v_n, u_1, u_2, ..., u_{\lfloor \frac{n}{2} \rfloor}\}$ be the geodetic set of AT_n. But $< S >$ has one component. Let $S' = S \cup \{v_3, v_5, ...v_{n-2}\}$. Clearly $< V(AT_n) - S' >$ is totally disconnected. Hence $g_{ss}(AT_n) = |\, S' \,| = n$. ☐

Theorem 2.20. *Let* $G = AT_n$ $(n \geqslant 6)$ *be the alternate triangular snake, then* $g_{1s}(G) = \lfloor \frac{n}{2} \rfloor + 1$.

Proof. Let $G = AT_n$. We have the following cases:
Case 1 Suppose n is even. Let $V(G) = \left\{v_1, v_2, ...v_n, u_1, u_2, ...u_{\frac{n}{2}}\right\}$.
Let $S = S_1 \cup S_2$ where

$$\begin{aligned} S_1 &= \{u_1, u_3, ..., u_{n-1}\} \\ S_2 &= \{v_n\} \end{aligned}$$

Here S is the edge geodetic set with minimum cardinality containing $\frac{n+2}{2}$ vertices. Then $< V-S >$ is disconnected. Therefore

$$\begin{aligned} g_{1s}(G) &= \mid S \mid \\ &= \frac{n}{2}+1 \end{aligned}$$

Case 2 Suppose n is odd. Let $V(G)=\left\{v_1,v_2,...v_n,u_1,u_2,...u_{\frac{n-1}{2}}\right\}$. Let $S=\{u_1,u_3,...,u_{n-2}, v_n\}$. Clearly S is the edge geodetic set with minimum cardinality containing $\lfloor\frac{n}{2}\rfloor+1$ vertices. But $< V-S >$ is disconnected. Therefore S is split edge geodetic set. Hence

$$\begin{aligned} g_{1s}(G) &= \mid S \mid \\ &= \lfloor\frac{n}{2}\rfloor+1 \end{aligned}$$

□

Definition 2.21. *The double alternate triangular snake DAT_n consists of two alternate triangular snake which have a common path.*

Theorem 2.22. *Let $G=DAT_n$ $(n\geqslant 4)$ be the double alternate triangular snake, then*

$$g_1(G)=\begin{cases} b+3 & if\, n\equiv 0(mod2),\\ b+2 & if\, n\equiv 1(mod2).\end{cases}$$

where b is the number of blocks.

Proof. Let $V(G)=\left\{v_i,u_j,w_j\right\}$ for $1\le i\le n, 1\le j\le\lfloor\frac{n}{2}\rfloor$ where v_i are the vertices of P_n and u_j,w_j are the vertices obtained from a path $P_n: v_1,v_2,...v_n$ by joining v_i and v_{i+1} alternatively. Then

$$V(G)=\begin{cases} 2n & \text{if n is even ,}\\ 2n-1 & \text{if n is odd .}\end{cases}$$

Let $B=\{B_1,B_2\}$ be the blocks of G, where $B_1=\{b_1,b_2,...,b_{\frac{n}{2}}\}$ and $B_2=\{b'_1,b'_2,...,b'_{\frac{n-1}{2}}\}$ such that $b_i=\{v_i,u_i,w_i,v_{i+1}\}$, $b'_i=\{v_{i+1},v_{i+2}\}$ and $\mid B\mid=b$.
Let us consider the following cases:
Case 1 when n is even. Let $S=\{u_1,u_2,...,u_{\frac{n}{2}},w_1,w_2,...,w_{\frac{n}{2}}\}$ be the geodetic set of G. But S is not edge geodetic set. Let $S'=S\cup\{v_1,v_n\}$. Clearly S' is an edge geodetic set. Hence $g_1(G)=b+3$.
Case 2 when n is odd. Let $S=\{u_1,u_2,...,u_{\frac{n}{2}},w_1,w_2,...,w_{\frac{n}{2}}\}$ be the geodetic set of G . Let $S'=S\cup\{v_1,v_n\}$. Clearly S' is an edge geodetic set. Hence $g_1(G)=b+2$. □

Corollary 2.23. *For the double alternate triangular snake $G=DAT_n$ $(n\geqslant 4)$,*

$$eg_r(G)=\begin{cases} b+3 & if \quad n\equiv 0(mod2)\,,\\ b+2 & if \quad n\equiv 1(mod2).\end{cases}$$

where b is the number of blocks.

Corollary 2.24. *For the double alternate triangular snake $G=DAT_n$ $(n\geqslant 5)$,*

$$g_{1t}(G)=\begin{cases} \frac{3n}{2} & if\, n\, is\, even\,,\\ \frac{3n-1}{2} & if\, n\, is\, odd.\end{cases}$$

where b is the number of blocks.

Theorem 2.25. *The strong split geodetic number of double alternate triangular snake $G=DAT_n$ $(n\geqslant 5)$ is,*

$$g_{ss}(G)=\begin{cases} \frac{3n}{2} & if\, n\, is\, even\,,\\ \frac{3n-1}{2} & if\, n\, is\, odd.\end{cases}$$

Proof. Let $G = DAT_n$. Let $V(G)=\{v_1,v_2,...,v_n,u_1,u_2,...,u_{\frac{n}{2}},w_1,w_2,...,w_{\frac{n}{2}}\}$ be the vertex set of G. We discuss the following cases:
Case 1 Suppose n is even. Consider $S=\{S_1,S_2,S_3\}$ where $S_1=\cup u_i/1\le i\le\frac{n}{2}$ and $S_2=\cup w_i/1\le i\le\frac{n}{2}$. Now, $< V-S >$ contains the set of vertices $\{v_i\}$ for $1\le i\le n$ such that $deg(v_i)\neq 0$. Then S is not strong split geodetic set. Let $S'=\{v_k/1\le k\le n-1\}$ which are all non adjacent vertices. Clearly $< V-S' >$ is totally disconnected. Therefore, $g_{ss}(G)=\mid S'\mid=\frac{3n}{2}$.
Case 2 Suppose n is odd. Let $S=\{u_1,u_2,...,u_{\frac{n}{2}},w_1,w_2,...,w_{\frac{n}{2}},v_n\}$ be the minimum geodetic set of G. Let $S'=S\cup\{v_k\}$ for $1\le k\le n-1$. Clearly induced subgraph $< V-S' >$ has isolated vertices. Therefore $g_{ss}(G)=\mid S'\mid=\frac{3n-1}{2}$. □

Theorem 2.26. *Let $G=DAT_n$ $(n\geqslant 6)$ be the double alternate triangular snake, then*

$$g_{1s}(G)=\begin{cases} n+3 & if\, n\, is\, even\,,\\ n+2 & if\, n\, is\, odd\,.\end{cases}$$

Proof. Let G be the double alternate triangular snake with $V(G)=\{v_1,v_2,...,v_n,u_1,u_2,...,u_{\lfloor\frac{n}{2}\rfloor},w_1,w_2,...,w_{\lfloor\frac{n}{2}\rfloor}\}$ We discuss the following cases:
Case 1 For n is even, $\mid V(G)\mid=2n$. Let $S=\{v_1,v_n\}\cup S_1\cup S_2$ where $S_1=\cup u_j$ and $S_2=\cup w_j$ $/1\le j\le\lfloor\frac{n}{2}\rfloor$. Thus $I[S]=V(G)$. Clearly S is an edge geodetic set. But $< V-S >$ is connected. Consider $S'=S\cup\{v_k\}$, $3\le k\le n-2$ where v_k is any one internal vertex so that $< V-S' >$ is disconnected. Thus S' is the split edge geodetic set. Therefore

$$\begin{aligned} g_{1s}(G) &= \mid S'\mid \\ &= S+1 \\ &= 2+\frac{n}{2}+\frac{n}{2}+1 \\ &= n+3. \end{aligned}$$

Case 2 For n is odd, $\mid V(G)\mid=2n-1$. Let $S=\{v_1,v_n\}\cup\{u_j\}\cup\{w_j\}$. Thus $I[S]=V(G)$. Clearly S is an edge geodetic set. But $< V-S >$ is connected. Consider $S'=S\cup\{v_k\}$ for

$3 \le k \le n-2$ where $\{v_k\}$ is the only internal vertex so that $< V - S' >$ is disconnected. Therefore

$$\begin{aligned} g_{1s}(G) &= \mid S' \mid \\ &= S+1 \\ &= n+2. \end{aligned}$$

□

Example: For the Double alternate triangular snake graph DAT_7 given in Figure 2. The empty color vertices is its split edge geodetic set.

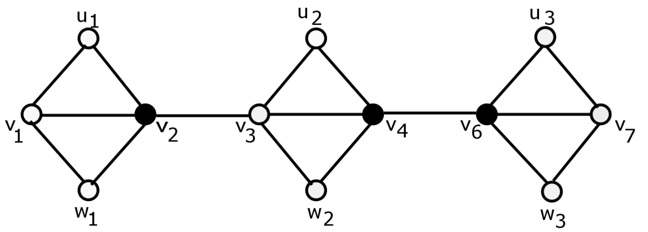

Figure 2. *G*

$S = \{v_1, u_1, u_2, u_3, w_1, w_2, w_3, v_7\}$ is the edge geodetic set so that $g_1(DAT_7) = 8$ and $S' = \{v_1, u_1, u_2, u_3, v_3, w_1, w_2, w_3, v_7\}$ is the split edge geodetic set. Therefore $g_{1s}(DAT_7) = 9$.

Definition 2.27. *A quadrilateral snake Q_n is obtained from a path P_n by joining v_i and v_{i+1} to new vertices u_i and w_i respectively and then joining u_i and w_i for $1 \le i \le n-1$ that is every edge of a path is replaced by a cycle C_4.*

Theorem 2.28. *For the quadrilateral snake $G = Q_n, (n \geqslant 2)$, $g_1(G) = d-2$ where d is the diameter of G.*

Proof. In order to obtain Q_n replace every edge of P_n by a cycle C_4. Let $\mid V(G) \mid = 3n-2$ and $\mid E(G) \mid = 4(n-1)$. Let $B = \{B_1, B_2, ..., B_{n-1}\}$ be the blocks of G. Let $\{u_i, w_i\}$ be the vertices of the block B_i for $1 \le i \le n-1$. Let $S = \{u_1, u_2, ..., u_{n-2}, w_{n-1}\}$ be the geodetic set of G. Clearly all the edges lie on any geodesic joining a pair of vertices of S and hence S is also an edge geodetic set of G. Since $d(u_i, w_{n-1}) = diam(G) = n+1$ we have $g_1(G) = \mid S \mid = d-2$. □

Corollary 2.29. *For the quadrilateral snake $G = Q_n$ $(n \geqslant 3)$, $egr(G) = d-2$ where d is the diameter of G.*

Corollary 2.30. *For the quadrilateral snake $G = Q_n$ $(n \geqslant 2)$, $g_{ns}(G) = d-2$ where d is the diameter of G.*

Theorem 2.31. *Let $G = Q_n$ $(n \geqslant 3)$ be the quadrilateral snake then $g_{ss}(G) = b+n-2$ where b is the number of blocks.*

Proof. Let $G = Q_n$. By Definition 2.27, $V(G) = \{v_1, v_2, ..., v_n, u_1, u_2, ..., u_{n-1}, w_1, w_2, ..., w_{n-1}\}$. Let $B = \{B_1, B_2, ...B_{n-1}\}$ be the blocks of G where each block contains 4 vertices such that 3 vertices are of degree 2 and one vertex is of maximum degree 4 which is a common vertex for adjacent blocks and $\mid B \mid = b$. Let $S = \{u_1, u_2, ..., u_{n-2}, w_{n-1}\}$ be the geodetic set. But induced subgraph $< V - S >$ is connected. Let $S' = S \cup \{v_k / 2 \le k \le n-1\}$ where $\triangle(v_k) = 4$. Clearly induced subgraph $< V - S' >$ is an independent set. Hence $g_{ss}(G) = \mid S' \mid = b+n-2$. □

Theorem 2.32. *For the quadrilateral snake $G = Q_n$ $(n \geqslant 4)$, $g_{1s}(G) = n$.*

Proof. Let $G = Q_n$. Let $V(G) = V_1 \cup V_2 \cup V_3$ where $V_1 = \{v_i / 1 \le i \le n\}, V_2 = \{u_j / 1 \le j \le n-1\}, V_3 = \{w_j / 1 \le j \le n-1\}$. Then $\mid V(G) \mid = 3n-2$ and $\mid E(G) \mid = 4(n-1)$. Let $S = \{u_1, u_2, ...u_j, v_n\}$ be the edge geodetic set of G. Clearly, $< V - S >$ has two components. Therefore $g_{1s}(G) = \mid S \mid = n-1+1 = n$. □

Definition 2.33. *The double quadrilateral snake DQ_n is obtained by path P_n by joining v_i and v_{i+1} to new vertices u_i, w_i for $i = 1, 2, ...n-1$ in upward direction and u'_i, w'_i for $i = 1, 2, ...n-1$ in downward direction. Let $V(G) = \{v_1, v_2, ...v_n, u_1, u_2, ...u_{n-1}, w_1, w_2, ...w_{n-1}, u'_1, u'_2, ...u'_{n-1}, w'_1, w'_2, ...w'_{n-1}\}$.*

Theorem 2.34. *For the double quadrilateral snake DQ_n $(n \geqslant 3)$, $g_1(DQ_n) = \frac{m}{7} + n - 1$ where m is the number of edges in DQ_n.*

Proof. Let $G = DQ_n$. By Definition 2.33, $\mid V(DQ_n) \mid = 5n-4$ and $\mid E(DQ_n) \mid = 7(n-1)$. Let $\{B_1, B_2, ..., B_{n-1}\}$ be the blocks of DQ_n. Now, consider $S = \{u_j, w'_j\}$ such that in each block $d(u_j, w'_j) = 3$ be the geodetic set of G. Since every edge of G lies on a geodesic joining u_j and v'_j, then S is also an edge geodetic set of G. Therefore, $g_1(G) = \frac{m}{7} + n - 1$. □

Corollary 2.35. *For the double quadrilateral snake DQ_n $(n \geqslant 3)$, $egr(DQ_n) = \frac{m}{7} + n - 1$ where m is the number of edges in DQ_n.*

Corollary 2.36. *Let $G = DQ_n, (n \geqslant 3)$ be the double quadrilateral snake , then $g_{ns}(G) = \frac{m}{7} + n - 1$ where m is the number of edges in DQ_n.*

Theorem 2.37. *Let $G = DQ_n$ be the double quadrilateral snake $(n \geqslant 4)$, $g_{ss}(DQ_n) = 3n-3$.*

Proof. Let G be a double quadrilateral snake. Let $V(G) = \{v_1, v_2, ..., v_n, u_1, u_2, \ ..., u_{n-1}, w_1, w_2, ..., w_{n-1}\}$. Consider $S = \{u_1, u_2, ..., u_{n-2}, w_{n-1}, u'_1, u'_2, ..., u'_{n-2}, w'_{n-1}\}$ be the geodetic set of G. Let $S' = S \cup \{v_2, v_3, ..., v_{n-1}\}$ such that $I[S'] = V(G)$. Since $< V - S' >$ contains isolated vertices, then $g_{ss}(G) = \mid S' \mid = 3n-4$. □

Theorem 2.38. *For the double quadrilateral snake* $G = DQ_n$ $(n \geqslant 4)$, $g_{1s}(G) = 2n-1$.

Proof. Let $V(G) = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$ where $V_1 = \{v_1, v_2, ...v_n\}$, $V_2 = \{u_1, u_2, ...u_{n-1}\}$, $V_3 = \{w_1, w_2, ...w_{n-1}\}$, $V_4 = \{u_1^{'}, u_2^{'}, ...u_{n-1}^{'}\}$, $V_5 = \{w_1^{'}, w_2^{'}, ...w_{n-1}^{'}\}$.
Let $S = \{u_1, u_2, ...u_{n-1}, w_1^{'}, w_2^{'}, ...w_{n-1}^{'}\}$ be the edge geodetic set. Choose any two vertices of S such that $d(u_i, w_i^{'}) = 3$. Then $< V - S >$ is connected so that S is not split edge geodetic set. Let $S^{'} = S \cup \{v_k\}$ where $\{v_k / 2 \leq k \leq n-1\}$ is only one internal vertex of path P_n. But $< V - S >$ is disconnected, therefore $g_{1s}(G) = \mid S^{'} \mid = 2n - 2 + 1 = 2n - 1$.

□

Example: For a double quadrilateral snake graph DQ_5 given in Figure 3. The empty color vertices is its split edge geodetic set.

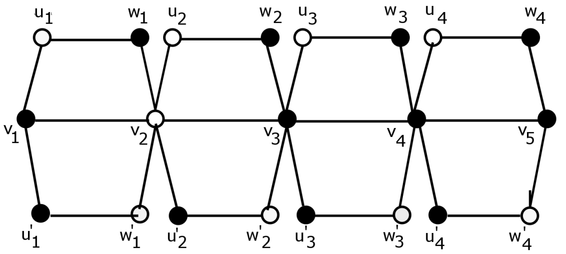

Figure 3. *G*

$S = \{u_1, u_2, u_3, u_4, w_1^{'}, w_2^{'}, w_3^{'}, w_4^{'}\}$ is the edge geodetic set so that $g_1(DQ_5) = 8$ and $S^{'} = \{v_2, u_1, u_2, u_3, u_4, w_1^{'}, w_2^{'}, w_3^{'}, w_4^{'}\}$ is the split edge geodetic set. Therefore $g_{1s}(DQ_5) = 9$.

Definition 2.39. *The alternate quadrilateral snake* AQ_n *is obtained from a path by joining* v_i *and* v_{i+1}*(alternatively) to new vertices* u_i *and* w_i *respectively and then joining* u_i *and* w_i.

Theorem 2.40. *Let* $G = AQ_n (n \geqslant 4)$ *be an alternate quadrilateral snake, then*

$$g_1(G) = \begin{cases} \lceil \frac{b}{2} \rceil & \textit{if } n \equiv 0(mod2)\,, \\ \frac{b+2}{2} & \textit{if } n \equiv 1(mod2)\,. \end{cases}$$

where b is the number of blocks.

Proof. Let $G = AQ_n$. Let $\{B_1, B_2\}$ be the number of blocks in G such that $\mid \{B_1, B_2\} \mid = b$. We observe that $B_1 = \{b_1, b_2, ..., b_{\lceil \frac{n}{2} \rceil}\}$ where each block $\{b_i / 1 \leq i \leq \lceil \frac{n}{2} \rceil\}$ is C_4 such that $b_1 = \{v_1, u_1, w_1, v_2\}, b_2 = \{v_3, u_2, w_2, v_4\}, ..., b_{\lceil \frac{n}{2} \rceil} = \{v_{n-1}, u_{\lceil \frac{n}{2} \rceil}, w_{\lceil \frac{n}{2} \rceil}, v_n\}$ and $B_2 = \{b_1^{'}, b_2^{'}, ..., b_{\lceil \frac{n}{2} \rceil}^{'}\}$ where each block $b_i^{'} / 1 \leq i \leq \lceil \frac{n}{2} \rceil$ is K_2 such that $b_1^{'} = \{v_1^{'}, u_1^{'}, w_1^{'}, v_2^{'}\}, b_2^{'} = \{v_3^{'}, u_2^{'}, w_2^{'}, v_4^{'}\}, ..., b_{\lceil \frac{n}{2} \rceil}^{'} = \{v_{n-1}^{'}, u_{\lceil \frac{n}{2} \rceil}^{'}, w_{\lceil \frac{n}{2} \rceil}^{'}, v_n^{'}\}$. We have the following cases:
Case 1 Let $n \equiv 0(mod2)$.
Let $S = \{u_1, u_2, ..., u_{\frac{n-2}{2}}, w_{\frac{n}{2}}\}$ be the geodetic set where $\{u_1, u_2, ..., u_{\frac{n-2}{2}}\}$ and $\{w_{\frac{n}{2}}\}$ are the vertices choosen from block B_1. Since every edge of G lies on a geodesic joining any two vertices in S, then $g_1(G) = \mid S \mid = \lceil \frac{b}{2} \rceil$.
Case 2 Let $n \equiv 1(mod2)$.
Let $S = \{u_1, u_2, ..., u_{\frac{n-3}{2}}, w_{\frac{n-1}{2}}, v_n\}$ be the minimum geodetic set where $\{u_1, u_2, .., u_{\frac{n-3}{2}}\}$ and $\{w_{\frac{n-1}{2}}\}$ are the vertices choosen from block B_1. Clearly S is an edge geodetic set of G. Therefore, $g_1(G) = \mid S \mid = \frac{b+2}{2}$. □

Corollary 2.41. *Let* $G = AQ_n (n \geqslant 4)$ *be an alternate quadrilateral snake , then*

$$eg_r(G) = \begin{cases} \lceil \frac{b}{2} \rceil & \textit{if } n \equiv 0(mod2)\,, \\ \frac{b+2}{2} & \textit{if } n \equiv 1(mod2)\,. \end{cases}$$

where b is the number of blocks.

Corollary 2.42. *For an alternate quadrilateral snake* $G = AQ_n (n \geqslant 4)$,

$$g_{ns}(G) = \begin{cases} \lceil \frac{b}{2} \rceil & \textit{if } n \equiv 0(mod2)\,, \\ \frac{b+2}{2} & \textit{if } n \equiv 1(mod2)\,. \end{cases}$$

where b is the number of blocks.

Theorem 2.43. *For an alternate quadrilateral snake* $AQ_n (n \geqslant 4)$ *, then*

$$g_{ss}(AQ_n) = \begin{cases} \alpha_0 & \textit{if n is even}\,, \\ \alpha_0 - 1 & \textit{if n is odd}\,. \end{cases}$$

Proof. Let $V(AQ_n) = \{v_1, v_2, ...v_n, u_1, u_2, ...u_{\frac{n}{2}}, w_1, w_2, ...w_{\frac{n}{2}}\}$ and α_0 is the vertex covering number of AQ_n.
We have the following cases:
Case 1 For n is even.
Let $S = \{u_1, u_2, ...u_{\frac{n-2}{2}}, w_{\frac{n}{2}}\}$ be the geodetic set of G. But $< V - S >$ is connected . Let $S^{'} = S \cup \{v_2, v_4, ..., v_{n-2}\} \cup \{v_{n-1}\}$. Clearly $< V - S^{'} >$ has isolated vertices. Hence,

$$\begin{aligned} g_{ss}(G) &= \mid S^{'} \mid \\ &= n \\ &= \alpha_0 \end{aligned}$$

Case 2 For n is odd.
Let $S = \{u_1, u_2, ...u_{\frac{n-3}{2}}, w_{\frac{n-1}{2}}, v_n\}$ be the minimum geodetic set of G. Let $S^{'} = S \cup \{v_2, v_4, ..., v_{n-2}\}$. Clearly $< V - S^{'} >$ is totally disconnected. Hence,

$$\begin{aligned} g_{ss}(G) &= \mid S^{'} \mid \\ &= n - 1 \\ &= \alpha_0 - 1 \end{aligned}$$

□

Theorem 2.44. *Let $G = AQ_n(n \geqslant 4)$ be an alternate quadrilateral snake , then*

$$g_{1s}(G) = \begin{cases} \frac{n+2}{2} & \text{if } n \equiv 0(mod2)\,, \\ \frac{n+3}{2} & \text{if } n \equiv 1(mod2)\,. \end{cases}$$

Proof. Let $G = AQ_n$ and by Definition 2.39

$$V(G) = \begin{cases} 2n & \text{if } n \equiv 0(mod2) \\ 2n-1 & \text{if } n \equiv 1(mod2) \end{cases}$$

Let $V(G) = \{V_1, V_2, V_3\}$ where $V_1 = \{v_1, v_2, ...v_n\}$, $V_2 = \{u_1, u_2, ...u_{\frac{n}{2}}\}$, $V_3 = \{w_1, w_2, ...w_{\frac{n}{2}}\}$. We have the following cases:

Case 1 Let n be even. Let $S = \{u_1, u_2, ...u_{\frac{n-2}{2}}, w_{\frac{n}{2}}\}$. Then S is an edge geodetic set. But $< V - S >$ is connected. Let $S^{'} = S \cup \{v_j\}$ where $\{v_j\}$ for $2 \leq j \leq n-1$ is any one internal vertex of P_n. But $< V - S^{'} >$ has two components. Therefore,

$$\begin{aligned} g_{1s}(G) &= \mid S^{'} \mid \\ &= S + \{v_j\} \\ &= \frac{n+2}{2} \end{aligned}$$

Case 2 Let n be odd. Let $S = \{u_1, u_2, ...u_{\frac{n-3}{2}}, w_{\frac{n}{2}}, v_n\}$. Clearly S is the minimum edge geodetic set. But $< V - S >$ is connected. Let $S^{'} = S \cup \{v_j\}$ where $\{v_j, 2 \leq j \leq n-1\}$ is any one internal vertex of P_n. Clearly, $< V - S^{'} >$ is disconnected, therefore

$$\begin{aligned} g_{1s}(G) &= \mid S^{'} \mid \\ &= S + \{v_j\} \\ &= \frac{n+3}{2} \end{aligned}$$

□

Example: For an alternate quadrilateral snake graph AQ_7 given in Figure 4. The empty color vertices is its split edge geodetic set.

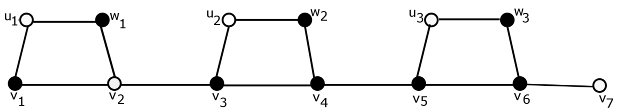

Figure 4. G

$S = \{u_1, u_2, u_3, v_7\}$ is the edge geodetic set so that $g_1(AQ_7) = 4$ and $S^{'} = \{v_2, u_1, u_2, u_3, v_7\}$ is the split edge geodetic set. Therefore $g_{1s}(AQ_7) = 5$.

Definition 2.45. *The double alternate quadrilateral snake DAQ_n consists of two alternate quadrilateral snakes that have a common path.*

Theorem 2.46. *Let $G = DAQ_n(n \geqslant 4)$ be double alternate quadrilateral snake , then $g_1(G) = n$.*

Proof. Let $G = DAQ_n$ be the graph obtained from joining v_i and v_{i+1} alternatively to new vertices $u_i, u_i^{'}$ and $w_i, w_i^{'}$ respectively. Then

$$V(G) = \begin{cases} 3n & \text{if } n \equiv 0(mod2) \\ 3n-2 & \text{if } n \equiv 1(mod2) \end{cases}$$

We have the following cases:

Case 1 Let n be even. Let $V(G) = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$ where $V_1 = \{v_1, v_2, ...v_n\}$, $V_2 = \{u_1, u_2, ...u_{\frac{n}{2}}\}$, $V_3 = \{w_1, w_2, ...w_{\frac{n}{2}}\}$, $V_4 = \{u_1^{'}, u_2^{'}, ...u_{\frac{n}{2}}^{'}\}$, $V_5 = \{w_1^{'}, w_2^{'}, ...w_{\frac{n}{2}}^{'}\}$ such that $u_i, u_i^{'}$ and $w_i, w_i^{'}$ are the new vertices added in upward and downward direction to v_i and v_{i+1} for $1 \leq i \leq n-1$. Let $S = \{u_1, u_2, ...u_{\frac{n}{2}}, w_1^{'}, w_2^{'}, ...w_{\frac{n}{2}}^{'}\}$. Choose the vertices of S such that $d(u_i, w_i^{'}) = 3$. Clearly S is the minimum edge geodetic set of G. Therefore,

$$\begin{aligned} g_1(G) &= \mid S^{'} \mid \\ &= \frac{n}{2} + \frac{n}{2} \\ &= n \end{aligned}$$

Case 2 Let n be odd. Let $V(G) = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$ where $V_1 = \{v_1, v_2, ...v_n\}$, $V_2 = \{u_1, u_2, ...u_{\frac{n-1}{2}}\}$, $V_3 = \{w_1, w_2, ...w_{\frac{n-1}{2}}\}$, $V_4 = \{u_1^{'}, u_2^{'}, ...u_{\frac{n-1}{2}}^{'}\}$, $V_5 = \{w_1^{'}, w_2^{'}, ...w_{\frac{n-1}{2}}^{'}\}$. Let $S = \{u_1, u_2, ...u_{\frac{n-1}{2}}, w_1^{'}, w_2^{'}, ...w_{\frac{n-1}{2}}^{'}, v_n\}$. Choose the vertices of S such that $d(u_i, w_i^{'}) = 3$. Therefore S is an edge geodetic set of G. Hence,

$$\begin{aligned} g_1(G) &= \mid S^{'} \mid \\ &= \frac{n-1}{2} + \frac{n-1}{2} + 1 \\ &= n \end{aligned}$$

□

Corollary 2.47. *Let $G = DAQ_n(n \geqslant 4)$ be double alternate quadrilateral snake , then $eg_r(G) = n$.*

Corollary 2.48. *For double alternate quadrilateral snake $DAQ_n(n \geqslant 4)$, $g_{ns}(DAQ_n) = n$.*

Theorem 2.49. *For double alternate quadrilateral snake $DAQ_n(n \geqslant 4)$,*

$$g_{ss}(DAQ_n) = \begin{cases} \frac{3n}{2} & \text{if n is even}\,, \\ \frac{3n-1}{2} & \text{if n is odd}\,. \end{cases}$$

Proof. Let $G = DAQ_n$. Let $V(G) = \{v_1, v_2, ..., v_n, u_1, u_2, u_{\frac{n}{2}}$ $w_1, w_2, ..., w_{\frac{n}{2}}, u'_1, u'_2, ..., u'_{\frac{n}{2}}, w'_1, w'_2, ..., w'_{\frac{n}{2}}\}$.
We discuss the following cases:
Case 1 Let n be even.
Consider $S = \{u_1, u_2, ...u_{\frac{n-2}{2}}, w_{\frac{n}{2}}, u'_1, u'_2, ...u'_{\frac{n-2}{2}}, w'_{\frac{n}{2}}\}$. Clearly S is a geodetic set of G. But $< V - S >$ is connected. Let $S' = S \cup \{v_2, v_4, ..., v_{n-1}\}$. Clearly $< V - S' >$ is an independent set and hence

$$\begin{aligned} g_{ss}(G) &= |S'| \\ &= \frac{3n}{2}. \end{aligned}$$

Case 2 Let n be odd.
Let $S = \{u_1, u_2, ...u_{\frac{n-1}{2}}, u'_1, u'_2, ...u'_{\frac{n-1}{2}}, v_n\}$ be the minimum geodetic set of G. But $< V - S >$ is connected. Consider $S' = S \cup \{v_2, v_4, ..., v_{n-1}\}$. Clearly $< V - S' >$ is an independent set and therefore,

$$\begin{aligned} g_{ss}(G) &= |S'| \\ &= \frac{3n-1}{2}. \end{aligned}$$

□

Theorem 2.50. *For double alternate quadrilateral snake* $DAQ_n (n \geqslant 4)$, $g_{1s}(G) = n + 1$.

Proof. Let $G = DAQ_n$. Let $V(G) = \{v_1, v_2, ..., v_n, u_1, u_2, u_{\frac{n}{2}},$ $w_1, w_2, ..., w_{\frac{n}{2}}, u'_1, u'_2, ..., u'_{\frac{n}{2}}, w'_1, w'_2, ..., w'_{\frac{n}{2}}\}$.
We have the following cases:
Case 1 Let n be even.
Let $S = \{u_1, u_2, ...u_{\frac{n}{2}}, w'_1, w'_2, ...w'_{\frac{n}{2}}\}$. Choose the vertices such that $d(u_i, w'_i) = 3$. Then S is an edge geodetic set. Let $S' = S \cup \{v_k\}$ where $\{v_k\}$ for $2 \leq k \leq n-1$ is only one internal vertex of P_n. Then $V - S'$ is disconnected. Therefore,

$$\begin{aligned} g_{1s}(G) &= |S'| \\ &= \frac{n}{2} + \frac{n}{2} + 1 \\ &= n + 1 \end{aligned}$$

Case 2 Let n be odd.
Let $S = \{u_1, u_2, ...u_{\frac{n-1}{2}}, w'_1, w'_2, ...w'_{\frac{n-1}{2}}, v_n\}$ such that $d(u_i, w'_i) = 3$. Clearly S is an edge geodetic set. But $V - S$ is connected. Hence S is not split edge geodetic set. Let $S' = S \cup \{v_k\}$ where $\{v_k\}$ for $2 \leq k \leq n-1$ is only one internal vertex of P_n. Since $< V - S' >$ is disconnected, then

$$\begin{aligned} g_{1s}(G) &= |S'| \\ &= \frac{n-1}{2} + \frac{n-1}{2} + 2 \\ &= n + 1 \end{aligned}$$

□

3. Conclusion

In this paper, edge geodetic parameters for some snake graphs like triangular snake graph, double triangular snake graph, alternate triangular snake graph, double alternate triangular snake graph, quadrilateral snake graph, double quadrilateral snake graph, alternate quadrilateral snake graph, double alternate quadrilateral snake graph are determined.

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