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The impact of aligned magnetic field on Walters Liquid B fluid over a stretching surface with Rosseland diffusion approximation

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Abstract

In the present article, we have investigated the influence of inclined magnetic field on Walter's Liquid B fluid flow through stretching sheet. Elastic deformation and Rosseland diffusion approximation for radiative heat flux are considered. With the help of similarity transformation the flow and energy equations are solved analytically by using confluent hypergeometric method. Results for various physical parameters are presented through graphs. It's also noted that the increasing values of elastic deformation parameter increase the local Nusselt number.

Keywords

Elastic Deformation, Inclined magnetic field, Rosseland Diffusion, Walter's Liquid B, Viscoelastic.

AMS Subject Classification

76D05, 76W05, 80A20.

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Contents

1. Introduction

The Walter's Liquid B fluid model is a non-Newtonian fluid model and it exhibit both viscous and elastic personality. In this fluid model with limiting viscosity at very low shear rates and short remembrance coefficient is one of the best models to describe the characteristics of a viscoelastic fluid. In this non-Newtonian fluid model can illustrate the behaviors of various polymeric liquid encountered in biotechnology and chemical engineering. Abdul Hakeem et al.[1] analyzed the both effects of non uniform heat source/sink and thermal radiation on Walters-B fluid flow through stretching sheet. Hussain and Ullah [2] studied the temperature dependent viscosity on Walters-B fluid flow through stretchable cylinder. Majeed et al. [3] analyzed numerically Walters-B fluid with various physical effects. MHD flow of Walters-B fluid over a nonlinear sheet discussed by Siddheshwar et al.[4].

Magnetohydrodynamic (MHD) qualities demonstrate the properties of electrically conducting fluids. MHD has numerous real-life applications in astrophysics, cosmology, geophysics, sensors, magnetic drug targeting and engineering. Inclined magnetic field effects on second grade nanofluid flow through stretching sheet are analyzed by Abdul Hakeem et al. [5]. Sheikholeslami et al. [6] investigated the non-uniform magnetic field effect on nanofluid. Jena et al. [7] studied the heat source/sink and chemical reaction effects on MHD viscoelastic fluid. Hayat et al. [8] discussed the Soret and Dufour effects in peristaltic motion of MHD couple stress fluid. Viscous dissipation effects in MHD 3D flow are analyzed by Muhammad et al. [9]. Daniel et al. [10] investigated thermal stratification effects on MHD radiative flow of nanofluid over nonlinear stretching sheet. In recent year, several researchers have investigated MHD effects in various physical phenomena [11-15].

Hence, main objective of the present article is to study Rosseland diffusion approximation and elastic deformation effects on Walters-B fluid flow through stretching sheet in the presence of inclined magnetic field are investigated. Analytical solutions are obtained for the transformed ODE's of momentum and energy PDE's by using confluent hypergeometric function. Prescribed surface temperature (PST) case is considered. Results for various physical parameters are presented through graphs.

2. Mathematical formulation

Consider a 2D steady laminar boundary layer flow of an incompressible Walters-B fluid (non-Newtonian) over a stretching sheet. Walter's B fluid flow is considered in x-direction, which is chosen along the stretching sheet and y is perpendicular it. Aligned magnetic field of strength B_0 applied along y direction, with acute angle anγ. At $\gamma = 90^o$ this magnetic field acts like transverse magnetic field (because $sin(90^o) = 1$). The magnetic Reynolds number is chosen very small. As a consequence, the induced magnetic field is lesser in comparison to the applied magnetic field . Thus, the induced magnetic field is not considered. The effects of viscous dissipation are further considered.

3. Flow Analysis

The equation governing the problem under consideration is given by

$$u_x + v_y = 0 \quad (3.1)$$

$$uu_x + vu_y = \vartheta u_{yy} - k_0 \left\{ uu_{xyy} + vu_{yyy} + u_x u_{yy} - u_y u_{xy} \right\} - \frac{\sigma B_0^2}{\rho}\ u\ sin^2\gamma \quad (3.2)$$

where u and v are the fluid velocity components of x and y direction, respectively, ϑ is the kinematic viscosity, ρ is the fluid density, k_0 is the elastic parameter and σ is the electrical conductivity.

The boundary conditions for the velocity field are given below

$$\begin{aligned} &u = ax \quad v = v_w \quad \text{at} \quad y = 0, \\ &u \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \quad (3.3)$$

We introduce the following similarity transformation and non-dimensional variables η and $f(\eta)$

$$u = axf_\eta, \quad v = -(a\vartheta)^{\frac{1}{2}} f, \quad \eta = \left(\frac{a}{\vartheta}\right)^{\frac{1}{2}} y \quad (3.4)$$

Using these similarity transformations in Eq. 1 is trivially satisfied and Eq. 2 non-dimensional form is:

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1 \left\{ 2f_\eta - f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} \right\} - Mnf_\eta sin^2\gamma \quad (3.5)$$

with corresponding boundary conditions

$$\begin{aligned} &f = 0, \ f_\eta = 1, \quad \text{at} \quad \eta = 0, \\ &f_\eta \to 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \quad (3.6)$$

Here $Mn = \frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter. $k_1 = \frac{k_0 a}{\vartheta}$ is the viscoelastic parameter.

The solution of (5) subject to boundary conditions is (6) can be found in the form,

$$f(\eta) = \frac{1 - e^{-\alpha\eta}}{\alpha} \quad (3.7)$$

Where

$$\alpha = \sqrt{\frac{1 + Mn\ sin^2\gamma}{1 - k_1}} \quad (3.8)$$

The wall shearing stress on the surface of the stretching sheet is given by

$$\tau_w = \left[vu_y - k_0 \left(uu_{xy}^2 - 2u_x u_y \right) \right]_{y=0} \quad (3.9)$$

The local skin friction coefficient is given by

$$C_f = \frac{\tau_w}{\rho {u_w}^2} = {Re_x}^{-1/2} (1 - k_1) f_{\eta\eta}(0) \quad (3.10)$$

where $Re_x = \frac{xu_w}{\vartheta}$ is the Reynolds number.

4. Heat Transfer Analysis

The governing thermal boundary layer equation of incompressible Walter's B Liquid fluid is stated as follows:

$$\rho c_p (uT_x + vT_y) = kT_{yy} + \mu u_y^2 - \rho\delta k_o \left\{ u_y (uu_x + vu_y)_y \right\} - (q_r)_y \quad (4.1)$$

where k is the thermal conductivity, ρ is the density, T is the temperature and c_p is the specific heat of constant pressure.

The Rosseland diffusion approximation for radiation heat flux has given by

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}, \quad (4.2)$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Further, we assume that the temperature difference within in flow is such that T^4 may be expanded in a Taylor series. Hence expanding T^4 about T_∞ and neglecting higher order terms we get

$$T^4 \cong 4{T_\infty}^3 T - 3T_\infty^4. \quad (4.3)$$

The boundary conditions are given by

$$\begin{aligned} &T = T_w = T_\infty + A(\tfrac{x}{l})^2 \quad \text{at} \quad y = 0, \\ &T \to T_\infty \text{ as } \quad y \to \infty \end{aligned} \quad (4.4)$$

where T_w is the temperature of the sheet, T_∞ is the temperature of the fluid far away from the sheet and l is the characteristic length. Define the non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta) = (T - T_\infty)(T_w - T_\infty)^{-1}. \tag{4.5}$$

Now, we make use of the transformations given by (4), (12), (15) in (11). This leads to the non dimensional form of temperature equation as follows:

$$\begin{aligned}&\theta_{\eta\eta} + \omega Prf\theta_\eta - 2\omega Prf_\eta \theta = \\ &-EcPr\omega\left\{f_{\eta\eta}^2 - \delta k_1 f_{\eta\eta}(f_\eta f_{\eta\eta} - ff_{\eta\eta\eta})\right\}\end{aligned} \tag{4.6}$$

where $Pr = \dfrac{\mu c_p}{k}$ is the Prandtl number, $Ec = \dfrac{u_w^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $N = \dfrac{k^* k}{4\sigma^* {T_\infty}^3}$ is radiation parameter and $\omega = \left(\dfrac{3N}{3N+4}\right)$.

Therefore, the boundary conditions in (14) take the form,

$$\begin{aligned}&\theta(\eta) = 1 \text{ at } \eta = 0,\\ &\theta(\eta) \to 0 \text{ as } \eta \to \infty.\end{aligned} \tag{4.7}$$

The result of (16), subject to boundary conditions in (17) can be obtained in terms of confluent hypergeometric function as

$$\begin{aligned}\theta(\eta) = {} & c_1 e^{-\alpha\left(\frac{a_0+b_0}{2}\right)\eta} \\ & \times M\left(\tfrac{a_0+b_0-4}{2}, 1+b_0, \tfrac{-Pr}{\omega\alpha^2}e^{-\alpha\eta}\right) \\ & -c_2 e^{-2\alpha\eta}\end{aligned} \tag{4.8}$$

$$\begin{aligned}\text{where } a_0 &= Pr\left(\frac{\omega}{\alpha^2}\right),\\ b_0 &= a_0,\\ c_2 &= \frac{-EcPr\omega(1-k_1)}{2(2-a_0)}\\ c_1 &= \frac{1+c_2}{M\left(\frac{a_0+b_0-4}{2}, 1+b_0, \frac{-Pr\omega}{\alpha^2}\right)}\end{aligned}$$

The non-dimensional wall temperature gradient obtained as follows:

$$\begin{aligned}\theta_\eta(0) = {} & -c_1\alpha\left(\frac{a_0+b_0}{2}\right) \\ & \times M\left(\frac{a_0+b_0-4}{2}, 1+b_0, \frac{-Pr\omega}{\alpha^2}\right) \\ & +c_1\frac{Pr\omega}{\alpha}\left(\frac{a_0+b_0-4}{2(1+b_0)}\right) \\ & M\left(\frac{a_0+b_0-2}{2}, 2+b_0, \frac{-Pr\omega}{\alpha^2}\right) \\ & +2c_2\alpha\end{aligned}$$

The local heat flux can be expressed as

$$\begin{aligned}q_w &= -\left(k + \frac{16\sigma^* T_\infty^3}{3kk^*}\right)\left(\frac{\partial T}{\partial y}\right)_{y=0} \\ &= -k\sqrt{\frac{a}{\nu}}(T_w - T_\infty)\omega\theta_\eta(0).\end{aligned} \tag{4.9}$$

The local Nusselt number is defined as

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)}.$$

In the present case it is derived as

$$Nu_x Re_x^{-1/2} = -\theta_\eta(0).$$

5. Results and Discussion

In this new study we have investigated the impact of aligned magnetic field on Walter's liquid B fluid over a stretching surface with Rosseland diffusion approximation. The effects of viscous dissipation are further considered. We have compared the values of $-\theta_\eta(0)$ with these of Turkyilmazoglu [13]. The comparison is found to be in good agreement as shown in Table 1.

5.1 Results for velocity profile

The aligned magnetic field and viscoelastic parameter impacts on velocity profile are depicted in Fig. 1. The Walter's B liquid fluid velocity decreases with increasing values of aligned magnetic field. It is clear that the increasing of aligned magnetic parameter slows down the fluid motion. It is also observed that getting higher of viscoelastic gives lower velocity in Walter's B liquid fluid area.

5.2 Results for Temperature profile

The effect of angle and magnetic parameters on temperature profile is illustrated in Fig. 2. It is clear that the rising values of aligned magnetic field increases the thickness of the thermal boundary layer. Because, Lorentz force is managed this resists the fluid flow due to which more heat is produced. The Walter's liquid B fluid flow which is suppressed by the viscous action gets a push from the magnetic field which counteracts the viscous effects. The combined effect of magnetic and aligned angle parameter increases the thermal boundary layer thickness.

The impact of viscoelastic and elastic deformation parameters on the temperature profile is shown in Fig. 3. It is observed that the thermal boundary layer thickness increase with viscoelastic parameter and decreases with elastic deformation parameter. Effects of radiation parameter, Prandtl number and Eckert number on temperature profile are shown in Fig. 4. It is noted that the increment of Eckert number increases the thermal boundary layer. The temperature profile always found to be decreased in the presence of thermal radiation and Prandtl number.

5.3 Results for local skin friction coefficient

The effects of magnetic parameter with aligned angle and viscoelastic parameter on the local skin friction coefficient are shown in Fig. 5. The magnetic parameter Mn is taken as the x-axis and the local skin friction coefficient is taken as the y-axis. It is clear that the local skin friction coefficient decreases with the increasing values of viscoelastic and angle parameter.

5.4 Results for local Nusselt number

The combined effects of the magnetic parameter with an aligned angle, viscoelastic, Prandtl number, Eckert number and radiation parameters on the local Nusselt number are demonstrated in Figs 6-7 respectively. The magnetic parameter Mn is taken as the x-axis and the local Nusselt number is taken as the y-axis. Fig. 6 shows that the effect of viscoelastic, elastic deformation and angle parameters on the local Nusselt number. The increasing values of angle and viscoelastic are decreased the local Nusselt number. It's also noted that the increasing values of elastic deformation parameter increase the local Nusselt number. Effect of Prandtl number, Eckert number and radiation parameter on the local Nusselt number are shown in Fig. 7. The combined effects of Prandtl number and radiation parameter are increasing the local Nusselt number. It's noted that the increasing values of Eckert number decrease the local Nusselt number.

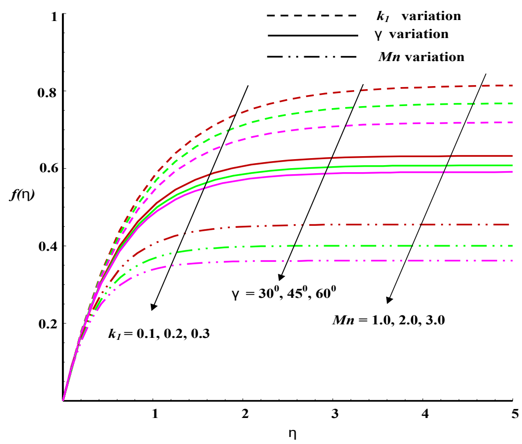

Figure 1. Effect of aligned angle, magnetic and viscoelastic parameters on velocity profile $f(\eta)$ with $k_1 = 0.5$, $Mn = 0.5$ and $\gamma = 45^0$.

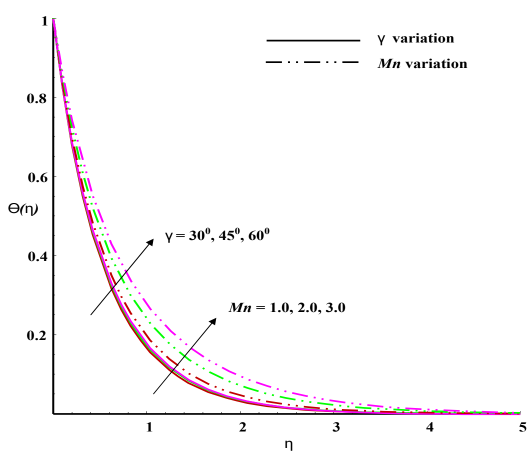

Figure 2. Effect of aligned angle and magnetic on temperature profile $\theta(\eta)$ with $k_1 = 0.5$, $Mn = 0.5$, $Pr = 6$, $N = 1$, $Ec = 0.5$ and $\gamma = 45^0$.

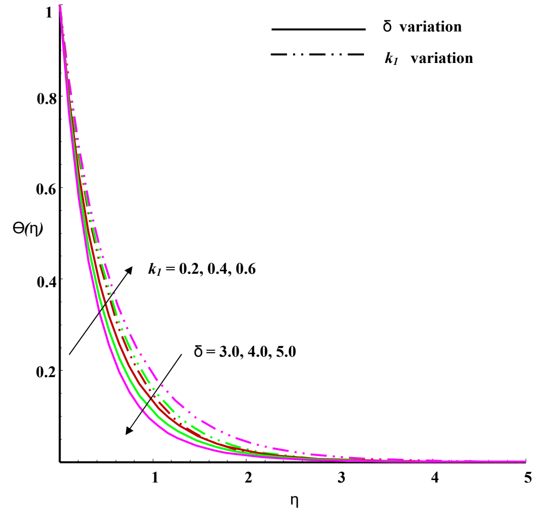

Figure 3. Effect of viscoelastic and elastic deformation on temperature profile $\theta(\eta)$ with $k_1 = 0.5$, $Mn = 0.5$, $Pr = 6$, $N = 1$, $Ec = 0.5$ and $\gamma = 45^0$.

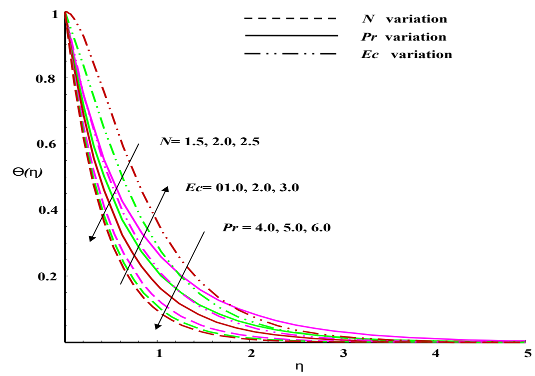

Figure 4. Effect of radiation parameter, Prandtl number and Eckert number on temperature profile $\theta(\eta)$ with $k_1 = 0.5$, $Mn = 0.5$, $Pr = 6$, $N = 1$, $Ec = 0.5$ and $\gamma = 45^0$.

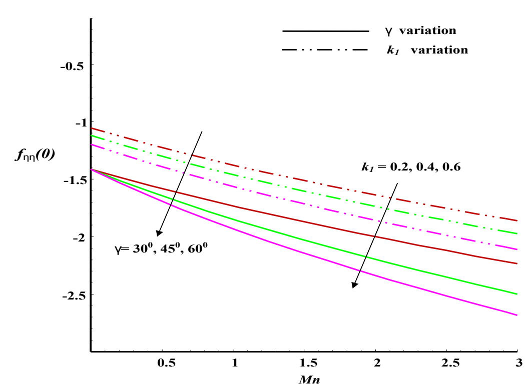

Figure 5. Effect of aligned angle, viscoelastic and magnetic on the local skin friction coefficient $f_{\eta\eta}(0)$ with $k_1 = 0.5$ and $\gamma = 45^0$.

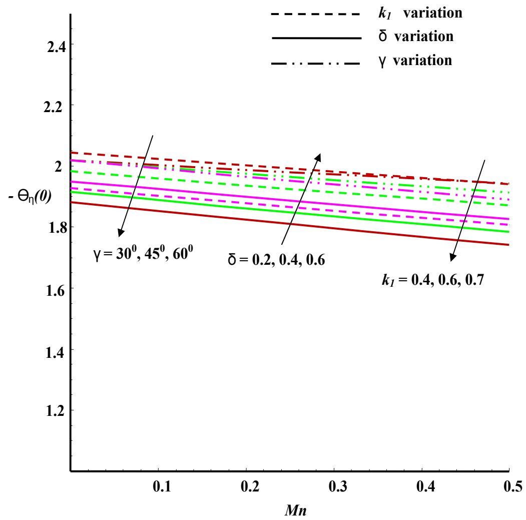

Figure 6. Effect of aligned angle, viscoelastic and elastic deformation parameters on the local Nusselt number $\theta_\eta(0)$ with $k_1 = 0.5$, $Pr = 6$, $N = 1$, $Ec = 0.5$ and $\gamma = 45^0$.

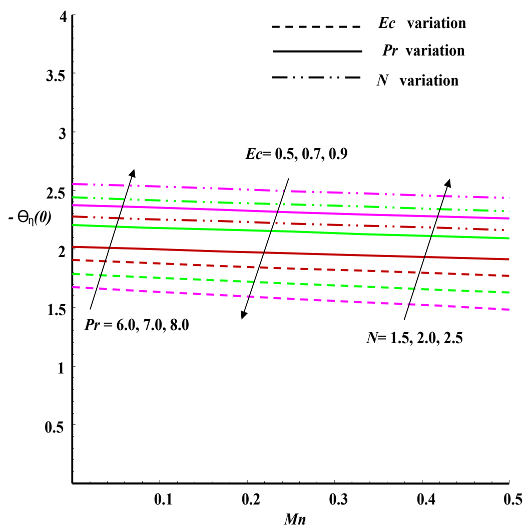

Figure 7. Effect of radiation parameter, Prandtl number and Eckert number on the local Nusselt number $\theta_\eta(0)$ with $k_1 = 0.5$, $Pr = 6$, $N = 1$, $Ec = 0.5$ and $\gamma = 45^0$.

6. Conclusion

An aligned magnetic field impact on Walter's B Liquid fluid flow over a stretching sheet with thermal radiation. An analytical solutions are obtained for governing momentum and energy equations and some specific results are obtained as follows.

- The momentum boundary layer of the non-Newtonian Walter's B Liquid fluid reduces with the increasing aligned angle of magnetic field, viscoelastic parameter and magnetic parameter.
- The inclined angle of the magnetic field acting an essential roles in controlling the magnetic field strength and the effects of Lorentz force on the Walter's B liquid fluid flow area.
- The thermal boundary layer enhances with the increasing values of aligned angle of magnetic field, viscoelastic parameters and Eckert number and decreases with Prandtl number and radiation parameter.
- The local skin friction coefficient decreases with enhanced values of viscoelastic parameter and aligned magnetic field.
- The elastic deformation and thermal radiation parameters increases the local Nusselt number.

Table 1. Comparison of $-\theta_\eta(0)$

Mn	Pr	Turkyilmazoglu [13]	Present Result
0	1	1.33333	1.33333
	5	3.31648	3.31648
1	1	1.21577	1.21577
	5	———	3.20720

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