



Stability analysis of a predator (bird) –prey (fish) harvesting model in the reserved and unreserved area

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Abstract

The extreme and unsustainable abuse of marine assets needs to prompt the advancement of a marine reserve as a fisheries management instrument. In this paper, a prey-predator fishery model in existence of bird predator, with prey dispersal in a two-patch environment has been proposed and examined. Holling type-I predator functional response to prey density has been considered for this research work. The harvesting is applied on prey in an unreserved area as well as on predator due to a commercial value. The dynamics of the proposed framework has been examined locally as well as globally. Finally, theoretical results so acquired have been confirmed with the support of numerical simulations through MATLAB.

Keywords

Prey-predator; Global stability; Reserve-unreserved area; Holling type-I; Lyapunov function; MPAs.

AMS Subject Classification

35Q61, 44A10, 44A15, 44A20, 44A30, 44A35, 81V10.

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1. Introduction

In the past few decades, the dynamics of interacting biological species have been examined from various angles. Numerous species have turned out to be threatened or endangered, and numerous others are on the verge of disappearance as a consequence of different reasons like overexploitation, over-predation, environmental pollution, mismanagement of

natural resources etc. To save these species, marine protected and marine reserve areas have been proposed as the most essential instrument to preserve the marine life and sustain the ecosystem. Beverton and Holt were the first in considering the idea of marine reserves. Clark [3] introduced the concept of economic and biological aspects of inexhaustible assets of multispecies fisheries. Recently, it has demonstrated by Dubey [5] that the reserved area has a stabilizing effect on the predator-prey dynamics. It is confirmed that regardless of whether the fishery is exploited constantly in the unreserved zone, fishery population can be kept up at an appropriate equilibrium level in the natural surroundings. Kar and Misra [17] have contemplated that the interior equilibrium level never disturbed in the absence of a predator, continuous harvesting and presence of a predator in the unreserved area. Dubey [6] proved that in ecology and evolution, the reserved zone plays a very significant role. By creation of a reserved zone in the habitat, provides the opportunity for the growth of the prey species without any external disturbances, where the predator has no access or chance of settling. In this way, the prey species can be preserved at a proper level. Kar and Chaudhuri [10] have investigated a prey-predator fishery system, where

only the prey species were liable to harvesting, by taking tax assessment as a control instrument. They looked for an optimal tax policy and an interior equilibrium corresponding to given tax policy. As per observational information for Lake Kasumigaura in Japan, Kitabatake [15] built up a dynamic model for fishery resources with a predator-prey relationship.

Marine reserves secure the species inside the reserve area along with increase fish richness in adjoining areas. Amit Sharma and Bhanu Gupta [29] studied the dynamics of fishery resource with reserve area in the presence of bird predator. Various possibilities of the biological and bionomic equilibrium of the system have been discussed. An optimal harvesting policy has been established using Pontryagin’s maximum principle. Yunfei et al. [28] investigated that marine reserves ensured the sustainability of the system. An appropriate equilibrium level of prey population is always maintained in the presence of predators as well as in the absence of predators in the unreserved zone. All the reserved and unreserved area models in an aquatic habitat have been motivated by the existence of Marine National Park, Kenya, a fully protected coral reef marine reserve comprising approximately 30% of former fishing ground and Marine National Park in the Iroise sea, a coastal sea west of Brittany (France).

By the creation of artificial boundary in the form of fencing of appropriate mesh size, predator’s passageway can be restricted to the reserved zone. Latest researchers found that MPAs are an exceptionally viable instrument for improving yield and in addition affirmation of stocks and maintainability of jeopardized species.

All the parameters of the system (2.1) are assumed to be positive and defined in the following Table:

Variable/Parameter	Description
$x_1(t)$	biomass density of the prey species inside the unreserved area
$x_2(t)$	biomass density of the prey species inside the reserved area
$y(t)$	biomass density of the bird predator
r	the intrinsic growth rate of the prey species inside the unreserved area
s	the intrinsic growth rate of the prey species inside the reserved area
K_1	carrying capacity of prey inside the unreserved area
m_1, m_2	migration rate from the unreserved area to reserved area and reserved area to unreserved area respectively
E_1, E_2	harvesting efforts applied to the prey (fishes) and the bird predator respectively
d	the natural death rate of a predator (bird)
$q_1 \& q_2$	the coefficient of catchability of prey in unreserved and predator area respectively
m	capturing rate of prey in reserved and unreserved area
α_1	conversion rate of prey to predator in unreserved and reserved area

In above model, it is assumed that if there is no migration of fish population from the reserved zone to the unreserved zone i.e. ($m_2 = 0$) and $r - m_1 - q_1 E_1$, then $\frac{dx_1}{dt} < 0$, In this case, fish species will be wiped out from the unreserved area. Correspondingly, if there is no immigration of the fish pop-

ulation from unreserved area to reserved area i.e. ($m_1 = 0$ and $s - m_2 < 0$), then $\frac{dx_2}{dt} < 0$ holds consequently, fishes will extinct from the reserved area. To protect the prey (fishes) species from extinction, migration of prey (fish) species from both the patches are essential. Therefore, throughout our

2. Formulation of the model

Consider a habitat in a biological system with prey (fishes) dispersal in a two-patch environment, one is assumed to be a free fishing zone and other is a reserved zone, where fishing and other additional activities are confined. Both zones are supposed to be homogeneous. Moreover, there is a bird predator in the framework which may move in both the reserved and unreserved areas of prey. We assume that the prey (fishes) species migrate between the two zones randomly. The harvesting is applied to prey in an unreserved area area as well as on the predator. It is assumed that capturing rates are same from both the reserves for the predator. The logistic growth is assumed only for prey in unreserved area. Keeping all the assumptions in view, a model is regulated by the following ordinary differential equations.

$$\begin{aligned}
 \frac{dx_1}{dt} &= rx_1 \left(1 - \frac{x_1}{K_1} \right) - m_1 x_1 + m_2 x_2 - m x_1 y - q_1 E_1 x_1 \\
 \frac{dx_2}{dt} &= s x_2 + m_1 x_1 - m_2 x_2 - m x_2 y \\
 \frac{dy}{dt} &= -d y + \alpha_1 (x_1 y + x_2 y) - q_2 E_2 y
 \end{aligned}
 \tag{2.1}$$



analysis, we assume that

$$r - m_1 - q_1E_1 > 0 \quad \text{and} \quad s - m_2 > 0 \quad (2.2)$$

3. Existence of equilibria

Following are the two possible steady states of the dynamical system of Eq. (2.1)

I $P_0(0, 0, 0)$, which always exist; (extinction of all species)

II $P_1(x_1^*, x_2^*, y^*)$, (The interior equilibrium point)

For the interior equilibrium point $P_1(x_1^*, x_2^*, y^*)$

On solving the 3rd equation of differential equations of (2.1) for non-zero point, we get

$$x_2 = \left[\frac{q_2E_2 + d}{\alpha_1} - x_1 \right] \quad (3.1)$$

On substituting the value of x_2 in differential equation of (2.1), the value of y is given by

$$y = \frac{1}{mx_1} \left[rx_1 - m_1x_1 - m_2x_1 - q_1E_1x_1 + \frac{m_2(q_2E_2 + d)}{\alpha_1} - \frac{rx_1^2}{K_1} \right] \quad (3.2)$$

For simplification let

$$\frac{q_2E_2 + d}{\alpha_1} = E_3.$$

Now substituting the value of x_2 from (3.1) and y from (3.2) in differential equation of (2.1), we obtain following cubic equation in terms of x_1

$$T_1x_1^3 + T_2x_1^2 + T_3x_1 + T_4 = 0, \quad (3.3)$$

where

$$\begin{aligned} T_1 &= \left(\frac{r}{K_1} \right) \\ T_2 &= \left(q_1E_1 + s - r - \frac{r}{K_1}E_3 \right) \\ T_3 &= ((r - m_1 - q_1E_1)E_3 - sE_3 - E_3m_2) \\ T_4 &= E_3^2m_2, \end{aligned}$$

assume

$$F(x_1) = T_1x_1^3 + T_2x_1^2 + T_3x_1 + T_4 \quad (3.4)$$

It is obvious that

$$F(0) = T_4 > 0$$

and if

$$F(K_1) = T_1K_1^3 + T_2K_1^2 + T_3K_1 + T_4 < 0 \quad (3.5)$$

then there exists a positive value of x_1 (say x_1^*) in the interval $[0, K_1]$. Now, the sufficient condition for x_1^* to be unique is

$$F'(x_1^*) = 3T_1(x_1^*)^2 + 2T_2(x_1^*) + T_3 < 0 \quad (3.6)$$

value of x_2^* and y^* will be given by

$$\begin{aligned} x_2^* &= \left[\frac{q_2E_2 + d}{\alpha_1} - x_1^* \right] \\ y^* &= \frac{1}{mx_1^*} \left[rx_1^* - m_2x_1^* - q_1E_1x_1^* - m_1x_1^* + \frac{m_2(q_2E_2 + d)}{\alpha_1} - \frac{r(x_1^*)^2}{K_1} \right] \end{aligned}$$

Thus x_2^* and Y^* will be positive iff following inequality holds

$$x_1^* < \min \left[\frac{q_2E_2 + d}{\alpha_1}, \frac{(r - m_1 - q_1E_1 - m_2)K_1}{r} \right] \quad (3.7)$$

Hence the equilibrium $P_1(x_1^*, x_2^*, y^*)$ exist, provided condition (3.7) is satisfied.

4. Stability analysis

The variational matrix of the system of equations (2.1) is

$$J_{(x_1, x_2, y)} = \begin{bmatrix} r - \frac{2rx_1}{K_1} - m_1 - q_1E_1 - my & m_2 & -mx_1 \\ m_1 & s - m_2 - my & -mx_2 \\ \alpha_1 y & \alpha_1 y & -d + \alpha_1 x_1 + \alpha_1 x_2 - q_2E_2 \end{bmatrix}$$

Theorem 4.1. *If the equilibrium point $P_0(0, 0, 0)$ exist then it will be always unstable.*

Proof.

$$\begin{vmatrix} r - m_1 - q_1E_1 - \lambda & m_2 & 0 \\ m_1 & s - m_2 - \lambda & 0 \\ 0 & 0 & -d - q_2E_2 - \lambda \end{vmatrix} = 0$$

The characteristic equation of system (2.1) at $P_0(0, 0, 0)$ is given by

$$\begin{aligned} \Rightarrow (\lambda + d + q_2E_2) [\lambda^2 - \lambda(r - m_1 - q_1E_1 + s - m_2) \\ + (r - m_1 - q_1E_1)(s - m_2) - m_1m_2] = 0 \end{aligned}$$

One of the eigenvalue is $\lambda_1 = -(d + q_2E_2) < 0$

Other two eigenvalues are given by

$$\begin{aligned} \lambda^2 - \lambda(r - m_1 - q_1E_1 + s - m_2) \\ + ((r - q_1E_1)(s - m_2) - m_1s) = 0 \end{aligned}$$

As $(r - m_1 - q_1E_1) + (s - m_2) > 0$ (Assumptions)

So, all the eigenvalues of the above characteristic equation are not negative as there is at least one change of sign. Therefore the equilibrium point $P_0(0, 0, 0)$ is always unstable.

Hence, $P_0(0, 0, 0)$ is unstable. \square



Biological meaning: It is concluded that even if the system is exploited continuously in the unreserved zone, the prey or the predator population persist and are not extinct for sufficiently large time.

Theorem 4.2. For the system (2.1), if the interior equilibrium point $P_1(x_1^*, x_2^*, y^*)$ exist, is always locally asymptotically stable.

Proof. The characteristic equation of the variational matrix of the system (2.1) at P_1 is

$$\begin{vmatrix} r - \frac{2r}{K_1}x_1^* - m_1 - q_1E_1 - my^* & m_2 & -mx_1^* \\ m_1 & s - m_2 - my^* & -mx_2^* \\ \alpha_1y^* & \alpha_1y^* & -d + \alpha_1x_1^* + \alpha_1x_2^* - q_2E_2 \end{vmatrix} = 0$$

that is

$$\begin{vmatrix} -\frac{rx_1^*}{K_1} - \frac{m_2x_2^*}{x_1^*} - \lambda & m_2 & -mx_1^* \\ m_1 & -\frac{m_1x_1^*}{x_2^*} - \lambda & -mx_2^* \\ \alpha_1y^* & \alpha_1y^* & -\lambda \end{vmatrix} = 0$$

$$\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0 \tag{4.1}$$

where

$$\begin{aligned} C_1 &= \frac{rx_1^*}{K_1} + \frac{m_2x_2^*}{x_1^*} + \frac{m_1x_1^*}{x_2^*} > 0 \\ C_2 &= m\alpha_1y^*(x_2^* + x_1^*) + \left(\frac{rx_1^*}{K_1}\right) \left(\frac{m_1x_1^*}{x_2^*}\right) \\ C_3 &= \left(\frac{rx_1^*}{K_1} + \frac{m_2x_2^*}{x_1^*}\right) m\alpha_1x_2^*y^* \\ &\quad + m\alpha_1y^*(m_2x_2^* + m_1x_1^*) \\ &\quad + \frac{m\alpha_1m_1x_1^{*2}y^*}{x_2^*} > 0 \\ C_1C_2 - C_3 &= \frac{rm_1x_1^*}{K_2x_2^*} \left(\frac{rx_1^*}{K_1} + \frac{m_2x_2^*}{x_1^*} + \frac{m_1x_1^*}{x_2^*}\right) \\ &\quad + \frac{m\alpha_1ry^*x_2^{*2}}{K_1} > 0 \end{aligned}$$

As $C_1 > 0, C_3 > 0$ and $C_1C_2 - C_3 > 0$

Therefore by Ruth-Hurwitz criterion, P_1 always locally asymptotically stable. \square

5. Global stability

In this section, we consider the global stability of the system of equations (2.1) at interior equilibrium point $P_1(x_1^*, x_2^*, y^*)$ by constructing a suitable Lyapunov function mentioned as below:

$$\begin{aligned} V(x_1, x_2, y) &= \left(x_1 - x_1^* - x_1^* \log \frac{x_1}{x_1^*}\right) \\ &\quad + h_1 \left(x_2 - x_2^* - x_2^* \log \frac{x_2}{x_2^*}\right) \\ &\quad + h_2 \left(y - y^* - y^* \log \frac{y}{y^*}\right) \end{aligned}$$

$$\begin{aligned} \frac{dV}{dt} &= \frac{(x_1 - x_1^*)}{x_1} \frac{dx_1}{dt} \\ &\quad + h_1 \frac{(x_2 - x_2^*)}{x_2} \frac{dx_2}{dt} \\ &\quad + h_2 \frac{(y - y^*)}{y} \frac{dy}{dt} \end{aligned}$$

$$h_1 = \left(\frac{x_2^*}{x_2}\right) \left(\frac{m_2}{m_1}\right),$$

$$h_2 = \frac{m}{\alpha_1}$$

After simplification we get

$$\begin{aligned} \frac{dV}{dt} &= -(x_1 - x_1^*)^2 \frac{r}{K_1} - \frac{m_2(x_1^*x_2 - x_1x_2^*)^2}{x_1x_1^*x_2} \\ &\quad + m(y - y^*)(x_2 - x_2^*) \left[1 - \frac{m_2x_2^*}{m_1x_1^*}\right] \end{aligned}$$

Hence

$$\frac{dV}{dt} < 0$$

provided

$$(y - y^*)(x_2 - x_2^*) \left[1 - \frac{m_2x_2^*}{m_1x_1^*}\right] \leq 0 \tag{5.1}$$

So, V is negative definite, provided (5.1) holds.

Thus equilibrium point of the system of equations (2.1) is globally asymptotically stable for the aforesaid condition. Moreover if

$$m_1x_1^* = m_2x_2^*$$

i.e. if the number of prey migrated from unreserved area to reserve area are same that of the number of prey migrated from reserved area to unreserved area then the system will become globally stable.

6. Numerical simulations

In order to investigate the dynamics of the system (2.1) with help of numerical simulation, we choose a different set of parameters. Let

$$\begin{aligned} r &= 3, s = 2, m_1 = 2, m_2 = 1, m = 0.5, q_1 = 0.2 \\ q_2 &= 0.1, E_1 = 1.5, E_2 = 0.08, d = 1.5, K_1 = 10, \alpha_1 = 0.2. \end{aligned} \tag{6.1}$$



for this set of parameters, equilibrium point $P_1(2.4876, 5.0524, 3.9695)$ exist and locally stable as it satisfies existence condition (3.7) and stability condition of Theorem 4.2 (see Fig. 1).

Further taking another set of parameters

$$r = 3, s = 2, m_1 = 1, m_2 = 0.5, m = 0.5, q_1 = 0.2$$

$$q_2 = 0.1, E_1 = 1.5, E_2 = 0.08, d = 1.5, K_1 = 10, \alpha_1 = 0.2. \tag{6.2}$$

Here it is observed that as migration rate of the fishes from unreserved to reserved area and vice versa decreases, then population of prey in unreserved area decreases whereas population of prey in reserved area and predator increases that is depicted by $P_1(2.4685, 5.0715, 3.9734)$ at $m_1 = 1, m_2 = 0.5, P_1(1.8863, 5.6537, 4.0671)$ at $m_1 = 0.7, m_2 = 0.2$ (see Fig. 2).

Taking the same parameters as (6.2) except increasing the carrying capacity of prey in unreserved area we get required equilibrium point $P_1(2.9953, 4.5447, 4.3182)$ at $K_1 = 30$ and $P_1(3.1383, 4.4017, 4.4260)$ at $K_1 = 50$. Here it is noticed that density of prey in unreserved area and predator increases on the other hand density of prey in reserved area decreases (see Fig. 3).

For the same set of parameters values as in (6.2) except $E_1 = 0.5$ and $E_2 = 0.05$ then the equilibrium point $P_1(2.6337, 4.8913, 4.0770)$ exists and also become stable. On further reducing the harvesting efforts rate to $E_1 = 0.1$ and $E_2 = 0.05$. It is observed that density of prey in unreserved area as well as predator increases whereas density of prey in reserved area decreases (see Fig. 4).

The extensive simulation is done and it is investigated that only P_1 and P_0 exist and P_1 is locally as well as globally stable.

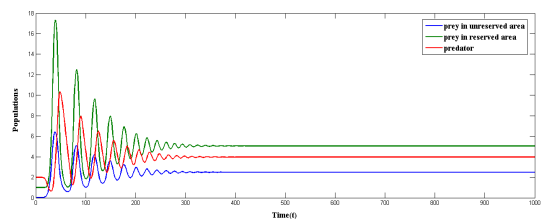


Figure 1(b). Time series plot of $x_1(t), x_2(t)$ and $y(t)$ of P_1 for data set (6.1)

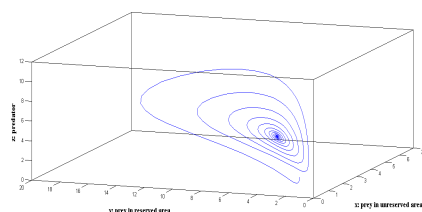


Figure 2. The phase diagram showing the local stability of P_1 for the data set (6.1) [$x(t)$: prey in unreserved area; $y(t)$: prey in reserved area; $z(t)$: predator]

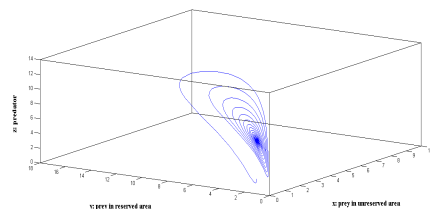


Figure 3. The phase diagram of $x_1(t), x_2(t)$ and $y(t)$ of P_1 for data set (6.1) except $m_1 = 1, m_2 = 0.5, K = 30$

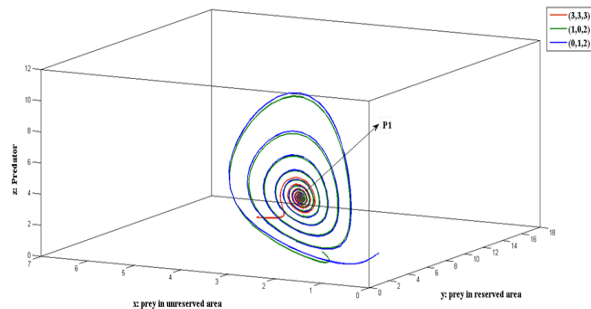


Figure 1(a). The phase diagram showing the global stability of P_1 for the data set (6.1) [$x_1(t)$: prey in unreserved area; $x_2(t)$: prey in reserved area; $y(t)$: predator]

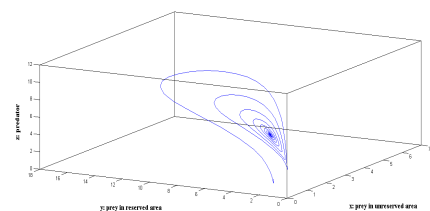


Figure 4. The phase diagram showing the local stability of P_1 for the data set (6.2) except $E_1 = 0.5, E_2 = 0.05$



7. Conclusion

A prey-predator fishery model in the presence of bird predator, with prey dispersal in a two-patch environment, has been proposed and investigated in the present paper. The harvesting is applied on prey (fishes) in an unreserved area as well as on the predator (bird). A threshold for existence, local along with the global stability at various equilibrium points has been inspected. It has been observed that global stability of interior equilibrium point P1 exists under certain conditions. From the numerical simulation, it has been verified that decrease of “ E_1 ” and “ E_2 ” (the harvesting efforts of prey in the unreserved zone and predator) cause a decrease in the population of the prey species in reserved area whereas increase in the biomass density of prey in unreserved area as well as predator. Further by numerical simulation, it is exposed that increase of carrying capacity of prey in an unreserved area is responsible for the decrease of the population of prey in reserved area whereas increase in the population of the prey species in unreserved area and predator. Moreover it is verified that as migration rate of the fishes from unreserved to reserved area and vice versa decreases, then population density of prey in unreserved area decreases whereas population density of prey in reserved area and predator increases.

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