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Propagation of disease from exotic infected predator to native population-A prey predator model

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Abstract

In this paper, a prey predator model for native population with SI infection in exotic population is developed and analyzed. A model with prey predator interaction in native population and exotic predator having the risk of infection is suggested to observe the transmission of disease from exotic predators to native population. Disease free equilibrium points (in presence and absence of predator) and endemic equilibrium points are calculated. Conditions for the existence and boundedness of equilibrium points have been derived. The local stability analysis of the model system around the all biologically feasible equilibrium points is discussed. We perform global dynamics of the model using Lyapunov theorem for endemic equilibrium point. We compare the growth of population in terms of ecological sensitive parameters predation rate (η_3), carrying capacity of environment (K) and transmission rate of disease (β) with the help of suitable graphs.

Keywords

Prey predator model, SI model, Stability Analysis, Descartes' rule of signs, Hurwitz criteria and Lyapunov theorem.

AMS Subject Classification

34D20, 34D23, 37C75, 93D05.

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Contents

1. Introduction

In modern era, Prey predator interaction is one of the challenging issue among researchers. Its complication is raised in presence of disease either in prey or in predator or in both. We cannot ignore these natural phenomena of regulating population structure [1]. The disease spread in prey population, generally through contact with infected prey, whereas in predator population occurred either through consumption of infected preys [3–5] or by contact with infected predator [3, 6].

Many researchers have suggested several models to analysis effect of disease in interacting species system. Haque (2010) studied the SIS predator–prey model with infection spreading through the predator species only. It was shown that infection in the predator species may save the prey from death even if the basic reproduction number was less than one, for which the prey to be able to occupy the predator. Pal et al. (2014) gave a predator–prey model with disease in predator species only. They showed that for some values of the predation rate all species could be survived and the disease did not transmit in the predator population. Han et al. (2001) studied four predator prey models in which disease spreads in both the prey and predator. They showed that when the disease exists in the prey population and also the predators feed sufficiently to survive, then disease will also persist in the predator population. Venturino (2002) gave two mathematical models with disease in the predators. Disease transmission involved both mass action and standard incidence rates, respectively. In the two models, it was assumed that the disease spreads among predators only and the infection in individuals do not reproduce. Stability analysis of the solutions of the two models was done to see the effect of the disease in the predator species and on the ecological system, also the sound prey can affect the dynamics of the disease in the predator population. Zhang and Sun (2005) suggested a predator–prey model with disease in the predator. General functional response and sufficient conditions were found out for the permanence of the ecological system.

Keeping in the view above discussion, we have concentrate to frame and analysis a model to see the effect of disease spread to native population from exotic infected predators. This paper is arranged in following manner. Section 2 includes formulation of model with help basic assumptions and non linear differential equations. Section 3 contains bounded region for the solution of the model. Section 4 has conditions for existence of biological feasible disease free and endemic equilibrium points. Section 5 carries conditions for local and global stability of model system around equilibrium points. In section 6 numerical simulation for all equilibrium points has been performed and also results have been depicted by graphs. In section 7 conclusion for growth of all the species with respect to sensitive parameters is discussed to support the analysis.

2. Mathematical Model

In this section, we have taken Native prey population (P), Native predator population (Q), Exotic susceptible predator population (Q_s) and Exotic infected predator population (Q_i) as interacting species.In our model, Prey predator type of interaction is considered for native population. Native prey population (P) grows logistically with term $rP\left(1-\frac{P}{K}\right)$ and this population is decreased by term $\frac{\eta_1 PQ}{1+k_1P+k_2Q}+\frac{\eta_2 PQ_S}{1+k_1P+k_2Q_S}+\frac{\eta_3 PQ_i}{1+k_1P+k_2Q_i}$. Native predator, Exotic susceptible predator and Exotic infected predator consume native susceptible preys. η_1, η_2 and η_3 denotes predation rate of Q, Q_s and Q_i on preys P, respectively. Since the infected predator Q_i is weaker than uninfected predators Q and Q_s. so we have assumed $\eta_3<\eta_1$ and $\eta_3<\eta_2$. Native predator population (Q) is increased by predation of preys with $\frac{\alpha\eta_1 PQ}{1+k_1P+k_2Q}$ and decrease by natural death rate dQ. Exotic predator population (Q_s) has recruitment rate Δ, which is increased by $\frac{\alpha\eta_2 PQ_S}{1+k_1P+k_2Q_S}$. It is assumed that disease transmits only from exotic infected predator to exotic susceptible predator. Suppose β is the transmission rate of disease. In this model, cause of infection and prevalence are ignored. Hence this population is decreased by $\beta Q_s Q_i$ and natural death rate dQ_S. Exotic infected predator

Table 1. Description of parameters

Parameters	Description
P	Native prey population.
Q	Native predator population.
Q_s	Exotic susceptible predator population.
Q_i	Exotic infected predator population.
r	Intrinsic growth rate of prey population.
K	The carrying capacity of the environment
k_1	Half saturation constant.
k_2	Magnitude of interference among predators.
α	Conversion efficiency. $(0<\alpha<1)$
β	Disease transmission rate
Δ	Recruitment rate of exotic predator population.
η_1	Search rate of exotic prey by native predators.
η_2	Search rate of exotic prey by exotic susceptible predators.
η_3	Search rate of exotic prey by exotic infected predators. $(\eta_1$ *and* $\eta_2>\eta_3)$
d	Natural death rate of predator.
σ	Disease induced death rate of infected predator population.

population (Q_i) is increased due to infection $\beta Q_S Q_i$ and predation $\frac{\alpha\eta_3 PQ_i}{1+k_1P+k_2Q_i}$. It is decreased by natural death rate dQ_i and disease induced death rate σQ_i. Recovery and immunity of infected predators are neglected.

On the basis of above discursion, we have developed a mathematical model with the help of following system of ordinary differential equations given below;

$$\frac{dP}{dt}=rP\left(1-\frac{P}{K}\right)-\frac{\eta_1 PQ}{1+k_1P+k_2Q}-\frac{\eta_2 PQ_S}{1+k_1P+k_2Q_S}-\frac{\eta_3 PQ_i}{1+k_1P+k_2Q_i}$$

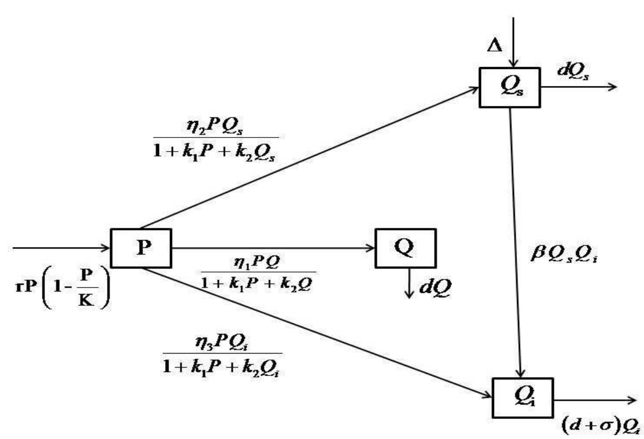

Figure 1. Diagramic representation of the proposed model

$$\frac{dQ}{dt} = \frac{\alpha\eta_1 PQ}{1+k_1P+k_2Q} - dQ \tag{2.1}$$

$$\frac{dQ_S}{dt} = \Delta - \beta Q_S Q_i + \frac{\alpha\eta_2 PQ_S}{1+k_1P+k_2Q_S} - dQ_S$$

$$\frac{dQ_i}{dt} = \beta Q_S Q_i + \frac{\alpha\eta_3 PQ_i}{1+k_1P+k_2Q_i} - (d+\sigma)Q_i$$

Associated initial conditions for the above model are as follows:
$P(0) > 0,\ Q(0) > 0,\ Q_s(0) > 0$ and $Q_i(0) > 0$.

3. Bounded Region

To find out the bounded region for the solution of system (2.1) let us assume,
$V(P, Q, Q_s, Q_i) = P + Q + Q_s + Q_i$
differentiating V with respect to t and using system (2.1). We have,

$$\frac{dV}{dt} \leq rP\left(1-\frac{P}{K}\right) + \Delta - d\left(Q+Q_S+Q_i\right) - \sigma Q_i$$

$$\leq P\left(r\left(1-\frac{P}{K}\right)+1\right) + \Delta - P - d\left(Q+Q_S+Q_i\right) - \sigma Q_i$$

$$\leq P(r+1) + \Delta - P - d\left(Q+Q_S+Q_i\right) - \sigma Q_i.$$

In particular, $\limsup_{t\to\infty} P(t) \geq \overset{*}{K}$, since $\frac{dP}{dt} \leq r\left(1-\frac{P}{K}\right)$,
where $\overset{*}{K} = \max\{P(0), K\}$ Thus,
$P(t)$ is bounded and defined on $[0,\infty)\, \forall t \geq 0$.
$\frac{dV}{dt} + mV \leq (r+1)\overset{*}{K} + \Delta$,
where $m = \min\{1, d, d+\sigma\}$.
now applying tools of the theory of differential inequality we get,
$0 < V(P, Q, Q_s, Q_i) \leq \frac{\Delta+\overset{*}{K}(r+1)}{m} + e^{-mt}V(0)$
which gives $0 < V(P, Q, Q_s, Q_i) \leq \frac{\Delta+\overset{*}{K}(r+1)}{m}$ as $t \to \infty$
So all the solutions of system (2.1) with respect to initial values are confined i.e. uniformly bounded in the region.

$$\Gamma_\varepsilon = \{(P, Q, Q_s, Q_i) \in R_+^4 :$$

$$P + Q + Q_s + Q_i \leq \frac{\Delta+\overset{*}{K}(r+1)}{m} + \varepsilon\}. \tag{3.1}$$

4. Equilibrium points and their existence

The system possesses following feasible biological equilibrium points given below;

4.1 The trivial equilibrium point

The trivial equilibrium point of system (2.1) is $B_0(0,0,0,0)$.

4.2 Disease-free equilibrium point without Predator

Disease-free equilibrium point without Predator of system (2.1) is $B_1(K, 0, 0, 0)$.

4.3 Disease-free equilibrium point with Predator

Disease-free equilibrium point with Predator of system (2.1) is $B_2\left(\hat{P}, \hat{Q}, \hat{Q}_s, 0\right)$
where
$\hat{Q} = \frac{(\alpha\eta_1 - dk_1)\hat{P} - d}{dk_2}$,
$\hat{P}$ and $\hat{Q}_s$ can be evaluated by the following set of two equations
$c_1\hat{P}^2 + c_2\hat{P} + c_3\hat{Q}_s + c_4 = 0$
$e_1\hat{Q}_s^2 + e_2\hat{Q}_s\hat{P} + e_3\hat{Q}_s + e_4\hat{P} + e_5 = 0$
Where
$c_1 = -\frac{\alpha r}{K},\ c_2 = \left(\alpha r - \frac{(\alpha\eta_1 - dk_1)}{k_2}\right),\ c_3 = -d,\ c_4 = \left(\Delta + \frac{d}{k_2}\right)$,
$e_1 = -dk_2, e_2 = (\alpha\eta_2 - dk_1),\ e_3 = (\Delta k_2 - d),\ e_4 = \Delta k_1$ and $e_5 = \Delta$.
using Descartes' rule of signs, we can easily conclude that $\hat{P}$ and $\hat{Q}_s$ have at least one positive value. Thus equilibrium point B_2 exists if $\hat{P} > \frac{d}{(\alpha\eta_1 - dk_1)}$ and $\alpha\eta_1 > dk_1$.

4.4 Endemic equilibrium point

Endemic equilibrium point of system (2.1) is $B_3\left(\overset{*}{P}, \overset{*}{Q}, \overset{*}{Q}_s, \overset{*}{Q}_i\right)$
where
$\overset{*}{Q} = \frac{(\alpha\eta_1 - dk_1)\overset{*}{P} - d}{dk_2}$ and $\overset{*}{Q}_i = \frac{1}{k_2}\left[\frac{\alpha\eta_3\overset{*}{P}}{\left((d+\sigma) - \beta\overset{*}{Q}_s\right)} - \left(1 + k_1\overset{*}{P}\right)\right]$,
$\overset{*}{P}$ and $\overset{*}{Q}_s$ can be calculated by the following set of two equations
$f_1\overset{*}{P}^2\overset{*}{Q}_s + f_2\overset{*}{P}^2 + f_3\overset{*}{Q}_s^2 + f_4\overset{*}{P}\overset{*}{Q}_s + f_5\overset{*}{P} + f_6\overset{*}{Q}_s + f_7 = 0$
$g_1\overset{*}{Q}_s^2\overset{*}{P}^2 + g_2\overset{*}{Q}_s^3\overset{*}{P} + g_3\overset{*}{Q}_s\overset{*}{P}^2 + g_4\overset{*}{P}\overset{*}{Q}_s^2 + g_5\overset{*}{Q}_s^3 + g_6\overset{*}{Q}_s\overset{*}{P} + g_7\overset{*}{Q}_S^2 + g_8\overset{*}{Q}_S + g_9\overset{*}{P} + g_{10} = 0$
where
$f_1 = \frac{r\alpha\beta k_2}{K},\ f_2 = -\frac{r\alpha(d+\sigma)k_2}{K},\ f_3 = dk_2\beta$,
$f_4 = -\beta\left(k_2 r\alpha - (\alpha\eta_1 - dk_1) + (d+\sigma)k_1\right)$,
$f_5 = \left(\left(k_2 r\alpha - (\alpha\eta_1 - dk_1) + (d+\sigma)k_1\right)(d+\sigma) - (d+\sigma)\alpha\eta_3\right)$,
$f_6 = -\left(d(d+\sigma)k_2 + (k_2\Delta + 2d + \sigma)\beta\right)$,
$f_7 = (k_2\Delta + 2d + \sigma)(d+\sigma)$,
$g_1 = -\beta^2 k_1^2, g_2 = -\beta^2 k_1 k_2, g_3 = \left((d+\sigma)k_1 - \alpha\eta_3\right)\beta k_1$,
$g_4 = -\left((\eta_2 + \eta_3)\alpha k_2 - k_1 k_2 d + 2k_1\beta - (d+\sigma)k_1 k_2\right)\beta$,
$g_5 = \{k_2 d - \beta\}\beta k_2$,
$g_6 = (\alpha\eta_2 - k_1 d)(d+\sigma)k_2 + 2k_1(d+\sigma)\beta - \alpha\beta\eta_3 - \Delta\beta k_1 k_2$,
$g_7 = -\left\{d(d+\sigma)k_2{}^2 + (d+\sigma)\beta k_2 + (k_2\Delta - d)\beta k_2 + k_2\beta^2\right\}$,
$g_8 = \{(k_2\Delta - d)(d+\sigma)k_2 + (d+\sigma)\beta - \Delta\beta k_2\}$,
$g_9 = k_1\Delta(d+\sigma)k_2, g_{10} = \Delta(d+\sigma)k_2$.
using Descartes' rule of signs, we reach to the conclusion that $\overset{*}{P}$ and $\overset{*}{Q}_s$ have at least one positive value.

Thus equilibrium point B_3 exists if
$\overset{*}{P} > \frac{d}{(\alpha\eta_1 - dk_1)}, \alpha\eta_1 > dk_1, \frac{(d+\sigma)}{\beta} > \overset{*}{Q}_s > \frac{k_1(d+\sigma)-\alpha\eta_3}{k_1\beta}$.

5. Stability Analysis

5.1 Local stability analysis

To observe local stability of system (2.1) around all feasible points, first we calculate variational matrix and using stability theorem we determine the stability of model system.

5.1.1 Local stability behaviour of the system around B_0

The variational matrix of the system (2.1) around $B_0(0,0,0,0)$ is given by;

$$J_0 = \begin{bmatrix} r & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & -d & 0 \\ 0 & 0 & 0 & -(d+\sigma) \end{bmatrix}$$

Eigen values of matrix J_0 are $r, -d, -d$ and $-(d+\sigma)$. since $r > 0$ i.e. one eigen value is positive Hence system (2.1) is always unstable around B_0.

5.1.2 Local stability behaviour of the system around B_1

The variational matrix of the system (2.1) around $B_1\ (K,0,0,0)$ is given by;

$$J_1 = \begin{bmatrix} -r & -\frac{\eta_1 K}{1+k_1K} & -\frac{\eta_2 K}{1+k_1K} & -\frac{\eta_3 K}{1+k_1K} \\ 0 & \frac{\alpha\eta_1 K}{1+k_1K} - d & 0 & 0 \\ 0 & 0 & \frac{\alpha\eta_2 K}{1+k_1K} - d & 0 \\ 0 & 0 & 0 & \frac{\alpha\eta_3 K}{1+k_1K} - (d+\sigma) \end{bmatrix}$$

Eigen values of matrix J_1 are $-r$, $\frac{\alpha\eta_1 K}{1+k_1K} - d$, $\frac{\alpha\eta_2 K}{1+k_1K} - d$ and $\frac{\alpha\eta_3 K}{1+k_1K} - (d+\sigma)$.

Hence system (2.1) is locally stable if $\eta_1 < \frac{d(1+k_1K)}{\alpha K}$, $\eta_2 < \frac{d(1+k_1K)}{\alpha K}$ and $\eta_3 < \frac{(d+\sigma)(1+k_1K)}{\alpha K}$ otherwise unstable.

5.1.3 Local stability behaviour of the system around B_2

The variational matrix of the system (2.1) around $B_2\left(\hat{P}, \hat{Q}, \hat{Q}_s, 0\right)$ is given by;

$$J_2 = \begin{bmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44} \end{bmatrix}$$

where

$$V_{11} = r - \frac{2r\hat{P}}{K} - \frac{\eta_1\hat{Q}}{1+k_1\hat{P}+k_2\hat{Q}} + \frac{\eta_1 k_1\hat{P}\hat{Q}}{\left(1+k_1\hat{P}+k_2\hat{Q}\right)^2} - \frac{\eta_2\hat{Q}_s}{1+k_1\hat{P}+k_2\hat{Q}_s} + \frac{\eta_2 k_1\hat{P}\hat{Q}_s}{\left(1+k_1\hat{P}+k_2\hat{Q}_s\right)^2},$$

$$V_{12} = -\frac{\eta_1\hat{P}}{1+k_1\hat{P}+k_2\hat{Q}} + \frac{\eta_1 k_2\hat{P}\hat{Q}}{\left(1+k_1\hat{P}+k_2\hat{Q}\right)^2},$$

$$V_{13} = -\frac{\eta_2\hat{P}}{1+k_1\hat{P}+k_2\hat{Q}_s} + \frac{\eta_2 k_2\hat{P}\hat{Q}_s}{\left(1+k_1\hat{P}+k_2\hat{Q}_s\right)^2}, V_{14} = -\frac{\eta_3\hat{P}}{1+k_1\hat{P}},$$

$$V_{21} = \frac{\alpha\eta_1\hat{Q}}{1+k_1\hat{P}+k_2\hat{Q}} - \frac{\alpha k_1\eta_1\hat{P}\hat{Q}}{\left(1+k_1\hat{P}+k_2\hat{Q}\right)^2}, V_{23} = 0,$$

$$V_{24} = 0, V_{22} = \frac{\alpha\eta_1\hat{P}}{1+k_1\hat{P}+k_2\hat{Q}} - \frac{\alpha k_2\eta_1\hat{P}\hat{Q}}{\left(1+k_1\hat{P}+k_2\hat{Q}\right)^2} - d,$$

$$V_{31} = \frac{\alpha\eta_2\hat{Q}_s}{1+k_1\hat{P}+k_2\hat{Q}_s} - \frac{\alpha k_1\eta_2\hat{P}\hat{Q}_s}{\left(1+k_1\hat{P}+k_2\hat{Q}_s\right)^2}, V_{32} = 0,$$

$$V_{33} = \frac{\alpha\eta_2\hat{P}}{1+k_1\hat{P}+k_2\hat{Q}_s} - \frac{\alpha\eta_2 k_2\hat{P}\hat{Q}_s}{\left(1+k_1\hat{P}+k_2\hat{Q}_s\right)^2} - d, V_{34} = -\beta\hat{Q}_s,$$

$$V_{41} = 0, V_{42} = 0, V_{43} = 0, V_{44} = \beta\hat{Q}_s + \frac{\alpha\eta_3\hat{P}}{1+k_1\hat{P}} - (d+\sigma).$$

The Eigen equation for J_2 is given by
$\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0$
where
$A_1 = -(V_{11} + V_{22} + V_{33} + V_{44})$,
$A_2 = V_{11}V_{22} + V_{11}V_{33} + V_{11}V_{44} + V_{22}V_{33} + V_{22}V_{44} + V_{33}V_{44} - V_{12}V_{21} - V_{13}V_{31} - V_{34}V_{43}$,
$A_3 = V_{12}V_{33}V_{21} + V_{12}V_{44}V_{21} + V_{13}V_{44}V_{31} - V_{11}V_{22}V_{33} - V_{11}V_{33}V_{44} - V_{22}V_{33}V_{44} - V_{11}V_{22}V_{44}$,
$A_4 = V_{11}V_{22}V_{33}V_{44} - V_{13}V_{22}V_{31}V_{44} - V_{12}V_{21}V_{33}V_{44}$.
using Routh-Hurwitz criteria, system (2.1) is stable around B_2 if $A_1, A_2, A_3, A_4 > 0$ and $(A_1.A_2 - A_3)A_3 - A_1^2.A_4 > 0$ otherwise unstable.

5.1.4 Local stability behaviour of the system around B_3

The variational matrix of the system (2.1) around $B_3\left(\overset{*}{P}, \overset{*}{Q}, \overset{*}{Q}_s, \overset{*}{Q}_i\right)$ is given by;

$$J_3 = \begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ U_{21} & U_{22} & U_{23} & U_{24} \\ U_{31} & U_{32} & U_{33} & U_{34} \\ U_{41} & U_{42} & U_{43} & U_{44} \end{bmatrix}$$

where

$$U_{11} = r - \frac{2r\overset{*}{P}}{K} - \frac{\eta_1\overset{*}{Q}}{1+k_1\overset{*}{P}+k_2\overset{*}{Q}} + \frac{\eta_1 k_1\overset{*}{P}\overset{*}{Q}}{\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}\right)^2} - \frac{\eta_2\overset{*}{Q}_s}{1+k_1\overset{*}{P}+k_2\overset{*}{Q}_s} + \frac{\eta_2 k_1\overset{*}{P}\overset{*}{Q}_s}{\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}_s\right)^2} - \frac{\eta_3\overset{*}{Q}_i}{1+k_1\overset{*}{P}+k_2\overset{*}{Q}_i} + \frac{\eta_3 k_1\overset{*}{P}\overset{*}{Q}_i}{\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}_i\right)^2},$$

$$U_{12} = -\frac{\eta_1\overset{*}{P}}{1+k_1\overset{*}{P}+k_2\overset{*}{Q}} + \frac{\eta_1 k_2\overset{*}{P}\overset{*}{Q}}{\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}\right)^2},$$

$$U_{13} = -\frac{\eta_2\overset{*}{P}}{1+k_1\overset{*}{P}+k_2\overset{*}{Q}_s} + \frac{\eta_2 k_2\overset{*}{P}\overset{*}{Q}_s}{\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}_s\right)^2},$$

$$U_{14} = -\frac{\eta_3\overset{*}{P}}{1+k_1\overset{*}{P}+k_2\overset{*}{Q}_i} + \frac{\eta_3 k_2\overset{*}{P}\overset{*}{Q}_i}{\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}_i\right)^2},$$

$U_{21}=\frac{\alpha\eta_1\overset{*}{Q}}{1+k_1\overset{*}{P}+k_2\overset{*}{Q}}-\frac{\alpha k_1\eta_1\overset{*}{P}\overset{*}{Q}}{\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}\right)^2}$,
$U_{22}=\frac{\alpha\eta_1 P}{1+k_1P+k_2Q}-\frac{\alpha k_2\eta_1 PQ}{(1+k_1P+k_2Q)^2}-d, U_{23}=0, U_{24}=0$,
$U_{31}=\frac{\alpha\eta_2 Q_S}{1+k_1P+k_2Q_S}-\frac{\alpha k_1\eta_2 PQ_S}{(1+k_1P+k_2Q_S)^2}, U_{32}=0$,
$U_{33}=-\beta Q_i+\frac{\alpha\eta_2 P}{1+k_1P+k_2Q_S}-\frac{\alpha\eta_2 k_2 PQ_S}{(1+k_1P+k_2Q_S)^2}-d, U_{34}=-\beta Q_S$,
$U_{41}=\frac{\alpha\eta_3 Q_i}{1+k_1P+k_2Q_i}-\frac{\alpha\eta_3 k_1 PQ_i}{(1+k_1P+k_2Q_i)^2}, U_{42}=0, U_{43}=\beta Q_i$,
$U_{44}=\beta Q_S+\frac{\alpha\eta_3 P}{1+k_1P+k_2Q_i}-\frac{\alpha\eta_3 k_2 PQ_i}{(1+k_1P+k_2Q_i)^2}-(d+\sigma)$.
The Eigen equation for J_2 is given by;
$\lambda^4+D_1\lambda^3+D_2\lambda^2+D_3\lambda+D_4=0$
where
$D_1=-(U_{11}+U_{22}+U_{33}+U_{44})$,
$D_2=U_{11}U_{22}+U_{11}U_{33}+U_{11}U_{44}+U_{22}U_{33}+U_{22}U_{44}+U_{33}U_{44}-U_{12}U_{21}-U_{13}U_{31}-U_{14}U_{41}-U_{34}U_{43}$,
$D_3=U_{34}U_{11}U_{43}+c+U_{14}U_{22}U_{41}+U_{34}U_{22}U_{43}+U_{12}U_{33}U_{21}+U_{14}U_{33}U_{41}+U_{12}U_{44}U_{21}+U_{13}U_{44}U_{31}-U_{11}U_{22}U_{33}-U_{11}U_{33}U_{44}-U_{22}U_{33}U_{44}-U_{11}U_{22}U_{44}-U_{13}U_{34}U_{41}-U_{14}U_{31}U_{43}$,
$D_4=U_{14}U_{22}U_{33}U_{41}+U_{13}U_{22}U_{34}U_{41}+U_{14}U_{22}U_{31}U_{43}+U_{12}U_{21}U_{34}U_{43}+U_{11}U_{22}U_{33}U_{44}-U_{11}U_{22}U_{34}U_{43}-U_{13}U_{22}U_{31}U_{44}-U_{12}U_{21}U_{33}U_{44}$.
using Routh-Hurwitz criteria, system (2.1) is stable at D_3 if $D_1,D_2,D_3,D_4>0$ and $(D_1.D_2-D_3)D_3-D_1^2.D_4>0$ otherwise unstable.

5.2 Global stability behaviour of system around B_3

To determine global stability of system (2.1) around $B_3\left(\overset{*}{P},\overset{*}{Q},\overset{*}{Q}_s,\overset{*}{Q}_i\right)$, we consider, positive definite function $W(P,Q,Q_s,Q_i) is given by$

$$W(P,Q,Q_s,Q_i)=\left(P-\overset{*}{P}-\overset{*}{P}\log\frac{P}{\overset{*}{P}}\right)+\left(Q-\overset{*}{Q}-\overset{*}{Q}\log\frac{Q}{\overset{*}{Q}}\right)+\left(Q_s-\overset{*}{Q}_s-\overset{*}{Q}_s\log\frac{Q_s}{\overset{*}{Q}_s}\right)+\left(Q_i-\overset{*}{Q}_i-\overset{*}{Q}_i\log\frac{Q_i}{\overset{*}{Q}_i}\right).$$

differentiating W with respect to t and using system (2.1), we get following expression given below;

$$\overset{\bullet}{W}=\left(P-\overset{*}{P}\right)\left(r\left(1-\frac{P}{K}\right)-\frac{\eta_1 Q}{1+k_1P+k_2Q}-\frac{\eta_2 Q_S}{1+k_1P+k_2Q_S}-\frac{\eta_3 Q_i}{1+k_1P+k_2Q_i}\right)+\left(Q-\overset{*}{Q}\right)\left(\frac{\alpha\eta_1 P}{1+k_1P+k_2Q}-d\right)+\left(Q_s-\overset{*}{Q}_s\right)\left(\frac{\Delta}{Q_s}-\beta Q_i+\frac{\alpha\eta_2 P}{1+k_1P+k_2Q_s}-d\right)+\left(Q_i-\overset{*}{Q}_i\right)\left(\beta Q_S+\frac{\alpha\eta_3 P}{1+k_1P+k_2Q_i}-(d+\sigma)\right)$$

$$\overset{\bullet}{W}=\left(P-\overset{*}{P}\right)\left(-\frac{r}{K}\left(P-\overset{*}{P}\right)-\eta_1\left[\frac{\left(1+k_1\overset{*}{P}\right)\left(Q-\overset{*}{Q}\right)-k_1\overset{*}{Q}\left(P-\overset{*}{P}\right)}{(1+k_1P+k_2Q)\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}\right)}\right]-\eta_2\left[\frac{\left(1+k_1\overset{*}{P}\right)\left(Q_s-\overset{*}{Q}_s\right)-k_1\overset{*}{Q}_s\left(P-\overset{*}{P}\right)}{(1+k_1P+k_2Q_s)\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}_s\right)}\right]-\eta_3\left[\frac{\left(1+k_1\overset{*}{P}\right)\left(Q_i-\overset{*}{Q}_i\right)-k_1\overset{*}{Q}_i\left(P-\overset{*}{P}\right)}{(1+k_1P+k_2Q_i)\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}_i\right)}\right]\right)$$
$$+\left(Q-\overset{*}{Q}\right)\left[\alpha\eta_1\left(\frac{\left(1+k_2\overset{*}{Q}\right)\left(P-\overset{*}{P}\right)-k_2\overset{*}{P}\left(Q-\overset{*}{Q}\right)}{(1+k_1P+k_2Q)\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}\right)}\right)\right]$$
$$+\left(Q_s-\overset{*}{Q}_s\right)\left(-\frac{\Delta}{Q_s\overset{*}{Q}_s}\left(Q_s-\overset{*}{Q}_s\right)-\beta\left(Q_i-\overset{*}{Q}_i\right)+\alpha\eta_2\left[\frac{\left(1+k_2\overset{*}{Q}_s\right)\left(P-\overset{*}{P}\right)-k_2\overset{*}{P}\left(Q_s-\overset{*}{Q}_s\right)}{(1+k_1P+k_2Q_s)\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}_s\right)}\right]\right)$$
$$+\left(Q_i-\overset{*}{Q}_i\right)\left(\beta\left(Q_s-\overset{*}{Q}_s\right)+\alpha\eta_3\left[\frac{\left(1+k_2\overset{*}{Q}_i\right)\left(P-\overset{*}{P}\right)-k_2\overset{*}{P}\left(Q_i-\overset{*}{Q}_i\right)}{(1+k_1P+k_2Q_i)\left(1+k_1\overset{*}{P}+k_2\overset{*}{Q}_i\right)}\right]\right)$$

Consequently, we get following expression given below;

$$\overset{\bullet}{W}=-\left[\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q}_s}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q}_i}{C\overset{*}{C}}\right)\left(P-\overset{*}{P}\right)^2+\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)\left(Q-\overset{*}{Q}\right)^2+\left(\frac{\Delta}{Q_s\overset{*}{Q}_s}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)\left(Q_s-\overset{*}{Q}_s\right)^2+\left(\frac{\alpha\eta_3k_2\overset{*}{P}}{B\overset{*}{B}}\right)\left(Q_i-\overset{*}{Q}_i\right)^2\right.$$
$$+\left(\frac{\eta_1\left(1+k_1\overset{*}{P}\right)-\alpha\eta_1\left(1+k_2\overset{*}{Q}\right)}{A\overset{*}{A}}\right)\left(P-\overset{*}{P}\right)\left(Q-\overset{*}{Q}\right)+\left(\frac{\eta_2\left(1+k_1\overset{*}{P}\right)-\alpha\eta_2\left(1+k_2\overset{*}{Q}_s\right)}{B\overset{*}{B}}\right)\left(P-\overset{*}{P}\right)\left(Q_s-\overset{*}{Q}_s\right)$$
$$\left.+\left(\frac{\eta_3\left(1+k_1\overset{*}{P}\right)-\alpha\eta_3\left(1+k_2\overset{*}{Q}_i\right)}{C\overset{*}{C}}\right)\left(P-\overset{*}{P}\right)\left(Q_i-\overset{*}{Q}_i\right)\right]$$

The above expression can be written as $L'^TM'L'$, where $L'=\left(P-\overset{*}{P},Q-\overset{*}{Q},Q_s-\overset{*}{Q}_s,Q_i-\overset{*}{Q}_i\right)$ and

$$M'=\begin{bmatrix} M'_{PP} & M'_{PQ} & M'_{PQ_s} & M'_{PQ_i} \\ M'_{PQ} & M'_{QQ} & M'_{Q_sQ} & M'_{Q_iQ} \\ M'_{PQ_s} & M'_{QQ_s} & M'_{Q_sQ_s} & M'_{Q_iQ_s} \\ M'_{PQ_i} & M'_{QQ_i} & M'_{Q_sQ_i} & M'_{Q_iQ_i} \end{bmatrix}$$

where,
$M'_{PP}=\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q}_s}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q}_i}{C\overset{*}{C}}\right)$, $M'_{QQ}=\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)$,
$M'_{Q_sQ_s}=\left(\frac{\Delta}{Q_s\overset{*}{Q}_s}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)$, $M'_{Q_iQ_i}=\left(\frac{\alpha\eta_3k_2\overset{*}{P}}{C\overset{*}{C}}\right)$,

$$M'_{QQ_s}=M'_{Q_sQ}=0, M'_{PQ}=M'_{QP}=\frac{1}{2}\left(\frac{\eta_1\left(1+k_1\overset{*}{P}\right)-\alpha\eta_1\left(1+k_2\overset{*}{Q}\right)}{A\overset{*}{A}}\right),$$

$$M'_{PQ_s}=M'_{Q_sP}=\frac{1}{2}\left(\frac{\eta_2\left(1+k_1\overset{*}{P}\right)-\alpha\eta_2\left(1+k_2\overset{*}{Q_s}\right)}{B\overset{*}{B}}\right),$$

$$M'_{PQ_i}=M'_{Q_iP}=\frac{1}{2}\left(\frac{\eta_3\left(1+k_1\overset{*}{P_i}\right)-\alpha\eta_3\left(1+k_2\overset{*}{Q_i}\right)}{C\overset{*}{C}}\right),$$

$$M'_{Q_sQ_i}=M'_{Q_iQ_s}=0, M'_{QQ_i}=M'_{Q_iQ}=0.$$

Therefore, $\overset{\bullet}{W}=\frac{dW}{dt}$ is negative definite if the symmetric matrix M' is positive definite. Which is possible when the entire principal minors of M' are positive.

$$P'_1=M'_{PP}=\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q_s}}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q_i}}{C\overset{*}{C}}\right),$$

$$P'_2=M'_{PP}.M'_{QQ}-{M'_{PQ}}^2$$
$$=\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q_s}}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q_i}}{C\overset{*}{C}}\right)\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)$$
$$-\frac{1}{4}\left(\frac{\eta_1\left(1+k_1\overset{*}{P}\right)-\alpha\eta_1\left(1+k_2\overset{*}{Q}\right)}{A\overset{*}{A}}\right)^2,$$

$$P'_3=M'_{PP}.M'_{QQ}.M'_{Q_sQ_s}-M'_{QQ}.{M'_{PQ_s}}^2-M'_{Q_sQ_s}.{M'_{PQ}}^2$$
$$=\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q_s}}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q_i}}{C\overset{*}{C}}\right)\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)$$
$$\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)$$
$$-\frac{1}{4}\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)\left(\frac{\eta_2\left(1+k_1\overset{*}{P}\right)-\alpha\eta_2\left(1+k_2\overset{*}{Q_s}\right)}{B\overset{*}{B}}\right)^2$$
$$-\frac{1}{4}\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)\left(\frac{\eta_1\left(1+k_1\overset{*}{P}\right)-\alpha\eta_1\left(1+k_2\overset{*}{Q}\right)}{A\overset{*}{A}}\right)^2$$

and

$$P'_4={M'_{PQ}}^2.M'_{Q_sQ_s}.M'_{Q_iQ_i}+{M'_{PQ_s}}^2.M'_{QQ}.M'_{Q_iQ_i}$$
$$+{M'_{PQ_i}}^2.M'_{QQ}.M'_{Q_sQ_s}-M'_{PP}.M'_{QQ}.M'_{Q_sQ_s}.M'_{Q_iQ_i}$$
$$=\frac{1}{4}\left(\frac{\eta_1\left(1+k_1\overset{*}{P}\right)-\alpha\eta_1\left(1+k_2\overset{*}{Q}\right)}{A\overset{*}{A}}\right)^2\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)\left(\frac{\alpha\eta_3k_2\overset{*}{P}}{C\overset{*}{C}}\right)$$
$$+\frac{1}{4}\left(\frac{\eta_2\left(1+k_1\overset{*}{P}\right)-\alpha\eta_2\left(1+k_2\overset{*}{Q_s}\right)}{B\overset{*}{B}}\right)^2\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)\left(\frac{\alpha\eta_3k_2\overset{*}{P}}{C\overset{*}{C}}\right)$$
$$+\frac{1}{4}\left(\frac{\eta_3\left(1+k_1\overset{*}{P_i}\right)-\alpha\eta_3\left(1+k_2\overset{*}{Q_i}\right)}{C\overset{*}{C}}\right)^2\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)$$
$$-\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q_s}}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q_i}}{C\overset{*}{C}}\right)\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)$$
$$\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)\left(\frac{\alpha\eta_3k_2\overset{*}{P}}{C\overset{*}{C}}\right).$$

$P'_1>0$ if $\frac{r}{K}>\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q_s}}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q_i}}{C\overset{*}{C}}$,

Table 2. Values of Parameter

Parameters	Description	Parameter value
P	Native prey population	-
Q	Native predator population	-
Q_s	Exotic susceptible predator population	-
Q_i	Exotic infected predator population	-
r	Intrinsic growth rate of prey population	5
k	The carrying capacity of the environment	800
k_1	Half saturation constant	0.9
k_2	Magnitude of interference among predators	2
α	Conversion efficiency $(0<\alpha<1)$	0.7
β	Disease transmission rate	0.04
Δ	Recruitment rate of exotic predator population	30
η_1	Search rate of exotic prey by native predators $(\eta_1 and \eta_2>\eta_3)$	0.65
η_2	Search rate of exotic prey by exotic susceptible predators	0.52
η_3	Search rate of exotic prey by exotic infected predators	0.29
d	Natural death rate of predator	0.5
σ	Disease induced death rate of infected predator population	0.25

$P'_2>0$ if $\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q_s}}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q_i}}{C\overset{*}{C}}\right)\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)$
$$>\frac{1}{4}\left(\frac{\eta_1\left(1+k_1\overset{*}{P}\right)-\alpha\eta_1\left(1+k_2\overset{*}{Q}\right)}{A\overset{*}{A}}\right)^2,$$

$P'_3>0$ if $\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q_s}}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q_i}}{C\overset{*}{C}}\right)\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)$
$$\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)$$
$$>\frac{1}{4}\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)\left(\frac{\eta_2\left(1+k_1\overset{*}{P}\right)-\alpha\eta_2\left(1+k_2\overset{*}{Q_s}\right)}{B\overset{*}{B}}\right)^2$$
$$+\frac{1}{4}\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)\left(\frac{\eta_1\left(1+k_1\overset{*}{P}\right)-\alpha\eta_1\left(1+k_2\overset{*}{Q}\right)}{A\overset{*}{A}}\right)^2 \text{ and}$$

$P'_4>0$ if
$$\frac{1}{4}\left(\frac{\eta_1\left(1+k_1\overset{*}{P}\right)-\alpha\eta_1\left(1+k_2\overset{*}{Q}\right)}{A\overset{*}{A}}\right)^2\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)\left(\frac{\alpha\eta_3k_2\overset{*}{P}}{C\overset{*}{C}}\right)$$
$$+\frac{1}{4}\left(\frac{\eta_2\left(1+k_1\overset{*}{P}\right)-\alpha\eta_2\left(1+k_2\overset{*}{Q_s}\right)}{B\overset{*}{B}}\right)^2\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)\left(\frac{\alpha\eta_3k_2\overset{*}{P}}{C\overset{*}{C}}\right)$$
$$+\frac{1}{4}\left(\frac{\eta_3\left(1+k_1\overset{*}{P_i}\right)-\alpha\eta_3\left(1+k_2\overset{*}{Q_i}\right)}{C\overset{*}{C}}\right)^2\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)$$
$$>\left(\frac{r}{K}-\frac{\eta_1k_1\overset{*}{Q}}{A\overset{*}{A}}-\frac{\eta_2k_1\overset{*}{Q_s}}{B\overset{*}{B}}-\frac{\eta_3k_1\overset{*}{Q_i}}{C\overset{*}{C}}\right)\left(\frac{\alpha\eta_1k_2\overset{*}{P}}{A\overset{*}{A}}\right)$$
$$\left(\frac{\Delta}{Q_s\overset{*}{Q_s}}+\frac{\alpha\eta_2k_2\overset{*}{P}}{B\overset{*}{B}}\right)\left(\frac{\alpha\eta_3k_2\overset{*}{P}}{C\overset{*}{C}}\right).$$

Table 3. Equilibrium points for various values of β

Transmission rate of disease(β)	Equilibrium points
0.04	$B_3\,(795, 3.47,\ 13.8,\ 51.4)$
0.06	$B_3\,(795.4,\ 3.47,\ 9.22,\ 52.4)$
0.08	$B_3\,(795.7,\ 3.48,\ 6.921,\ 52.08)$

Table 4. Equilibrium points for various values of η_3

Predation rate (η_3)	Equilibrium points
0.29	$B_3\,(795, 3,\ 13,\ 51)$
0.2	$B_3\,(795.5,\ 3.47,\ 15.3,\ 46)$
0.1	$B_3\,(796,\ 3.4,\ 17.0,\ 41.2)$

Thus if previous conditions hold then (B_4) is stable, otherwise unstable.

6. Numerical Simulation

The main object of this section is to observe the dynamical behaviour of the system for various values of parameters and calculate equilibrium points. Here, we have performed numerical simulations using MATLAB R2014a (32-bit) and Wolfram Mathematica 8.0 softwares for system (2.1). Predation rate (η_3), carrying capacity of environment (K) and transmission rate of disease (β) are significant parameters from study point of view. For validity of the results of the system (2.1), we choose a set of biologically feasible parameter values, which are given in Table 2.
We have obtained a set of invariant equilibrium points for various values of β under the fixed value of $\eta_3 = 0.29$ and $K = 800$ started in Table 3 given below;

We have found out a set of invariant equilibrium points for various values of η_3 under the fixed value of $\beta = 0.04$ and $K = 800$ listed in Table 4 given below;

We have carried out a set of invariant equilibrium points for various values of K under the fixed value of $\beta = 0.04$and $\eta_3 = 0.29$ putted in Table 5 given below;

Thus, on this section we have observed the dynamic behavior of model system (2.1) for various values of η_3, β and K.

7. Conclusion

Predation is an important factor that regulates prey population. The fatal disease can harm population that decreases

Table 5. Equilibrium points for various values of K

Carrying capacity of environment (K)	Equilibrium points
600	$B_3\,(595, 2.47,\ 14.01,\ 50.60)$
800	$B_3\,(795,\ 3.47,\ 13.8,\ 51.4)$
1000	$B_3\,(994,\ 4.4,\ 13.7,\ 52.03)$

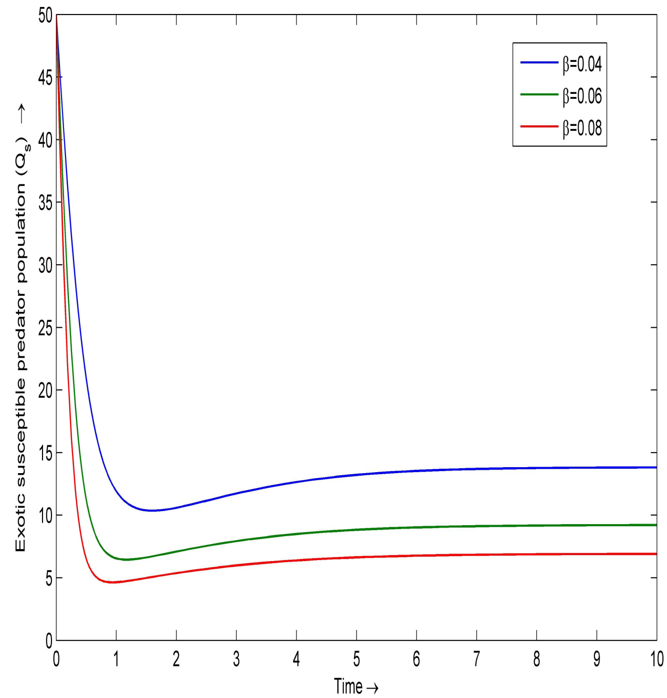

Figure 2. Plot between Time (t) and exotic susceptible predator (Q_s) for various values of β

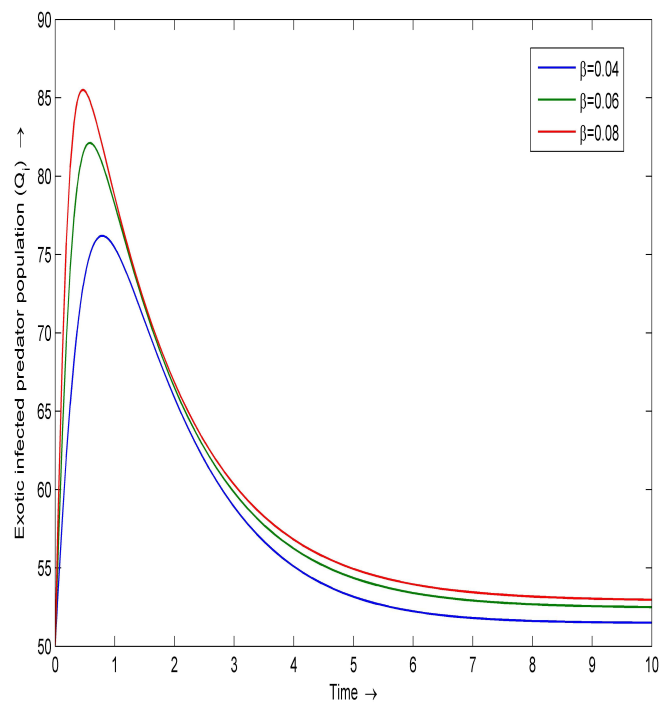

Figure 3. Plot between Time (t) and exotic infected predator (Q_i) for various values of β

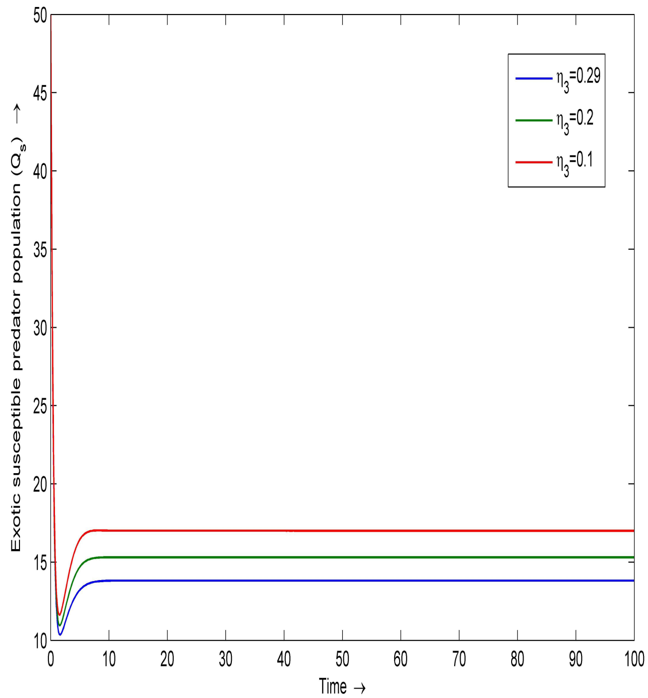

Figure 4. Plot between Time (t) and exotic susceptible predator (Q_s) for various values of η_3

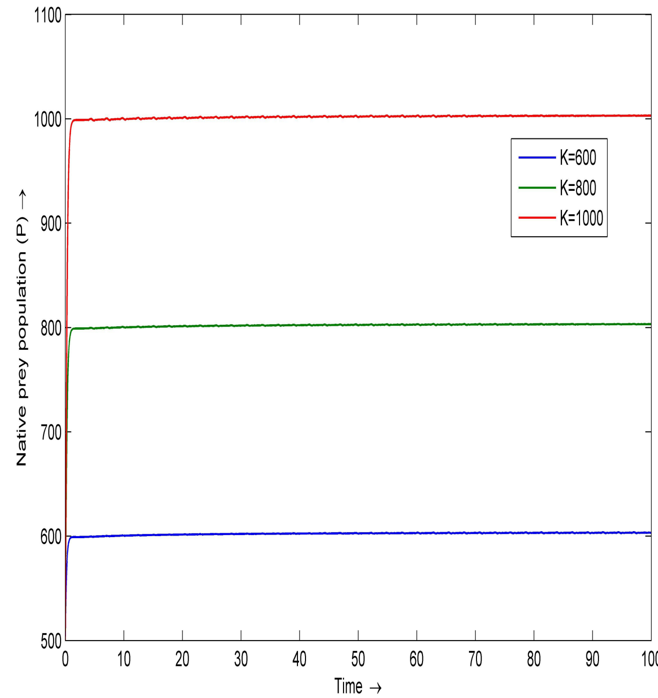

Figure 6. Plot between Time (t) and native preys (P) for various values of K

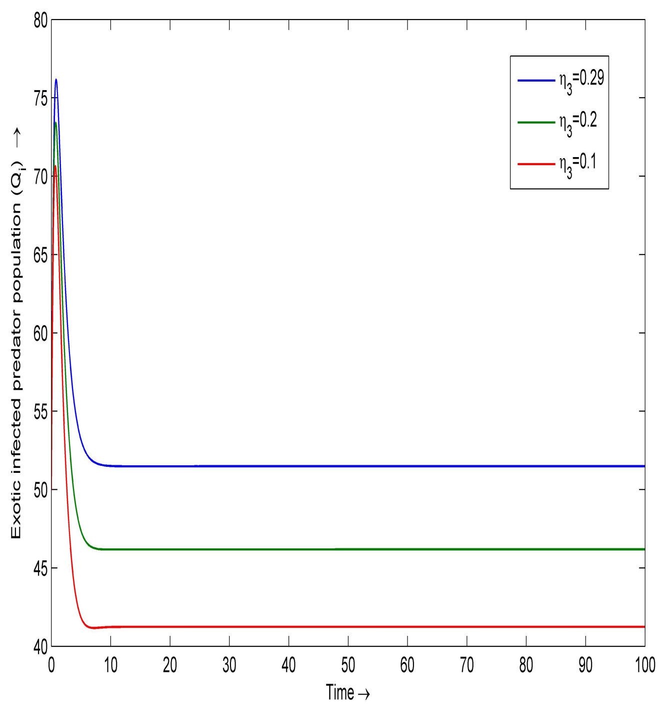

Figure 5. Plot between Time (t) and exotic infected predator (Q_i) for various values of η_3

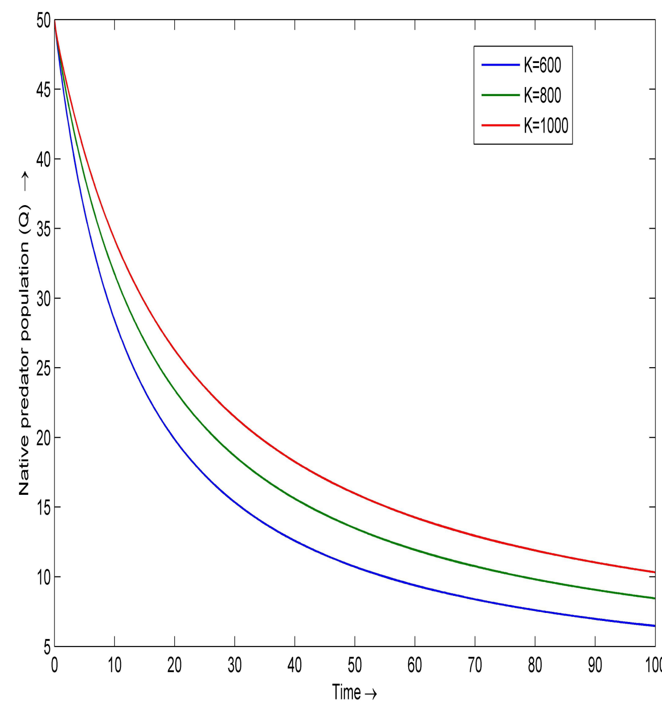

Figure 7. Plot between Time (t) and native predator (Q) for various values of K

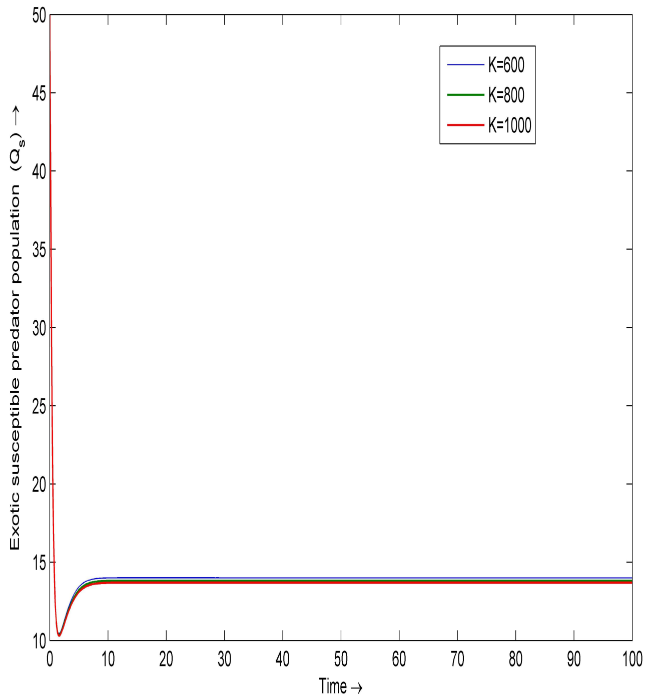

Figure 8. Plot between Time (t) and exotic susceptible predator (Q_s) for various values of K

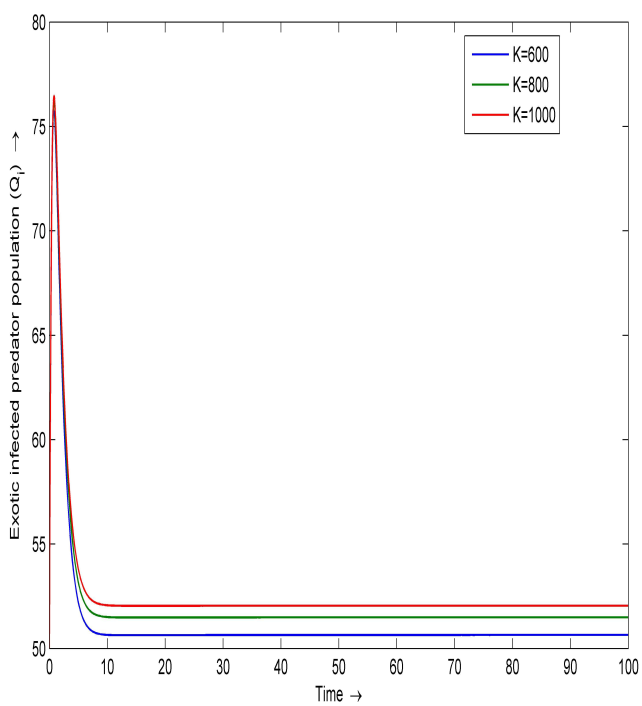

Figure 9. Plot between Time (t) and exotic infected predator (Q_i) for various values of K

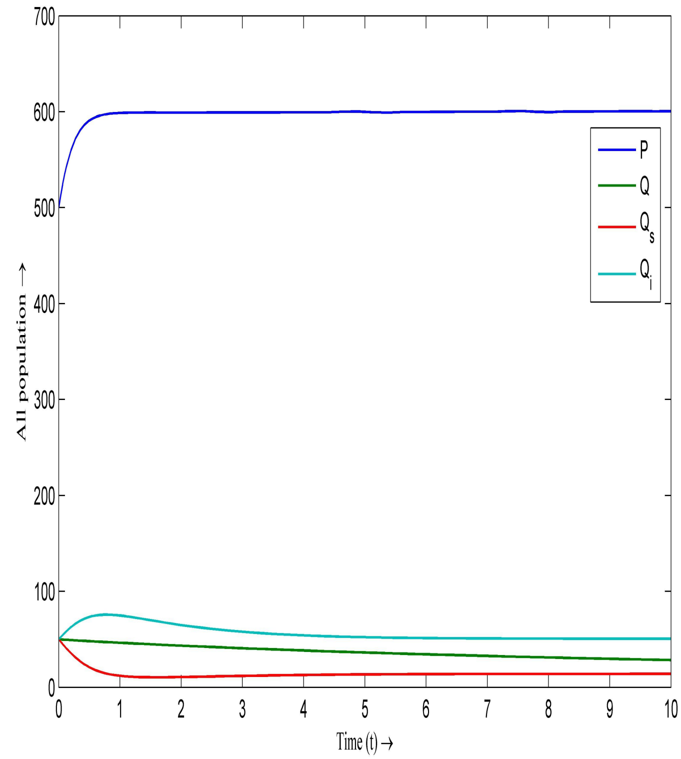

Figure 10. Plot between Time (t) and all population

the growth rate or increasing the death rate. In this paper, a non linear mathematical model with native population and endemic exotic predators was formed to study the transmission of disease. It is shown that three factors i.e. transmission rate of disease(β) ,predation rate (η_3) and carrying capacity of environment (K) which can be taken as sensitive parameters affects the community size. Keeping $\eta_3 = 0.29$ and $K = 800$ fixed, it was concluded that as β disease transmission rate of disease decreases exotic susceptible predator population increases (see Fig.2) and exotic infected predator population decreases (see Fig.3). Keeping $\beta = 0.04$ and $K = 800$ fixed, it was seen that as η_3 the predation rate of exotic infected predators, decreases exotic susceptible predator population increases (see Fig.4) and exotic infected predator population decreases (see Fig.5). Keeping $\eta_3 = 0.29$ and $\beta = 0.04$ fixed, it was observed that as K decreases native prey population, native predator population, exotic infected predator population decreases (see Figs.6,7,9) and there is not effect of change in K on exotic susceptible predators Q_s (see Fig.8). Dynamic of all the species is also depicted (see Fig.10). However, it is also argued that consumption of prey by infected predator may have positive or negative effect on community structure, depends upon infection severity. Special care or prevention should be given to species to save society from disease.

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