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Translations of intuitionistic fuzzy soft structure of B-algebras

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Abstract

In this paper, the idea of intuitionistic fuzzy soft subalgebras, intuitionistic fuzzy soft ideals, and intuitionistic fuzzy soft a-ideals of B-algebra are introduced with many associated properties interrogated. In connection to the intuitionistic fuzzy soft set theory, translations, extensions, and multiplications of intuitionistic fuzzy soft subalgebras, intuitionistic fuzzy ideals, and intuitionistic fuzzy a-ideals are introduced. Interconnected with intuitionistic fuzzy soft translations, intuitionistic fuzzy soft extensions, and intuitionistic fuzzy soft multiplications of intuitionistic fuzzy soft subalgebras, intuitionistic fuzzy soft ideals, and intuitionistic fuzzy soft a-ideals are investigated.

Keywords

B-algebra, IF soft subalgebra, IF soft ideal, IF soft a-ideal, IF soft translation, IF soft extension, IF soft multiplication.

AMS Subject Classification

06F35, 03G25, 03E72, 08A72.

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Contents

1. Introduction

In mathematics, the fuzzy sets are sets whose elements have degrees of existence. Lotfi A. Zadeh [1] was introduced Fuzzy sets in 1965 as an extension of the classical set. The existence values lie between the interval [0, 1]. In the latter, the membership values lie between either 0 or 1. The fuzzy set theory can be used in a wide range of domains in which information is imprecise, such as Bio-informatics. The other fundamental research in the field of interest for Atanassov [2] is Fuzzy sets, defined by Lotfi Zadeh. In a significant manner, he extended the concept of Intuitionistic Fuzzy Sets. Also, the operations and relations over intuitionistic fuzzy sets, part of which has analogies in the theory of fuzzy sets. Without analogous in traditional, fuzzy set theory is the operators on modal, topological, level. Neggers et al. [3] introduced several classes of algebras of interest such as BCH/BCI/BCK-algebras and some properties are investigated. Jun et al. [4] developed the fuzzification of normal B-subalgebras are defined, and properties are investigated.

Ahn et al. [5] classify the subalgebras by their family of level subalgebras in B-algebras. Senapati et al. [6] introduced the notions of fuzzy dot subalgebras, fuzzy normal dot subalgebras, and fuzzy dot ideals of B-algebras are introduced. They introduced the notion of fuzzy relations on the family of fuzzy dot subalgebras and fuzzy dot ideals of B-algebras. Hashemi [7] introduced the concepts of fuzzy translation to fuzzy associative ideals in BCK/BCI-algebras. The interrelated to fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy ideals are investigated. Senapati et al. [8] introduced the notion of fuzzy translation to fuzzy H-ideals in BCK/BCI-algebras. Senapati et al. [9] introduced the concepts of intuitionistic fuzzy translation to intuitionistic fuzzy H-ideals in BCK/BCI-algebras. Lee et al.

[10] discussed fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy subalgebras in BCI/BCK-algebras. Related to fuzzy translations, fuzzy extensions, and fuzzy multiplications are investigated. Kim et al. [11] defined the intuitionistic fuzzy subalgebras of B-algebras which are related to several classes of algebras such as BCI/BCK-algebras.

The soft set theory was proposed by Molodtsov [12] in 1999. Maji et al. [13-15] introduced the notion of intuitionistic fuzzy soft set and application of soft sets in a decision-making problem. Jun [16] introduced the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras. The notion of soft BCK/BCI-algebras was introduced by Jun et al. [17]. They proposed a fuzzy soft set of several kinds of theories in BCK/BCI-algebras. Also, the notions of fuzzy soft BCK/BCI-algebras, fuzzy soft ideals, and fuzzy soft p-ideals are introduced, and related properties are investigated. Senapati [18] introduced translations of intuitionistic fuzzy B-algebras. Motivated by this, we introduce the notion of translations of intuitionistic fuzzy soft structure of B-algebra and establish some of their basic properties. In this paper, IF soft translations (IFSTs), IF soft extensions (IFSEs), IF soft multiplications (IFSMs) of IF soft subalgebras (IFSSUs), IF soft ideals (IFSIDs), and IF soft a-ideals (IFSAIDs) of B-algebras are introduced.

2. Preliminaries

In this section, some basic aspects which are used to present the paper.

Definition 2.1. *A non-empty set X with a constant 0 and a binary operation $*$ is called a B-algebra if for every x, y, z $\in$ X satisfies the following axioms*
(B1) $x * x = 0$
(B2) $x * 0 = x$
(B3) $(x*y)*z = x*(z*(0*y))$.
For concise, we also call X a B-algebra.

Example 2.2 Let $X = \{0,1,2\}$ with the Cayley table given below:

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then X is a B-algebra.

Definition 2.3. *A non-empty subset S in X is called a subalgebra of X if $x * y \in S$ for any x, y $\in$ S.*

Definition 2.4. *A non-empty subset A in X is called an ideal of X if it satisfies (i) $0 \in A$ and (ii) $x * y \in A$ and $y \in A$ imply $x \in A$.*

Definition 2.5. *A non-empty subset A in X is called an a-ideal of X if it satisfies (i) and (iii) $(x * z) * (0 * y) \in A$ and $z \in A$ imply $y * x \in A$.*

Definition 2.6. *A pair (M,A) is called soft set over X if and only if M is a mapping into the set of all subsets of the set X.*

Definition 2.7. *A fuzzy set A of X is of the form $A = \{(x, \sigma_A(x)) : x \in X\}$, where $\sigma_A(x) : X \to [0,1]$ is called the degree of existence of the element x in the set A and $0 \le \sigma_A(x) \le 1$.*

Definition 2.8. *Let X is primary set under consideration and let E act as a set factors. Let $\mathscr{F}(X)$ represent the set of all fuzzy sets in X. Then $(\tilde{M}, A)$ is called a fuzzy soft set over X and $A \subseteq E$, where $\tilde{M} : A \to \mathscr{F}(X)$.*

Definition 2.9. *A fuzzy soft set $\tilde{M}[\delta]$ in X is called a fuzzy soft subalgebra of X if for every $x, y \in X$ and $\delta \in A$ satisfies*
(FSSU1) $\sigma_{\tilde{M}[\delta]}(0) \ge \sigma_{\tilde{M}[\delta]}(x)$ *and*
(FSSU2) $\sigma_{\tilde{M}[\delta]}(x*y) \ge \sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{M}[\delta]}(y)$.

Definition 2.10. *A fuzzy soft set $\tilde{M}[\delta]$ in X is called a fuzzy soft ideal of X if for every $x, y \in X$ and $\delta \in A$ satisfies*
(FSID1) $\sigma_{\tilde{M}[\delta]}(0) \ge \sigma_{\tilde{M}[\delta]}(x)$ *and*
(FSID2) $\sigma_{\tilde{M}[\delta]}(x) \ge \sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]}(y)$.

Definition 2.11. *A fuzzy soft set $\tilde{M}[\delta]$ in X is called a fuzzy soft a-ideal of X if for every $x, y, z \in X$ and $\delta \in A$ satisfies*
(FSAID1) $\sigma_{\tilde{M}[\delta]}(0) \ge \sigma_{\tilde{M}[\delta]}(x)$ *and*
(FSAID2) $\sigma_{\tilde{M}[\delta]}(y*x) \ge \sigma_{\tilde{M}[\delta]}((x*z)*(0*y)) \wedge \sigma_{\tilde{M}[\delta]}(z)$.

Definition 2.12. *Let $\tilde{M}[\delta]$ be a fuzzy soft subset of X and let $\alpha \in [0, 1 - sup\{\sigma_{\tilde{M}[\delta]}(x) : x \in X\}]$. A mapping $(\sigma_{\tilde{M}[\delta]})^T_\alpha : X \to [0,1]$ is called fuzzy soft α translation of $\tilde{M}[\delta]$ if it satisfies $(\sigma_{\tilde{M}[\delta]})^T_\alpha(x) = \sigma_{\tilde{M}[\delta]}(x) + \alpha$ for every $x \in X$ and $\delta \in$A.*

Definition 2.13. *An intuitionistic fuzzy set (IFS) A is of the form $A = \{(x, \sigma_A(x), \tau_A(x)) : x \in X\}$, where $\sigma_A(x) : X \to [0,1]$, $\tau_A(x) : X \to [0,1]$ with the condition $0 \le \sigma_A(x) + \tau_A(x) \le 1$ for all $x \in X$. The number $\sigma_A(x), \tau_A(x)$ means the degree of existence and non-existence of the element x in the set A.*

Definition 2.14. *Let X is a primary set under consideration and let E act as a set of factors. Let $\mathscr{I}\mathscr{F}(X)$ represent the set of all intuitionistic fuzzy set X. Then $(\tilde{M}, A)$ is called an intuitionistic fuzzy soft set over X and $A \subseteq E$, where $\tilde{M} : A \to \mathscr{I}\mathscr{F}(X)$.*

Definition 2.15. *Let $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X$ and $\delta \in A\}$ and $\tilde{N}[\delta] = \{(x, \sigma_{\tilde{N}[\delta]}(x), \tau_{\tilde{N}[\delta]}(x)) : x \in X$ and $\delta \in A\}$ be two intuitionistic fuzzy soft set on X. Then the intersetion and union of $\tilde{M}[\delta]$ and $\tilde{N}[\delta]$ are denoted by $\tilde{M}[\delta] \cap \tilde{N}[\delta]$ and $\tilde{M}[\delta] \cup \tilde{N}[\delta]$ respectively and is given by*

$$\tilde{M}[\delta] \cap \tilde{N}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{N}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{N}[\delta]}(x)) : x \in X \text{ and } \delta \in A\},$$

$$\tilde{M}[\delta] \cup \tilde{N}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x) \vee \sigma_{\tilde{N}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{N}[\delta]}(x)) : x \in X \text{ and } \delta \in A\}.$$

Definition 2.16. *An intuitionistic fuzzy soft set $\tilde{M}[\delta]$ in X is called an intuitionistic fuzzy soft set subalgebra of X if for every $x, y \in X$ and $\delta \in A$ satisfies*

(IFSSU1) $\sigma_{\tilde{M}[\delta]}(0) \geq \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(0) \leq \tau_{\tilde{M}[\delta]}(x)$,
(IFSSU2) $\sigma_{\tilde{M}[\delta]}(x*y) \geq \sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{M}[\delta]}(y)$ and
(IFSSU3) $\tau_{\tilde{M}[\delta]}(x*y) \leq \tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{M}[\delta]}(y)$.

Definition 2.17. *An intuitionistic fuzzy soft set $\tilde{M}[\delta]$ in X is called an intuitionistic fuzzy soft ideal of X if for every $x, y \in X$ and $\delta \in A$ satisfies*

(IFSSU1) $\sigma_{\tilde{M}[\delta]}(0) \geq \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(0) \leq \tau_{\tilde{M}[\delta]}(x)$,
(IFSSU2) $\sigma_{\tilde{M}[\delta]}(x) \geq \sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]}(y)$ and
(IFSSU3) $\tau_{\tilde{M}[\delta]}(x) \leq \tau_{\tilde{M}[\delta]}(x*y) \vee \tau_{\tilde{M}[\delta]}(y)$.

Definition 2.18. *An intuitionistic fuzzy soft set $\tilde{M}[\delta]$ in X is called an intuitionistic fuzzy soft a-ideal of X if for every $x, y, z \in X$ and $\delta \in A$ satisfies*

(IFSAID1) $\sigma_{\tilde{M}[\delta]}(0) \geq \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(0) \leq \tau_{\tilde{M}[\delta]}(x)$,
(IFSAID2) $\sigma_{\tilde{M}[\delta]}(y*x) \geq \sigma_{\tilde{M}[\delta]}((x*z)*(0*y)) \wedge \sigma_{\tilde{M}[\delta]}(z)$ and
(IFSAID3) $\tau_{\tilde{M}[\delta]}(y*x) \leq \tau_{\tilde{M}[\delta]}((x*z)*(0*y)) \vee \tau_{\tilde{M}[\delta]}(z)$.

3. Translations of intuitionistic fuzzy soft subalgebras

For shortness, we shall use the notation $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ for the IFSS $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X$ and $\delta \in A\}$. During the whole of this paper, we take $\Im = inf\{\tau_{\tilde{M}[\delta]}(x) : x \in X$ and $\delta \in A\}$ for every IFSS $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ of X.

Definition 3.1. *Let $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ be an IFSS of X and $\alpha \in [0, \Im]$. An object having the form $\tilde{M}[\delta]^T_\alpha = ((\sigma_{\tilde{M}[\delta]})^T_\alpha, (\tau_{\tilde{M}[\delta]})^T_\alpha)$ is called an intuitionistic fuzzy soft α-translation (IFSAT) of $\tilde{M}[\delta]$ if $(\sigma_{\tilde{M}[\delta]})^T_\alpha(x) = (\sigma_{\tilde{M}[\delta]})(x) + \alpha$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(x) = (\sigma_{\tilde{M}[\delta]})(x) - \alpha$ for every $x, y \in X, \delta \in A$.*

Theorem 3.2. *Let $\tilde{M}[\delta]$ be an IFSSU of X and let $\alpha \in [0, \Im]$. Then the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSSU of X.*

Proof. Let $x, y \in X$ and $\delta \in A$. Then $(\sigma_{\tilde{M}[\delta]})^T_\alpha(x*y) = (\sigma_{\tilde{M}[\delta]})(x*y) + \alpha \geq \big((\sigma_{\tilde{M}[\delta]})(x) \wedge \sigma_{\tilde{M}[\delta]}(y)\big) + \alpha = (\sigma_{\tilde{M}[\delta]}(x) + \alpha) \wedge (\sigma_{\tilde{M}[\delta]}(y) + \alpha) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(x) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(y)$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(x*y) = (\tau_{\tilde{M}[\delta]})(x*y) - \alpha \leq \big((\tau_{\tilde{M}[\delta]})(x) \vee \tau_{\tilde{M}[\delta]}(y)\big) - \alpha = (\tau_{\tilde{M}[\delta]}(x) - \alpha) \vee (\tau_{\tilde{M}[\delta]}(y) - \alpha) = (\tau_{\tilde{M}[\delta]})^T_\alpha(x) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(y)$. Hence, the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSSU of X. □

Theorem 3.3 *Let $\tilde{M}[\delta]$ be an IFSS of X such that IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSSU of X for a particular $\alpha \in [0, \Im]$. Then $\tilde{M}[\delta]$ is an IFSSU of X.*

Proof. Assume that $\tilde{M}[\delta]^T_\alpha = \{(x, (\sigma_{\tilde{M}[\delta]})^T_\alpha(x), (\tau_{\tilde{M}[\delta]})^T_\alpha(x)) : x \in X$ and $\delta \in A\}$ is an IFSSU of X for a particular $\alpha \in [0, \Im]$. Let $x, y \in X$ and $\delta \in A$. Then $\sigma_{\tilde{M}[\delta]}(x*y) + \alpha = (\sigma_{\tilde{M}[\delta]})^T_\alpha(x*y)) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha(x)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(y)) = (\sigma_{\tilde{M}[\delta]}(x) + \alpha) \wedge (\sigma_{\tilde{M}[\delta]}(y) + \alpha) = (\sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{M}[\delta]}(y)) + \alpha$ and $\tau_{\tilde{M}[\delta]}(x*y) - \alpha = (\tau_{\tilde{M}[\delta]})^T_\alpha(x*y)) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha(x)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(y)) = (\tau_{\tilde{M}[\delta]}(x) - \alpha) \wedge (\tau_{\tilde{M}[\delta]}(y) - \alpha) = (\tau_{\tilde{M}[\delta]}(x) \wedge \tau_{\tilde{M}[\delta]}(y)) - \alpha$ which implies that $\sigma_{\tilde{M}[\delta]}(x*y) \geq \sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{M}[\delta]}(y)$ and $\tau_{\tilde{M}[\delta]}(x*y) \leq \tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{M}[\delta]}(y)$ for every $x, y \in X$ and $\delta \in A$. Hence, $\tilde{M}[\delta]$ is an IFSSU of X. □

Definition 3.4. *Let $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ and $\tilde{N}[\delta] = (\sigma_{\tilde{N}[\delta]}, \tau_{\tilde{N}[\delta]})$ be IFSSs of X. If $\tilde{M}[\delta] \leq \tilde{N}[\delta]$ i.e, $\sigma_{\tilde{M}[\delta]}(x) \leq \sigma_{\tilde{N}[\delta]}(x)$ and $\tau_{\tilde{M}[\delta]}(x) \geq \tau_{\tilde{N}[\delta]}(x)$ for every $x \in X$ and $\delta \in A$, $\tilde{N}[\delta]$ is an intuitionistic fuzzy soft extension (IFSE) of $\tilde{M}[\delta]$.*

Definition 3.5. *Let $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ and $\tilde{N}[\delta] = (\sigma_{\tilde{N}[\delta]}, \tau_{\tilde{N}[\delta]})$ be IFSSs of X. Then $\tilde{N}[\delta]$ is called an intuitionistic fuzzy soft S-extension (IFSSE) of $\tilde{M}[\delta]$ if the following assertions are valid:*

(i) $\tilde{N}[\delta]$ is an IFSE of $\tilde{M}[\delta]$
(ii) if $\tilde{M}[\delta]$ is an IFSSU of X, then $\tilde{N}[\delta]$ is an IFSSU of X.

From the definition of IFSAT, we get $(\sigma_{\tilde{M}[\delta]})^T_\alpha(x) = (\sigma_{\tilde{M}[\delta]})(x) + \alpha$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(x)) = (\tau_{\tilde{M}[\delta]})(x) - \alpha$ for every $x \in X$ and $\delta \in A$.

Theorem 3.6. *Let $\tilde{M}[\delta]$ be an IFSS X and $\alpha \in [0, \Im]$. Then the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSSE of $\tilde{M}[\delta]$.*

The reverse of the Theorem 3.6 is not correct by the following example.

Example 3.7. Consider a B-algebra X=$\{0, 1, 2, 3, 4, 5\}$ with the Cayley table:

$*$	0	1	2	3	4	5
0	0	5	4	3	2	1
1	1	0	5	4	3	2
2	2	1	0	5	4	3
3	3	2	1	0	5	4
4	4	3	2	1	0	5
5	5	4	3	2	1	0

Let $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ be an IFSS in X by

X	0	1	2
$\tilde{M}[\delta]$	[0.5,0.3]	[0.2,0.4]	[0.5,0.3]

X	3	4	5
$\tilde{M}[\delta]$	[0.2,0.4]	[0.5,0.3]	[0.2,0.4]

Then $\tilde{M}[\delta]$ is an IFSSU of X. Let $\tilde{N}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ be an IFSS in X by

X	0	1	2
$\tilde{N}[\delta]$	[0.65,0.22]	[0.33,0.36]	[0.65,0.22]

X	3	4	5
$\tilde{N}[\delta]$	[0.33,0.36]	[0.65,0.22]	[0.33,0.36]

Then $\tilde{N}[\delta]$ is an IFSSE of $\tilde{M}[\delta]$. But it is not IFSAT $(\tilde{M}[\delta])^T_\alpha$ of $\tilde{M}[\delta]$ for any $\alpha \in [0,\Im]$ and $\delta \in A$.

Clearly, the intersection of IFSSEs of an IFSSU $\tilde{M}[\delta]$ of an IFSSE of $\tilde{M}[\delta]$. But the union of IFSSEs of an IFSSU $\tilde{M}[\delta]$ of X is not an IFSSE of $\tilde{M}[\delta]$ by the following example.

Example 3.8. Consider a B-algebra X = $\{0,1,2,3,4,5\}$ with the cayley table:

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

Let $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ be an IFSS in X by

X	0	1	2
$\tilde{M}[\delta]$	[0.6,0.2]	[0.4,0.3]	[0.4,0.3]

X	3	4	5
$\tilde{M}[\delta]$	[0.4,0.3]	[0.4,0.3]	[0.4,0.3]

Then $\tilde{M}[\delta]$ is an IFSSU of X. Let $\tilde{N}[\delta]$ and $\tilde{O}[\delta]$ be two IFSS in X by

X	0	1	2
$\tilde{N}[\delta]$	[0.7,0.1]	[0.5,0.2]	[0.5,0.2]

X	3	4	5
$\tilde{N}[\delta]$	[0.7,0.1]	[0.5,0.2]	[0.5,0.2]

and

X	0	1	2
$\tilde{O}[\delta]$	[0.8,0.2]	[0.6,0.3]	[0.6,0.3]

X	3	4	5
$\tilde{O}[\delta]$	[0.6,0.3]	[0.8,0.2]	[0.6,0.3]

respectively. Then $\tilde{N}[\delta]$ and $\tilde{O}[\delta]$ are IFFSSs of $\tilde{M}[\delta]$. Obviously, the union $\tilde{N}[\delta] \cup \tilde{O}[\delta]$ is an IFSE of $\tilde{M}[\delta]$, but it is not an IFSSE of $\tilde{M}[\delta]$ since

$$\sigma_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(4*3) \geq \sigma_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(4) \wedge \sigma_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(3)$$
$$\sigma_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(1) \geq 0.8 \wedge 0.7$$
$$0.6 \ngeq 0.7.$$

For an IFSS $\tilde{M}[\delta] = \{(x, \sigma_{\tilde{M}[\delta]}(x), \tau_{\tilde{M}[\delta]}(x)) : x \in X$ and $\delta \in A\}$ of X, $\alpha \in [0,\Im]$ and $t,s \in [0,1]$ with $t \geq \alpha$, let $U_\alpha\left(\sigma_{M[\delta]};t\right) = \{x : \sigma_{\tilde{M}[\delta]}(x) \geq t-\alpha\}$ and $L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right) = \{x : \tau_{\tilde{M}[\delta]}(x) \leq s+\alpha\}$. If $\tilde{M}[\delta]$ is an IFSSU of X, $U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$, $L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$ are subalgebras of X for every $t \in Im(\sigma_{\tilde{M}[\delta]})$, $s \in Im(\tau_{\tilde{M}[\delta]})$ with $t \geq \alpha$. But, if $\tilde{M}[\delta]$ is not an IFSSU of X, then $U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$ are not subalgebras of X by the following example.

Example 3.9. Consider a B-algebra X = $\{0,1,2,3,4,5\}$ in Example 3.8 and $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ be an IFSS in X by

X	0	1	2
$\tilde{M}[\delta]$	[0.66,0.26]	[0.43,0.52]	[0.43,0.52]

X	3	4	5
$\tilde{M}[\delta]$	[0.54,0.29]	[0.54,0.29]	[0.54,0.29]

since $\sigma_{\tilde{M}[\delta]}(4*3) \geq \sigma_{\tilde{M}[\delta]}(4) \wedge \sigma_{\tilde{M}[\delta]}(3)$
$$\sigma_{\tilde{M}[\delta]}(1) \geq 0.54 \wedge 0.54$$
$$0.43 \ngeq 0.54$$
and $\tau_{\tilde{M}[\delta]}(5*4) \leq \tau_{\tilde{M}[\delta]}(5) \wedge \tau_{\tilde{M}[\delta]}(4)$
$$\tau_{\tilde{M}[\delta]}(1) \leq 0.54 \wedge 0.54$$
$$0.52 \nleq 0.29.$$
Therefore, $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ is not an IFSSU of X.

For $\alpha = 0.16$, $t = 0.70$, $U_\alpha\left(\tau_{\tilde{M}[\delta]};t\right) = \{0,3,4,5\}$ is not a subalgebra of X since $3*4 = 2 \notin U_\alpha\left(\tau_{M[\delta]};t\right)$.

For $\alpha = 0.16$ and s = 0.25, we obtain $L_\alpha(\tau_{\hat{M}[\delta]};s) = \{0,3,4,5\}$ which is not a subalgebra of X since $5*4 = 1 \notin L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$.

Theorem 3.10. *For $\alpha \in [0,\Im]$, let $\tilde{M}[\delta]^T_\alpha = ((\sigma_{\tilde{M}[\delta]})^T_\alpha, (\tau_{\tilde{M}[\delta]})^T_\alpha)$ be the IFSAT of $\tilde{M}[\delta]$. Then the following are equivalent:*
(i) $\tilde{M}[\delta]^T_\alpha = ((\sigma_{\tilde{M}[\delta]})^T_\alpha, (\tau_{\tilde{M}[\delta]})^T_\alpha)$ is an IFSSU of X.
(ii) $U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$ are subalgebra of X for $t \in Im(\sigma_{\tilde{M}[\delta]})$,$s \in Im(\tau_{\tilde{M}[\delta]})$ with $t \geq \alpha$.

Proof. Assume that $\tilde{M}[\delta]^T_\alpha$ is an IFSSU of X. Then $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha$ and $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha$ are fuzzy soft subalgebra of X. Let $x,y \in X$ and $\alpha \in A$ such that $x,y \in U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $t \in Im(\sigma_{\tilde{M}[\delta]})$ with $t \geq \alpha$. Then $\sigma_{\tilde{M}[\delta]}(x) \geq t-\alpha$ and $\sigma_{\tilde{M}[\delta]}(y) \geq t-\alpha$, i.e., $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) = \left(\sigma_{\tilde{M}[\delta]}(x)\right)+\alpha \geq t$ and $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(y) = \left(\sigma_{\tilde{M}[\delta]}(y)\right)+\alpha \geq t$. Since $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha$ is a fuzzy soft subalgebra of X, $\sigma_{\tilde{M}[\delta]}(x*y)+\alpha = \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(y) \geq t$,that is, $\sigma_{\tilde{M}[\delta]}(x*y) \geq t-\alpha$ so that $x*y \in U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$.

Again, let x,y $\in X$ and $\delta \in A$ be such that $x,y \in L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$ and $s \in Im(\tau_{\tilde{M}[\delta]})$. Then $\left(\tau_{\tilde{M}[\delta]}(x)\right) \leq s+\alpha$ and $\left(\tau_{\tilde{M}[\delta]}(y)\right) \leq s+\alpha$, i.e., $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) = \left(\tau_{\tilde{M}[\delta]}\right)(x)-\alpha \leq s$ and $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(y) = \left(\tau_{\tilde{M}[\delta]}\right)(y)-\alpha \leq s$. Since $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha$ is a fuzzy soft subalgebra of X, $\tau_{\tilde{M}[\delta]}(x*y)-\alpha = \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \leq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) \vee \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(y) \leq s$, that is, $\tau_{\tilde{M}[\delta]}(x*y) \leq s+\alpha$ so that $x*y \in L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$.Therefore, $U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$ are sub-algebras of X.

Conversely, suppose that $U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$ are sub-algebras of X for $t\in Im\left(\sigma_{\tilde{M}[\delta]}\right)$, $s\in Im\left(\tau_{\tilde{M}[\delta]}\right)$ with $t\geq\alpha$. If there exists $a,b\in X$ such that $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(a*b)<\beta\leq\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(a)\wedge\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(b)$, then $\sigma_{\tilde{M}[\delta]}(a)\geq\beta-\alpha$ and $\sigma_{\tilde{M}[\delta]}(b)\geq\beta-\alpha$, but $\sigma_{\tilde{M}[\delta]}(a*b)<\beta-\alpha$. This shows that $a\in U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $b\in U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$, but $a*b\notin U_\alpha\left(\sigma_{\tilde{M}[\delta]};t\right)$. This is a contradiction, therefore $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x*y)\geq\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x)\wedge\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(y)$ for every $x,y\in X$ and $\delta\in A$.

Again, assume that there exist $c,d\in X$ such that $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(c*d)>\gamma\geq\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(c)\vee\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(d)$. Then $\tau_{\tilde{M}[\delta]}(c)\leq\gamma+\alpha$ and $\tau_{\tilde{M}[\delta]}(d)\leq\gamma+\alpha$ but $\tau_{\tilde{M}[\delta]}(c*d)>\gamma+\alpha$. Hence, $c\in L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$ and $d\in L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$, but $c*d\notin L_\alpha\left(\tau_{\tilde{M}[\delta]};s\right)$. This is not possible and therefore, $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x*y)\leq\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x)\vee\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(y)$ for every $x,y\in X$ and $\delta\in A$. Consequently, $\tilde{M}[\delta]^T_\alpha=\left(\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha,\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha\right)$ is an IFSSU of X. □

Theorem 3.11. *Let $\tilde{M}[\delta]=\left(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]}\right)$ be an IFSSU of X and $\alpha,\beta\in[0,\Im]$. If $\alpha\geq\beta$, IFSAT $\tilde{M}[\delta]^T_\alpha=\left(\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha,\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha\right)$ of $\tilde{M}[\delta]$ is an IFSSE of the intuitionistic fuzzy soft β-translation (IFSBT) $\tilde{M}[\delta]^T_\beta=\left(\left(\sigma_{\tilde{M}[\delta]}\right)^T_\beta,\left(\tau_{\tilde{M}[\delta]}\right)^T_\beta\right)$ of $\tilde{M}[\delta]$.*

Proof. Straightforward. □

For every IFSSU $\tilde{M}[\delta]$ of X and $\beta\in[0,\Im]$, the IFSBT $\left(\tilde{M}[\delta]\right)^T_\beta$ of $\tilde{M}[\delta]$ is an IFSSU of X. If $\tilde{N}[\delta]$ is an IFSSE of $\left(\tilde{M}[\delta]\right)^T_\beta$, then there exists $\alpha\in[0,\Im]$ such that $\alpha\geq\beta$ and $\tilde{N}[\delta]\geq\left(\tilde{M}[\delta]\right)^T_\alpha$, that is $\sigma_{\tilde{N}[\delta]}(x)\geq\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha$ and $\tau_{\tilde{N}[\delta]}(x)\leq\left(\tau_{\tilde{N}[\delta]}\right)^T_\alpha$ for every $x,y\in X$ and $\delta\in A$.

Theorem 3.12. *Let $\tilde{M}[\delta]$ be an IFSSU of X and let $\beta\in[0,\Im]$. For every IFSSE $\tilde{N}[\delta]=\left(\sigma_{\tilde{N}[\delta]},\tau_{\tilde{N}[\delta]}\right)$ of the IFSBT $\tilde{M}[\delta]^T_\beta$ of $\tilde{M}[\delta]$, there exists $\alpha\in[0,\Im]$ such that $\alpha\geq\beta$ imply $\tilde{N}[\delta]$ is an IFSSE of the IFSAT $\tilde{M}[\delta]^T_\beta$ of $\tilde{M}[\delta]$.*

Let us clarify the Theorem 3.12 by the following example.

Example 3.13. Consider a B-algebra $X=\{0,1,2,3,4,5\}$ and $\tilde{M}[\delta]=\left(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]}\right)$ be an IFSS of X defined in Example 3.7. Then $\Im=0.4$. If $\beta=0.23$, then the IFSBT $\tilde{M}[\delta]^T_\beta$ of $\tilde{M}[\delta]$ is given by

X	0	1	2
$\tilde{M}[\delta]^T_\alpha$	[0.81, 0.15]	[0.51, 0.25]	[0.81, 0.15]

X	3	4	5
$\tilde{M}[\delta]^T_\alpha$	[0.51, 0.25]	[0.81, 0.15]	[0.51, 0.25]

Let $\tilde{N}[\delta]=\left(\sigma_{\tilde{N}[\delta]},\tau_{\tilde{N}[\delta]}\right)$ be an IFSS in X by

X	0	1	2
$\tilde{N}[\delta]$	[0.85, 0.10]	[0.57, 0.17]	[0.85, 0.10]

Then $\tilde{N}[\delta]$ is clearly an IFSSU of X which is an IFSSE of the IFSBT $\tilde{M}[\delta]^T_\beta$ of $\tilde{M}[\delta]$. But $\tilde{N}[\delta]$ is not an IFSAT of $\tilde{M}[\delta]$ for a particular $\alpha\in[0,\Im]$. If $\alpha=0.26>0.23=\beta$ and the IFAT $\tilde{M}[\delta]^T_\alpha=\left(\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha,\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha\right)$ of $\tilde{M}[\delta]$ is given as follows:

Note that $\tilde{N}[\delta](x)\geq\left(\tilde{M}[\delta]\right)^T_\alpha(x)$, that is $\sigma_{\tilde{N}[\delta]}(x)\geq\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x)$ and $\tau_{\tilde{N}[\delta]}(x)\leq\left(\tau_{\tilde{N}[\delta]}\right)^T_\alpha(x)$ for every $x\in X$ and $\delta\in A$, and hence, $\tilde{N}[\delta]$ is an IFSSE of the IFSAT $\tilde{M}[\delta]^T_\delta of\tilde{M}[\delta]$.

Definition 3.14. *Let $\tilde{M}[\delta]$ be an IFSS of X and $\rho\in[0,1]$. An object having the form $\tilde{M}[\delta]^m_\rho=\left(\left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho,\left(\tau_{\tilde{M}[\delta]}\right)^m_\rho\right)$ is called an intuitionistic fuzzy soft ρ-multiplication of $\tilde{M}[\delta]$ if $\left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho(x)=\sigma_{\tilde{M}[\delta]}(x)\bullet\rho$ and $\left(\tau_{\tilde{M}[\delta]}\right)^m_\rho(x)=\tau_{\tilde{M}[\delta]}(x)\bullet\rho$ for every $x\in X$ and $\delta\in A$.*

For any IFSS $\tilde{M}[\delta]=\left(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]}\right)$ of X, an intuitionistic fuzzy soft 0-multiplication $\tilde{M}[\delta]^m_0=\left(\left(\sigma_{\tilde{M}[\delta]}\right)^m_0,\left(\tau_{\tilde{M}[\delta]}\right)^m_0\right)$ of $\tilde{M}[\delta]$ is an IFSSU of X.

Theorem 3.15. *If $\tilde{M}[\delta]=\left(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]}\right)$ is an IFSSU of X, then the intutionistic fuzzy soft ρ-multiplication of $\tilde{M}[\delta]$ is an IFSSU of X for any $\rho\in[0,1]$.*

Proof. Straightforward. □

Theorem 3.16. *If $\tilde{M}[\delta]=\left(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]}\right)$ is any IFSS of X, then the following are equivalent :*

(i) $\tilde{M}[\delta]$ is an IFSSU of X.

(ii) for every $\rho\in[0,1]$, $\tilde{M}[\delta]^m_\rho$ is an IFSSU of X.

Proof Necessary follows from Theorem 3.15. For sufficient part let $\rho\in[0,1]$ such that $\tilde{M}[\delta]^m_\rho$ is an IFSSU of X. Then for every $x,y\in X$ and $\delta\in A$, $\sigma_{\tilde{M}[\delta]}(x*y)\bullet\rho=\left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho(x*y)\geq\left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho(x)\wedge\left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho(y)=\left(\sigma_{\tilde{M}[\delta]}(x)\bullet\rho\right)\wedge\left(\sigma_{\tilde{M}[\delta]}(y)\bullet\rho\right)$

X	3	4	5
$\tilde{N}[\delta]$	[0.57, 0.17]	[0.85, 0.10]	[0.57, 0.17]

X	0	1	2
$\tilde{M}[\delta]^T_\alpha$	[0.84, 0.12]	[0.54, 0.22]	[0.84, 0.12]

X	3	4	5
$\tilde{M}[\delta]^T_\alpha$	[0.54, 0.22]	[0.84, 0.12]	[0.54, 0.22]

$= \left(\sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{M}[\delta]}(y)\right) \bullet \rho$ and $\tau_{\tilde{M}[\delta]}(x*y) \bullet \rho = \left(\tau_{\tilde{M}[\delta]}\right)^m_\rho (x*y) \geq \left(\tau_{\tilde{M}[\delta]}\right)^m_\rho (x) \vee \left(\tau_{\tilde{M}[\delta]}\right)^m_\rho (y) = \left(\tau_{\tilde{M}[\delta]}(x) \bullet \rho\right) \vee \left(\tau_{\tilde{M}[\delta]}(y) \bullet \rho\right) = \left(\tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{M}[\delta]}(y)\right) \bullet \rho$. Therefore, $\sigma_{\tilde{M}[\delta]}(x*y) \geq \sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{M}[\delta]}(y)$ and $\tau_{\tilde{M}[\delta]}(x*y) \leq \tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{M}[\delta]}(y)$ for every $x, y \in X$ and $\delta \in A$ since $\rho \neq 0$. Hence, $\tilde{M}[\delta]$ is an IFSSU of X.

4. Translations of intuitionistic fuzzzy soft ideals

In this section, translations of intuitionistic fuzzy soft ideals are defined with few results studied.

Theorem 4.1. *If $\tilde{M}[\delta] = \left(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}\right)$ is an IFSIDs of X, then IFSAT $\tilde{M}[\delta]^T_\alpha = \left(\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha, \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha\right)$ of $\tilde{M}[\delta]$ is IFSIDs of X for a particular $\alpha \in [0, \Im]$.*

Proof. Let $\tilde{M}[\delta] = \left(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}\right)$ is an IFSIDs of X and $\alpha \in [0, \Im]$. Then $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (0) = \sigma_{\tilde{M}[\delta]}(0) + \alpha \geq \alpha_{\tilde{M}[\delta]}(x) + \alpha$ and $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (0) = \tau_{\tilde{M}[\delta]}(0) - \alpha \leq \tau_{\tilde{M}[\delta]}(x) - \alpha$ for every $x, y \in X$ *and* $\delta \in A$. Now, $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x) = \sigma_{\tilde{M}[\delta]}(x) + \alpha \geq \left(\sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]}(y)\right) + \alpha = \left(\sigma_{\tilde{M}[\delta]}(x*y) + \alpha\right) \wedge \left(\sigma_{\tilde{M}[\delta]}(y) + \alpha\right) = \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (y)$ and $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (x) = \tau_{\tilde{M}[\delta]}(x) - \alpha \leq \left(\tau_{\tilde{M}[\delta]}(x*y) \vee \tau_{\tilde{M}[\delta]}(y)\right) - \alpha = \left(\tau_{\tilde{M}[\delta]}(x*y) - \alpha\right) \vee \left(\tau_{\tilde{M}[\delta]}(y) - \alpha\right) = \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (x*y) \vee \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (y)$ for every $x, y \in X$ and $\delta \in A$. Hence, the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSID of X.

Theorem 4.2. *Let $\tilde{M}[\delta]$ be an IFSS of X such that the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSIDs of X for a particular $\alpha \in [0, \Im]$. Then $\tilde{M}[\delta]$ is an IFSID of X.*

Proof Assume that $\tilde{M}[\delta]^T_\alpha$ is an IFSID of X for any $\alpha \in [0, \Im]$. Let $x, y \in X$ and $\delta \in A$, $\sigma_{\tilde{M}[\delta]}(0) + \alpha = \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (0) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x) = \sigma_{\tilde{M}[\delta]}(x) + \alpha$ and $\tau_{\tilde{M}[\delta]}(0) - \alpha = \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (0) \leq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (x) = \tau_{\tilde{M}[\delta]}(x) - \alpha$ which implies $\sigma_{\tilde{M}[\delta]}(0) \geq \sigma_{\tilde{M}[\delta]}(x)$ and $\tau_{\tilde{M}[\delta]}(0) \leq \tau_{\tilde{M}[\delta]}(x)$. Now, we have $\sigma_{\tilde{M}[\delta]}(x) + \alpha = \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (y) = \left(\sigma_{\tilde{M}[\delta]}(x*y) + \alpha\right) \wedge \left(\sigma_{\tilde{M}[\delta]}(y) + \alpha\right) \sigma_{\tilde{M}[\delta]}(x) + \alpha \geq \left(\sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]}(y)\right) + \alpha$ and $\tau_{\tilde{M}[\delta]}(x) - \alpha = \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (x) \leq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (x*y) \vee \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (y) = \left(\tau_{\tilde{M}[\delta]}(x*y) - \alpha\right) \vee \left(\tau_{\tilde{M}[\delta]}(y) - \alpha\right) \tau_{\tilde{M}[\delta]}(x) - \alpha \leq \left(\tau_{\tilde{M}[\delta]}(x*y) \vee \tau_{\tilde{M}[\delta]}(y)\right) - \alpha$ which implies that $\sigma_{\tilde{M}[\delta]}(x) \geq \sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]}(y)$ and $\tau_{\tilde{M}[\delta]}(x) \leq \tau_{\tilde{M}[\delta]}(x*y) \vee \tau_{\tilde{M}[\delta]}(y)$ for every $x, y \in X$ and $\delta \in A$. Hence, $\tilde{M}[\delta]$ is an IFSID of X. □

Theorem 4.3. *Let the $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ be an IFSID of X for any $\alpha \in [0, \Im]$. Then the followings are valid:*

(i) If $x*y \leq z$, $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (z)$ and $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (x) \leq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (y) \vee \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (z)$,

(ii) If $x \leq y$, $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (y)$ and $\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (x) \leq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha (y)$

for every $x, y, z \in X$ and $\delta \in A$.

Proof (i) Assume that $x, y, z \in X$ and $\delta \in A$ such that $x*y \leq z$. Then

$$\begin{aligned}
\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x) &\geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (y) \\
&\geq \left[\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha ((x*y)*z) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (z)\right] \\
&\quad \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (y) \\
&= \left[\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (0) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (z)\right] \\
&\quad \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (y) \\
\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (x) &\geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha (z)
\end{aligned}$$

and

$$\begin{aligned}
\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) &\leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\\
&\leq \left[\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}((x*y)*z)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(z)\right]\\
&\quad\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\\
&= \left[\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(z)\right]\\
&\quad\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\\
\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) &\leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(z).
\end{aligned}$$

(ii) Again, take $x,y\in X$ and $\delta\in A$ such that $x\leq y$. Then

$$\begin{aligned}
\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) &\geq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\wedge\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\\
&= \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)\wedge\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)
\end{aligned}$$

$$\begin{aligned}
\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) &\geq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\ \textit{and}\\
\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) &\leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\\
&= \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\\
\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) &\leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x).
\end{aligned}$$

We now give a condition for the IFSAT $\tilde{M}[\delta]_{\alpha}^{T}$ of $\tilde{M}[\delta]$ which is an IFSID of X to be an IFSSU of X.

Theorem 4.4. *Assume that IFSAT $\tilde{M}[\delta]_{\alpha}^{T}$ of $\tilde{M}[\delta]$ is an IFSID of X for a particular $\alpha\in[0,\Im]$. If $x*y\leq x$, for every $x,y\in X$ and $\delta\in A$, then $\tilde{M}[\delta]_{\alpha}^{T}$ is an IFSSU of X.*

Proof Suppose that IFSAT $\tilde{M}[\delta]_{\alpha}^{T}$ of $\tilde{M}[\delta]$ is an IFSID of X. Assume that $(x*y)*x=0$ for every $x,y\in X$ and $\delta\in A$. Then $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\geq\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)\geq\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\wedge\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)$ $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\geq\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)\wedge\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)$ and $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\leq\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)\leq\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)$ $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\leq\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)$. Hence, $\tilde{M}[\delta]_{\alpha}^{T}$ is an IFSSU of X.

Theorem 4.5. *If $\tilde{M}[\delta]$ is an IFSS of X such that the IFSAT $\tilde{M}[\delta]_{\alpha}^{T}$ of $\tilde{M}[\delta]$ is an IFSID of X for $\alpha\in[0,\Im]$, then the sets $T_{\sigma_{\tilde{M}[\delta]}}=\left\{x:x\in X\text{ and }\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)=\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)\right\}$ and $T_{\tau_{\tilde{M}[\delta]}}=\left\{x:x\in X\text{ and }\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)=\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)\right\}$ are ideals of X.*

Proof Suppose that $\tilde{M}[\delta]_{\alpha}^{T}=\left(\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T},\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}\right)$ is an IFSID of X. Then $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ and $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ are fuzzy soft ideal of X. Obviously, $0\in T_{\sigma_{\tilde{M}[\delta]}},T_{\tau_{\tilde{M}[\delta]}}$. Let $x,y\in X$ and $\delta\in A$ such that $x*y\in T_{\sigma_{\tilde{M}[\delta]}}$ and $y\in T_{\sigma_{\tilde{M}[\delta]}}$. Then $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)=\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)=\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)$ and so $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)\geq\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y)\wedge\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)\geq\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)$. Since $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ is a fuzzy soft ideal of X, $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)=\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)$. This implies $\sigma_{\tilde{M}[\delta]}(x)+\alpha=\sigma_{\tilde{M}[\delta]}(0)+\alpha$ or $\sigma_{\tilde{M}[\delta]}(x)=\sigma_{\tilde{M}[\delta]}(0)$ so that $x\in T_{\sigma_{\tilde{M}[\delta]}}$. Therefore, $T_{\sigma_{\tilde{M}[\delta]}}$ is an ideal of X.

Repeatedly, let $a,b\in X$, $\delta\in A$ such that $a*b\in T_{\tau_{\tilde{M}[\delta]}}$, $b\in T_{\tau_{\tilde{M}[\delta]}}$ imply $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(a)=\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)=\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(b)$ and so $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(a)\leq\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(a*b)\vee\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(b)=\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)$. Since $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ is a fuzzy soft ideal of X, $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(a)=\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0)$ implies $\tau_{\tilde{M}[\delta]}(a)+\alpha=\tau_{\tilde{M}[\delta]}(0)+\alpha$ or $\tau_{\tilde{M}[\delta]}(a)=\tau_{\tilde{M}[\delta]}(0)$ so that $a\in T_{\tau_{\tilde{M}[\delta]}}$. Therefore, $T_{\tau_{\tilde{M}[\delta]}}$ is an ideal of X.

Definition 4.6. *Let $\tilde{M}[\delta]=\left(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]}\right)$ and $\tilde{N}[\delta]=\left(\sigma_{\tilde{N}[\delta]},\tau_{\tilde{N}[\delta]}\right)$ be IFSSs of X. Then $\tilde{N}[\delta]$ is called an intuitionistic fuzzy soft ideal extension (IFSIE) of $\tilde{M}[\delta]$ is the following are valid:*

(i) $\tilde{N}[\delta]$ is an IFSE of $\tilde{M}[\delta]$

(ii) If $\tilde{M}[\delta]$ is an IFSID of X, then $\tilde{N}[\delta]$ is an IFSID of X.

From the definition of IFSAT, we get $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)=\sigma_{\tilde{M}[\delta]}(x)+\alpha$ and $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)=\tau_{\tilde{M}[\delta]}(x)-\alpha$ for every $x,y\in X$ and $\delta\in A$.

Theorem 4.7. *Let $\tilde{M}[\delta]$ be an IFSS of X and $\alpha\in[0,\Im]$. Then the IFSAT $\tilde{M}[\delta]_{\alpha}^{T}$ of $\tilde{M}[\delta]$ is an IFSID extension of $\tilde{M}[\delta]$.*

An IFSIE of an IFSID $\tilde{M}[\delta]$ may not be represented as an IFSAT of $\tilde{M}[\delta]$, that is, the reverse of Theorem 4.7 is not correct by the following example.

Example 4.8. Let $X=\{0,1,2,3,4,5\}$ be a B-algebra in Example 3.7 and $\tilde{M}[\delta]=\left(\sigma_{\tilde{M}[\delta]},\tau_{\tilde{M}[\delta]}\right)$ be an IFSS in X by

X	0	1	2
$\tilde{M}[\delta]$	[0.59, 0.23]	[0.50, 0.36]	[0.41, 0.47]

X	3	4	5
$\tilde{M}[\delta]$	[0.41, 0.47]	[0.41, 0.47]	[0.41, 0.47]

Then $\tilde{M}[\delta]$ is an IFSID of X. Let $\tilde{N}[\delta] = \left(\sigma_{\tilde{N}[\delta]}, \tau_{\tilde{N}[\delta]}\right)$ be an IFSS in X by

X	0	1	2
$\tilde{N}[\delta]$	[0.62, 0.20]	[0.54, 0.31]	[0.43, 0.45]

X	3	4	5
$\tilde{N}[\delta]$	[0.43, 0.45]	[0.43, 0.45]	[0.43, 0.45]

Then $\tilde{N}[\delta]$ is an IFSIE of $\tilde{M}[\delta]$. But it is not IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ for a particular $\alpha \in [0, \Im]$ and $\delta \in A$.

Clearly, the intersection of IFSIEs of an IFSID $\tilde{M}[\delta]$ of X is an IFSIE of $\tilde{M}[\delta]$. But the union of IFSIEs of an IFSID $\tilde{M}[\delta]$ of X is not an IFSIE of $\tilde{M}[\delta]$ by the following example.

Example 4.9. Consider a B-algebra $X = \{0,1,2,3\}$ with the Cayley table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let $\tilde{M}[\delta] = \left(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}\right)$ be an IFSS in X by

X	0	1
$\tilde{M}[\delta]$	[0.62,0.34]	[0.40,0.55]

X	2	3
$\tilde{M}[\delta]$	[0.40,0.55]	[0.40,0.55]

Then $\tilde{M}[\delta]$ is an IFSID of X. Let $\tilde{N}[\delta]$ and $\tilde{O}[\delta]$ be two IFSSs in X by

X	0	1
$\tilde{N}[\delta]$	[0.69,0.26]	[0.52,0.31]

X	2	3
$\tilde{N}[\delta]$	[0.69,0.26]	[0.52,0.31]

and

X	0	1
$\tilde{O}[\delta]$	[0.63,0.28]	[0.63,0.28]

X	2	3
$\tilde{O}[\delta]$	[0.46,0.43]	[0.46,0.43]

respectively. Then $\tilde{N}[\delta]$ and $\tilde{O}[\delta]$ are IFSIEs of $\tilde{M}[\delta]$. Obviously, then union $\tilde{N}[\delta] \cup \tilde{O}[\delta]$ is an IFSIE of $\tilde{M}[\delta]$, but it is not an IFSIE of $\tilde{M}[\delta]$ since

$$\begin{aligned} \sigma_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(3) &\geq \sigma_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(3*1) \wedge \sigma_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(1) \\ 0.52 &\geq 0.69 \wedge 0.63 \\ 0.52 &\ngeq 0.63 \end{aligned}$$

and

$$\begin{aligned} \tau_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(3) &\leq \tau_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(3*1) \vee \tau_{\tilde{N}[\delta]\cup\tilde{O}[\delta]}(1) \\ 0.31 &\leq 0.26 \vee 0.28 \\ 0.31 &\nleq 0.28. \end{aligned}$$

If $\tilde{M}[\delta]$ is an IFSID of X, $U_\alpha\left(\sigma_{\tilde{M}[\delta]}; t\right)$ and $L_\alpha\left(\tau_{\tilde{M}[\delta]}; s\right)$ are ideals of X for every $t \in Im\left(\sigma_{\tilde{M}[\delta]}\right)$ and $s \in Im\left(\tau_{\tilde{M}[\delta]}\right)$ with $t \geq \alpha$. But, if $\tilde{M}[\delta]$ is an IFSID of X, then $U_\alpha\left(\sigma_{\tilde{M}[\delta]}; t\right)$ and $L_\alpha\left(\tau_{\tilde{M}[\delta]}; s\right)$ are not ideals of X by the following example.

Example 4.10. Consider a B-algebra $X = \{0,1,2,3\}$ in Example 4.9 and $\tilde{M}[\delta] = \left(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}\right)$ be an IFSS in X by

X	0	1
$\tilde{M}[\delta]$	[0.72,0.20]	[0.47,0.48]

X	2	3
$\tilde{M}[\delta]$	[0.56,0.33]	[0.56,0.33]

Since

$$\begin{aligned} \sigma_{\tilde{M}[\delta]}(1) &\geq \sigma_{\tilde{M}[\delta]}(1*2) \wedge \sigma_{\tilde{M}[\delta]}(2) \\ 0.47 &\geq 0.56 \wedge 0.56 \\ 0.47 &\ngeq 0.56 \end{aligned}$$

and

$$\begin{aligned} \tau_{\tilde{M}[\delta]}(1) &\leq \tau_{\tilde{M}[\delta]}(1*2) \vee \tau_{\tilde{M}[\delta]}(2) \\ 0.48 &\leq 0.33 \vee 0.33 \\ 0.48 &\nleq 0.33. \end{aligned}$$

Therefore, $\tilde{M}[\delta] = \left(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}\right)$ is not an IFSID of X. For $\alpha = 0.13, t = 0.70$ and $s = 0.25$, we obtain $U_\alpha\left(\sigma_{\tilde{M}[\delta]}; t\right) = L_\alpha\left(\tau_{\tilde{M}[\delta]}; s\right) = \{0,2,3\}$ which are not ideals of X since $2 * 3 = 1 \notin \{0,2,3\}$.

Theorem 4.11. *If $\tilde{M}[\delta]^T_\alpha = \left(\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha, \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha\right)$ is an IFSIDs of X, then $\left(\tilde{M}[\delta]^m\right)^T_\alpha = \left(\left(\sigma^m_{\tilde{M}[\delta]}\right)^T_\alpha, \left(\tau^m_{\tilde{M}[\delta]}\right)^T_\alpha\right)$ is an IFSIDs of X.*

Proof For every $x \in X$ and $\delta \in A$, $\left(\tilde{M}[\delta]^m\right)^T_\alpha$ is an IFSS in X defined by $\left(\tilde{M}[\delta]^m\right)^T_\alpha = \left(\left(\sigma^m_{\tilde{M}[\delta]}\right)^T_\alpha, \left(\tau^m_{\tilde{M}[\delta]}\right)^T_\alpha\right)$ where m is any non-negative integer. Let $\tilde{M}[\delta]^T_\alpha$ be an IFSIDs of X and $\alpha \in [0, \Im]$. Then

$$\left(\sigma^m_{\tilde{M}[\delta]}\right)^T_\alpha(0) = \sigma^m_{\tilde{M}[\delta]}(0) + \alpha \geq \sigma^m_{\tilde{M}[\delta]}(x) + \alpha = \left(\sigma^m_{\tilde{M}[\delta]}\right)^T_\alpha(x)$$

$$\text{and} \left(\tau^m_{\tilde{M}[\delta]}\right)^T_\alpha(0) = \tau^m_{\tilde{M}[\delta]}(0) - \alpha \leq \tau^m_{\tilde{M}[\delta]}(x) - \alpha = \left(\tau^m_{\tilde{M}[\delta]}\right)^T_\alpha(x)$$

for every $x \in X$ and $\delta \in A$. Now,

$$\begin{aligned}\left(\sigma^m_{\tilde{M}[\delta]}\right)^T_\alpha(x) = \sigma^m_{\tilde{M}[\delta]}(x) + \alpha &\geq \left(\sigma^m_{\tilde{M}[\delta]}(x*y) \wedge \sigma^m_{\tilde{M}[\delta]}(y)\right) + \alpha \\ &= \left(\sigma^m_{\tilde{M}[\delta]}(x*y) + \alpha\right) \wedge \left(\sigma^m_{\tilde{M}[\delta]}(y) + \alpha\right) \\ \left(\sigma^m_{\tilde{M}[\delta]}\right)^T_\alpha(x) &\geq \left(\sigma^m_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \wedge \left(\sigma^m_{\tilde{M}[\delta]}\right)^T_\alpha(y)\end{aligned}$$

and

$$\begin{aligned}\left(\tau^m_{\tilde{M}[\delta]}\right)^T_\alpha(x) = \tau^m_{\tilde{M}[\delta]}(x) - \alpha &\leq \left(\tau^m_{\tilde{M}[\delta]}(x*y) \vee \tau^m_{\tilde{M}[\delta]}(y)\right) - \alpha \\ &= \left(\tau^m_{\tilde{M}[\delta]}(x*y) + \alpha\right) \vee \left(\tau^m_{\tilde{M}[\delta]}(y) - \alpha\right) \\ \left(\tau^m_{\tilde{M}[\delta]}\right)^T_\alpha(x) &\leq \left(\tau^m_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \vee \left(\tau^m_{\tilde{M}[\delta]}\right)^T_\alpha(y)\end{aligned}$$

for every $x, y \in X$ and $\delta \in A$. Hence, $\left(\tilde{M}[\delta]^m\right)^T_\alpha$ is an IFSIDs of X.

Theorem 4.12. *If $\tilde{M}[\delta]^T_\alpha$ and $\left(\tilde{M}[\delta]^c\right)^T_\alpha$ are both IFSIDs of X and $\alpha \in [0, \Im]$, then $\tilde{M}[\delta]^T_\alpha$ is a constant function.*

Proof Let $\tilde{M}[\delta]^T_\alpha$ and $\left(\tilde{M}[\delta]^c\right)^T_\alpha$ be both IFSIDs of X and $\alpha \in [0, \Im]$.
(i) Let $x \in X$ and $\delta \in A$. Then

$$\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(0) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) \tag{4.1}$$

$$\begin{aligned}\text{and} \left(\sigma^c_{\tilde{M}[\delta]}\right)^T_\alpha(0) &\geq \left(\sigma^c_{\tilde{M}[\delta]}\right)^T_\alpha(x) \\ \Rightarrow 1 - \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(0) &\geq 1 - \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x)\end{aligned}$$

$$\Rightarrow \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(0) \leq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) \tag{4.2}$$

From Equations (4.1) and (4.2), we have

$$\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(0) = \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) \tag{4.3}$$

Again, let $x \in X$ and $\delta \in A$. Than

$$\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(0) \leq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) \tag{4.4}$$

$$\begin{aligned}\text{and} \left(\tau^c_{\tilde{M}[\delta]}\right)^T_\alpha(0) &\leq \left(\tau^c_{\tilde{M}[\delta]}\right)^T_\alpha(x) \\ \Rightarrow 1 - \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(0) &\leq 1 - \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x)\end{aligned}$$

$$\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(0) \geq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) \tag{4.5}$$

From Equations (4.4) and (4.5), we have

$$\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(0) = \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) \tag{4.6}$$

(ii) Let $x, y \in X$ and $\delta \in A$. Then

$$\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(y) \tag{4.7}$$

$$\begin{aligned}\text{and} \left(\sigma^c_{\tilde{M}[\delta]}\right)^T_\alpha(x) &\geq \left(\sigma^c_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \wedge \left(\sigma^c_{\tilde{M}[\delta]}\right)^T_\alpha(y) \\ \Rightarrow 1 - \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) &\geq \left(1 - \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x*y)\right) \wedge \left(1 - \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(y)\right)\end{aligned} \tag{4.8}$$

$$\Rightarrow \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) \leq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(y) \tag{4.9}$$

From Equations (4.7) and (4.9), we have

$$\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x) = \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(y) \tag{4.10}$$

(iii) Let $x, y \in X$ and $\delta \in A$. Then

$$\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) \leq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \vee \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(y) \tag{4.11}$$

$$\begin{aligned}\text{and} \left(\tau^c_{\tilde{M}[\delta]}\right)^T_\alpha(x) &\leq \left(\tau^c_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \vee \left(\tau^c_{\tilde{M}[\delta]}\right)^T_\alpha(y) \\ \Rightarrow 1 - \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) &\leq \left(1 - \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x*y)\right) \vee \left(1 - \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(y)\right)\end{aligned} \tag{4.12}$$

$$\Rightarrow \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) \geq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \vee \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(y) \tag{4.13}$$

From Equations (4.11) and (4.13), we have

$$\left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x) = \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x*y) \vee \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(y) \tag{4.14}$$

Hence (4.6),(4.10) and (4.14) $\Rightarrow \tilde{M}[\delta]_{\alpha}^{T}$ is a constant function.

Theorem 4.13. *Let $\tilde{M}[\delta]$ be an IFSS of X and $\alpha \in [0,\Im]$. Then the IFSAT of $\tilde{M}[\delta]$ is an IFSID of X if and only if $U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$ are ideals of X for $t \in Im\left(\sigma_{\tilde{M}[\delta]}\right)$, $s \in Im\left(\tau_{\tilde{M}[\delta]}\right)$ with $t \geq \alpha$.*

Proof Let $\tilde{M}[\delta]_{\alpha}^{T} = \left(\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}, \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}\right)$ be an IFSID of X. Then $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ and $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ are fuzzy soft ideals of X. Let $t \in Im\left(\sigma_{\tilde{M}[\delta]}\right)$, $s \in Im\left(\tau_{\tilde{M}[\delta]}\right)$ with $t \geq \alpha$. Since $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0) \geq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)$ for all $x \in X$ and $\delta \in A$, we have $\sigma_{\tilde{M}[\delta]}(0)+\alpha = \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0) \geq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) = \sigma_{\tilde{M}[\delta]}(x) + \alpha$ for $x \in U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$. Hence, $0 \in U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$. Let $x,y \in X$ such that $x*y, y \in U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$. Then $\sigma_{\tilde{M}[\delta]}(x*y) \geq t-\alpha$ and $\sigma_{\tilde{M}[\delta]}(y) \geq t-\alpha$, i.e., $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y) = \sigma_{\tilde{M}[\delta]}(x*y)+\alpha \geq t$ and $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y) = \sigma_{\tilde{M}[\delta]}(y)+\alpha \geq t$. Since $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ is a fuzzy soft ideal of X, we have $\sigma_{\tilde{M}[\delta]}(x)+\alpha = \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y) \geq t$ that is, $\sigma_{\tilde{M}[\delta]}(x) \geq t-\alpha$ so that $x \in U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$. Therefore, $U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$ is a fuzzy soft ideal of X.

Again, since $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0) \leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)$ for all $x \in X$, $\tau_{\tilde{M}[\delta]}(0)-\alpha = \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0) \leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) = \tau_{\tilde{M}[\delta]}(x)-\alpha$ for $x \in L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$. Hence, $0 \in L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$. Let $x,y \in X$ and $\delta \in A$ such that $x*y, y \in L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$. Then $\tau_{\tilde{M}[\delta]}(x*y) \leq s+\alpha$ and $\tau_{\tilde{M}[\delta]}(y) \leq s+\alpha$, i.e., $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y) = \tau_{\tilde{M}[\delta]}(x*y)-\alpha \leq s$ and $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)-\alpha \leq s$. Since $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ is a fuzzy soft ideal of X, we have $\tau_{\tilde{M}[\delta]}(x)-\alpha = \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) \leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y) \vee \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y) \leq s$, that is, $\tau_{\tilde{M}[\delta]}(x) \leq s+\alpha$ so that $x \in L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$. Hence, $L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$ is a fuzzy soft ideal of X.

Conversely, assume that $U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$ are ideals of X for $t \in Im\left(\sigma_{\tilde{M}[\delta]}\right)$, $s \in Im\left(\tau_{\tilde{M}[\delta]}\right)$ with $t \geq \alpha$. If there exists $i \in X$ such that $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0) < \lambda \leq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(i)$, then $\sigma_{\tilde{M}[\delta]}(i) \geq \lambda-\alpha$ but $\sigma_{\tilde{M}[\delta]}(0) < \lambda-\alpha$. This shows that $i \in U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $0 \notin U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$. Which is a contradiction and $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0) \geq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)$ for all $x \in X$ and $\delta \in A$.

Repeatedly, if there exists $j \in X$ such that $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0) > \varphi \geq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(j)$, then $\tau_{\tilde{M}[\delta]}(q) \leq \varphi+\alpha$ but $\tau_{\tilde{M}[\delta]}(0) > \varphi+\alpha$. This shows that $j \in L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$ and $0 \notin L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$. This is a contradiction and $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(0) \leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x)$ for all $x \in X$ and $\delta \in A$.

Let $k,\ell \in X$ and $\delta \in A$ such that $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(k) < \beta \leq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(k*\ell) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(\ell)$. Then $\sigma_{\tilde{M}[\delta]}(k*\ell) \geq \beta-\alpha$ and $\sigma_{\tilde{M}[\delta]}(\ell) \geq \beta-\alpha$, but $\sigma_{\tilde{M}[\delta]}(k) < \beta-\alpha$. This shows that $k*\ell \in U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$ and $\ell \in U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$, but $k \notin U_{\alpha}\left(\sigma_{\tilde{M}[\delta]};t\right)$. Which is a contradiction and $\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)$ for every $x,y \in X$ and $\delta \in A$.

Again, suppose that there exists $m,n \in X$ such that $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(m) > \gamma \geq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(m*n) \wedge \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(n)$. Then $\tau_{\tilde{M}[\delta]}(m*n) \leq \gamma+\alpha$ and $\tau_{\tilde{M}[\delta]}(n) \leq \gamma+\alpha$, but $\tau_{\tilde{M}[\delta]}(m) > \gamma+\alpha$. Hence, $m*n \in L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$ and $n \in L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$, but $m \notin L_{\alpha}\left(\tau_{\tilde{M}[\delta]};s\right)$. This is impossible and therefore, $\left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x) \leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(x*y) \vee \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}(y)$ for every $x,y \in X$ and $\delta \in A$. Consequently, $\tilde{M}[\delta]_{\alpha}^{T} = \left(\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}, \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}\right)$ is an IFSID of X.

Theorem 4.14. *Let $\tilde{M}[\delta] = \left(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}\right)$ be an IFSID of X and $\alpha,\beta \in [0,\Im]$. If $\alpha \leq \beta$, the IFSAT $\tilde{M}[\delta]_{\alpha}^{T} = \left(\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}, \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}\right)$ of $\tilde{M}[\delta]$ is an IFSIE of the IFSBT $\tilde{M}[\delta]_{\alpha}^{T} = \left(\left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}, \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}\right)$ of $\tilde{M}[\delta]$.*

Proof Straightforward

For every IFSID $\tilde{M}[\delta]$ of X and $\beta \in [0,\Im]$, the IFSBT $\tilde{M}[\delta]_{\alpha}^{T}$ of $\tilde{M}[\delta]$ is an IFSID of X. If $\tilde{N}[\delta] = \left(\sigma_{\tilde{N}[\delta]}, \tau_{\tilde{N}[\delta]}\right)$ is an IFSIE of $\tilde{M}[\delta]_{\alpha}^{T}$, then there exists $\alpha \in [0,\Im]$ such that $\alpha \geq \beta$ and $\tilde{N}[\delta] \geq \tilde{M}[\delta]_{\alpha}^{T}$ means that $\sigma_{\tilde{N}[\delta]}(x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ and $\tau_{\tilde{N}[\delta]}(x) \leq \left(\tau_{\tilde{M}[\delta]}\right)_{\alpha}^{T}$ for every $x,y \in X$ and $\delta \in A$.

Theorem 4.15. *Let $\tilde{M}[\delta]$ be an IFSID of X and $\beta \in [0,\Im]$. For every IFSIE $\tilde{N}[\delta] = \left(\sigma_{\tilde{N}[\delta]}, \tau_{\tilde{N}[\delta]}\right)$ of the IFSBT $\tilde{M}[\delta]_{\alpha}^{T}$ of $\tilde{M}[\delta]$, there exists $\alpha \in [0,\Im]$ such that $\alpha \geq \beta$ imply $\tilde{N}[\delta]$ is an IFSIE of the IFSAT $\tilde{M}[\delta]_{\alpha}^{T}$ of $\tilde{M}[\delta]$.*

Let us clarify Theorem 4.15 by the following example.

Example 4.16. Consider a B-algebra $X = \{0,1,2,3,4,5\}$ in Example 3.7 and $\tilde{M}[\delta] = \left(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}\right)$ be an IFSS in X by

X	0	1	2
$\tilde{M}[\delta]$	[0.69, 0.26]	[0.54, 0.37]	[0.71, 0.28]

X	3	4	5
$\tilde{M}[\delta]$	[0.54, 0.37]	[0.69, 0.26]	[0.54, 0.37]

Then $\tilde{M}[\delta]$ is an IFSID of X and $\Im = 0.28$. If we take $\beta = 0.11$, then the IFSBT $\tilde{M}[\delta]^T_\beta = \left(\left(\sigma_{\tilde{M}[\delta]}\right)^T_\beta, \left(\tau_{\tilde{M}[\delta]}\right)^T_\beta\right)$ of $\tilde{M}[\delta]$ is given by

X	0	1	2
$\tilde{M}[\delta]^T_\beta$	[0.80, 0.15]	[0.65, 0.26]	[0.80, 0.15]

X	3	4	5
$\tilde{M}[\delta]^T_\beta$	[0.65, 0.26]	[0.80, 0.15]	[0.65, 0.26]

Let $\tilde{N}[\delta] = \left(\sigma_{\tilde{N}[\delta]}, \tau_{\tilde{N}[\delta]}\right)$ be an IFSS in X by

X	0	1	2
$\tilde{N}[\delta]$	[0.85, 0.09]	[0.73, 0.17]	[0.85, 0.09]

X	3	4	5
$\tilde{N}[\delta]$	[0.73, 0.17]	[0.85, 0.09]	[0.73, 0.17]

Then $\tilde{N}[\delta]$ is clearly an IFSID of X, an IFSIE of the intuitionistic fuzzy soft β-translation $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$. But $\tilde{N}[\delta]$ is not an IFSAT of A for a particular $\alpha \in [0, \Im]$. If we take $\alpha = 0.15$ then $\alpha = 0.15 > 0.11 = \beta$ and the IFSAT $\tilde{M}[\delta]^T_\alpha = \left(\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha, \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha\right)$ of $\tilde{M}[\delta]$ is given as follows:

X	0	1	2
$\tilde{M}[\delta]^T_\alpha$	[0.84, 0.11]	[0.69, 0.22]	[0.84, 0.11]

X	3	4	5
$\tilde{M}[\delta]^T_\alpha$	[0.69, 0.22]	[0.84, 0.11]	[0.69, 0.22]

Note that $\tilde{N}[\delta](x) \geq \tilde{M}[\delta]^T_\alpha(x)$ that is $\sigma_{\tilde{N}[\delta]}(x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha$ and $\tau_{\tilde{N}[\delta]}(x) \leq \left(\tau_{\tilde{N}[\delta]}\right)^T_\alpha$ for every $x \in X$ and $\delta \in A$, Hence $\tilde{N}[\delta]$ is an IFSIE of the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$.

For any IFSS $\tilde{M}[\delta] = \left(\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]}\right)$ of X, an intuitionistic fuzzy soft 0-multiplication $\tilde{M}[\delta]^m_0 = \left(\left(\sigma_{\tilde{M}[\delta]}\right)^m_0, \left(\tau_{\tilde{M}[\delta]}\right)^m_0\right)$ of $\tilde{M}[\delta]$ is an IFSID of X.

Theorem 4.17. *If $\tilde{M}[\delta]$ is an IFSID of X, then the intuitionistic fuzzy soft ρ-multiplication of $\tilde{M}[\delta]$ is an IFSID of X for every $\rho \in [0,1]$.*

Proof Straightforward.

Theorem 4.18. *If $\tilde{M}[\delta]$ is any IFSS of X, then the following are equivalent:*

(i) $\tilde{M}[\delta]$ is an IFSID of X.

(ii) for a particular $\rho \in [0,1]$, $\tilde{M}[\delta]^m_\rho$ is an IFSID of X.

Proof Necessary follows from Theorem 4.17 For Sufficient part let $\rho \in [0,1]$ such that $\tilde{M}[\delta]^m_\rho = \left(\left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho, \left(\tau_{\tilde{M}[\delta]}\right)^m_\rho\right)$ is an IFSID of X. Then for every $x, y \in X$ and $\delta \in A$, $\sigma_{\tilde{M}[\delta]}(x) \bullet \rho = \left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho(x) \geq \left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho(x*y) \wedge \left(\sigma_{\tilde{M}[\delta]}\right)^m_\rho(y) = \left(\sigma_{\tilde{M}[\delta]}(x*y) \bullet \rho\right) \wedge \left(\sigma_{\tilde{M}[\delta]}(y) \bullet \rho\right) = \left(\sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]}(y)\right) \bullet \rho$ and $\tau_{\tilde{M}[\delta]}(x) \bullet \rho = \left(\tau_{\tilde{M}[\delta]}\right)^m_\rho(x) \leq \left(\tau_{\tilde{M}[\delta]}\right)^m_\rho(x*y) \vee \left(\tau_{\tilde{M}[\delta]}\right)^m_\rho(y) = \left(\tau_{\tilde{M}[\delta]}(x*y) \bullet \rho\right) \vee \left(\tau_{\tilde{M}[\delta]}(y) \bullet \rho\right) = \left(\tau_{\tilde{M}[\delta]}(x*y) \vee \tau_{\tilde{M}[\delta]}(y)\right) \bullet \rho$. Therefore, $\sigma_{\tilde{M}[\delta]}(x) \geq \sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]}(y)$ and $\tau_{\tilde{M}[\delta]}(x) \leq \tau_{\tilde{M}[\delta]}(x*y) \vee \tau_{\tilde{M}[\delta]}(y)$ for every $x, y \in X$ and $\alpha \in A$ since $\rho \neq 0$. Hence, $\tilde{M}[\delta]$ is an IFSID of X.

Theorem 4.19. *If $\tilde{M}[\delta]^T_\alpha$ and $\tilde{N}[\delta]^T_\alpha$ be IFSIDs of X and $\alpha \in [0, \Im]$, then $\tilde{M}[\delta]^T_\alpha \cap \tilde{N}[\delta]^T_\alpha$ also an IFSIDs of X.*

Proof Let $\tilde{M}[\delta]^T_\alpha$ and $\tilde{N}[\delta]^T_\alpha$ be two IFSIDS of X. Then for every $x \in X$ and $\delta \in A$, $\left(\sigma_{\tilde{M}[\delta]}\right)^T_\alpha(0) \geq \left(\tau_{\tilde{M}[\delta]}\right)^T_\alpha(x)$*and* $\left(\tau_{\tilde{N}[\delta]}\right)^T_\alpha(0) \geq \left(\tau_{\tilde{N}[\delta]}\right)^T_\alpha(x)$. Now $\left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(0) = \left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(0) + \alpha = \left(\sigma_{\tilde{M}[\delta]}(0) \wedge \sigma_{\tilde{N}[\delta]}(0)\right) + \alpha \geq \left(\sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{N}[\delta]}(x)\right) + \alpha \left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(0) \geq \left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(x)$. Again for any $x \in X$ and $\delta \in A$, we have $\left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(0) = \left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(0) - \alpha = \left(\tau_{\tilde{M}[\delta]}(0) \vee \tau_{\tilde{N}[\delta]}(0)\right) - \alpha \leq \left(\tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{N}[\delta]}(x)\right) - \alpha \left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(0) \leq \left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(x)$. Let $x, y \in X$ and $\delta \in A$, we have

$$
\begin{aligned}
&\left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(x) = \sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(x) + \alpha \\
&= \left(\sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{N}[\delta]}(x)\right) + \alpha \\
&\geq \left[\left(\sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]}(y)\right) \wedge \left(\sigma_{\tilde{N}[\delta]}(x*y) \wedge \sigma_{\tilde{N}[\delta]}(y)\right)\right] + \alpha \\
&= \left[\left(\sigma_{\tilde{M}[\delta]}(x*y) \wedge \sigma_{\tilde{N}[\delta]}(x*y)\right) \wedge \left(\sigma_{\tilde{M}[\delta]}(y) \wedge \sigma_{\tilde{N}[\delta]}(y)\right)\right] + \alpha \\
&= \left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(x*y) \wedge \sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(y)\right) + \alpha
\end{aligned}
$$

$= \left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(x*y)+\alpha\right) \wedge \left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(y)+\alpha\right)$
$\left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(x) \geq \left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(x*y) \wedge \left(\sigma_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(y)$

and
$\left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(x) = \tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(x)-\alpha$
$= \left(\tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{N}[\delta]}(x)\right)-\alpha$
$\leq \left[\left(\tau_{\tilde{M}[\delta]}(x*y) \vee \tau_{\tilde{M}[\delta]}(y)\right) \vee \left(\tau_{\tilde{N}[\delta]}(x*y) \vee \tau_{\tilde{N}[\delta]}(y)\right)\right]-\alpha$
$= \left[\left(\tau_{\tilde{M}[\delta]}(x*y) \vee \tau_{\tilde{N}[\delta]}(x*y)\right) \vee \left(\tau_{\tilde{M}[\delta]}(y) \vee \tau_{\tilde{N}[\delta]}(y)\right)\right]-\alpha$
$= \left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(x*y) \vee \tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(y)\right)-\alpha$
$= \left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(x*y)-\alpha\right) \vee \left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}(y)-\alpha\right)$
$\left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(x) \leq \left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(x*y) \vee \left(\tau_{\tilde{M}[\delta]\cap\tilde{N}[\delta]}\right)^T_\alpha(y)$.

Hence, $\tilde{M}[\delta]^T_\alpha \cap \tilde{N}[\delta]^T_\alpha$ is an IFSID of X.

Theorem 4.20. *Let* $\tilde{M}[\delta_i]^T_\alpha = \left(\left(\sigma_{\tilde{M}[\delta_i]}\right)^T_\alpha, \left(\tau_{\tilde{M}[\delta_i]}\right)^T_\alpha i=1,2,\cdots\right)$ *be an IFSID of X. Then* $\bigcap_{i=1}^{n} \tilde{M}[\delta_i]^T_\alpha$ *is also an intuitionistic fuzzy soft ideals of X, where* $\bigcap \tilde{M}[\delta_i]^T_\alpha(x) = \left\{\left(x, \left(\wedge\sigma_{\tilde{M}[\delta_i]}(x) : i=1,2,\cdots\right), \left(\vee\sigma_{\tilde{M}[\delta_i]}(x) : i=1,2,\cdots\right)\right) : x \in X \text{ and } \delta_i \in \bigcap A_i\right\}$.

5. Translations of intuitionistic fuzzy soft a-ideals

In this section, translations of intuitionnistic fuzzy soft a-ideals are defined by a few of their results studied.

Theorem 5.1. *If* $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ *is an IFSAID of X, then IFSAT* $\tilde{M}[\delta]^T_\alpha = ((\sigma_{\tilde{M}[\delta]})^T_\alpha, (\tau_{\tilde{M}[\delta]})^T_\alpha)$ *of* $\tilde{M}[\delta]$ *is an IFSAID of X for a particular* $\alpha \in [0,\Im]$.

Proof. Let $\tilde{M}[\delta] = (\sigma_{\tilde{M}[\delta]}, \tau_{\tilde{M}[\delta]})$ be an IFSAID of X and $\alpha \in [0,\Im]$. Then $(\sigma_{\tilde{M}[\delta]})^T_\alpha(0) = \sigma_{\tilde{M}[\delta]}(0)+\alpha \geq \sigma_{\tilde{M}[\delta]}(x)+\alpha$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(0) = \tau_{\tilde{M}[\delta]}(0)-\alpha \leq \tau_{\tilde{M}[\delta]}(x)-\alpha$ for every $x \in A$ and $\delta \in A$. Now,
$(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) = \sigma_{\tilde{M}[\delta]}(y*x)+\alpha \geq (\sigma_{\tilde{M}[\delta]}((x*z)*(0*y)) \wedge \sigma_{\tilde{M}[\delta]}(z))+\alpha$
$= (\sigma_{\tilde{M}[\delta]}((x*z)*(0*y))+\alpha) \wedge (\sigma_{\tilde{M}[\delta]}(z)+\alpha)$
$(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) = (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z)$
and
$(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) = \tau_{\tilde{M}[\delta]}(y*x)-\alpha \leq (\tau_{\tilde{M}[\delta]}((x*z)*(0*y)) \vee \tau_{\tilde{M}[\delta]}(z))-\alpha$
$= (\tau_{\tilde{M}[\delta]}((x*z)*(0*y))-\alpha) \vee (\tau_{\tilde{M}[\delta]}(z)-\alpha)$
$(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) = (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\tau_{\tilde{M}[\delta]})^T_\alpha(z)$
for every $x,y \in X$ and $\delta \in A$. Hence, the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSAID of X. ☐

Theorem 5.2. *Let* $\tilde{M}[\delta]$ *be an IFSS of X such that the IFSAT* $\tilde{M}[\delta]^T_\alpha$ *of* $\tilde{M}[\delta]$ *is an IFSAID of X for a particular* $\alpha \in [0,\Im]$. *Then* $\tilde{M}[\delta]$ *is an IFSID of X.*

Proof. Assume that $\tilde{M}[\delta]^T_\alpha$ is an IFSAID of X for a particular $\alpha \in [0,\Im]$. Let $x,y \in X$ and $\delta \in A$,
$\sigma_{\tilde{M}[\delta]}(0)+\alpha = (\sigma_{\tilde{M}[\delta]})^T_\alpha(0) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha(x) = \sigma_{\tilde{M}[\delta]}(x)+\alpha$,
$\tau_{\tilde{M}[\delta]}(0)-\alpha = (\tau_{\tilde{M}[\delta]})^T_\alpha(0) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha(x) = \tau_{\tilde{M}[\delta]}(x)-\alpha$,
which implies $\sigma_{\tilde{M}[\delta]}(0) \geq \sigma_{\tilde{M}[\delta]}(x)$ and $\tau_{\tilde{M}[\delta]}(0) \leq \tau_{\tilde{M}[\delta]}(x)$. Now, we have
$\sigma_{\tilde{M}[\delta]}(y*x)+\alpha = (\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z)$
$= (\sigma_{\tilde{M}[\delta]}((x*z)*(0*y)+\alpha)) \wedge (\sigma_{\tilde{M}[\delta]}(z)+\alpha)$
$\sigma_{\tilde{M}[\delta]}(y*x)+\alpha \geq (\sigma_{\tilde{M}[\delta]}((x*z)*(0*y)) \wedge \sigma_{\tilde{M}[\delta]}(z))+\alpha$
and
$\tau_{\tilde{M}[\delta]}(y*x)-\alpha = (\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(z)$
$= (\tau_{\tilde{M}[\delta]}((x*z)*(0*y)-\alpha)) \vee (\tau_{\tilde{M}[\delta]}(z)-\alpha)$
$\tau_{\tilde{M}[\delta]}(y*x)-\alpha \leq (\tau_{\tilde{M}[\delta]}((x*z)*(0*y)) \vee \tau_{\tilde{M}[\delta]}(z))-\alpha$
which implies that $\sigma_{\tilde{M}[\delta]}(y*x) \geq (\sigma_{\tilde{M}[\delta]}((x*z)*(0*y)) \wedge \sigma_{\tilde{M}[\delta]}(z))$ and $\tau_{\tilde{M}[\delta]}(y*x) \leq (\tau_{\tilde{M}[\delta]}((x*z)*(0*y)) \vee \tau_{\tilde{M}[\delta]}(z))$ for every $x,y,z \in X$ and $\delta \in A$. Hence, $\tilde{M}[\delta]$ is an IFSID of X. ☐

Theorem 5.3. *If the IFSAT* $\tilde{M}[\delta]^T_\alpha$ *of* $\tilde{M}[\delta]$ *is an IFSAID of X for a particular* $\alpha \in [0,\Im]$, $\tilde{M}[\delta]^T_\alpha$ *is an IFSSU of X.*

Proof. Let IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSAID of X.Then $(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z)$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) \geq (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(z)$. Since $\tilde{M}[\delta]$ is a subalgebra,
$(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) = \sigma_{\tilde{M}[\delta]}(y*x)+\alpha \geq (\sigma_{\tilde{M}[\delta]}(x) \wedge \sigma_{\tilde{M}[\delta]}(y))+\alpha$
$= (\sigma_{\tilde{M}[\delta]}(x)+\alpha) \wedge (\sigma_{\tilde{M}[\delta]}(y)+\alpha)$
$(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha(x) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(y)$
and
$(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) = \tau_{\tilde{M}[\delta]}(y*x)-\alpha \leq (\tau_{\tilde{M}[\delta]}(x) \vee \tau_{\tilde{M}[\delta]}(y))-\alpha$
$= (\tau_{\tilde{M}[\delta]}(x)-\alpha) \vee (\tau_{\tilde{M}[\delta]}(y)-\alpha)$
$(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha(x) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(y)$
Therefore, $\tilde{M}[\delta]^T_\alpha$ is an IFSSU of X ☐

Theorem 5.4. *Let* $\tilde{M}[\delta]$ *be an IFSS of X such that the IFSAT* $\tilde{M}[\delta]^T_\alpha$ *of* $\tilde{M}[\delta]$ *is an IFSAID of X for a particular* $\alpha \in [0,\Im]$. *Then the sets* $T_{\tilde{M}[\delta]} = \{x \in X : (\sigma_{\tilde{M}[\delta]})^T_\alpha(x) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(0)$ *and* $(\tau_{\tilde{M}[\delta]})^T_\alpha(x) = (\tau_{\tilde{M}[\delta]})^T_\alpha(0)\}$ *is an a-ideal of X.*

Proof. Clearly, $0 \in T_{\tilde{M}[\delta]}$. Assume that $x,y,z \in X$ and $\delta \in A$ such that $(x*z)*(0*y), z \in T_{\tilde{M}[\delta]}$, then $(\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(0) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(z)$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) = (\tau_{\tilde{M}[\delta]})^T_\alpha(0) = (\tau_{\tilde{M}[\delta]})^T_\alpha(z)$.Thus, we have $(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z) = \sigma_{\tilde{M}[\delta]}(0)$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(z) = \tau_{\tilde{M}[\delta]}(0)$. Since, $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSAID of X, $(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) = \sigma_{\tilde{M}[\delta]}(0)$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) = \tau_{\tilde{M}[\delta]}(0)$. Therefore

$(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x)+\alpha = \sigma_{\tilde{M}[\delta]}(0)+\alpha$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x)-\alpha = \tau_{\tilde{M}[\delta]}(0)-\alpha$, which implies $\sigma_{\tilde{M}[\delta]}(y*x) = \sigma_{\tilde{M}[\delta]}(0)$ and $\tau_{\tilde{M}[\delta]}(y*x) = \tau_{\tilde{M}[\delta]}(0)$, so that $y*x \in T_{\tilde{M}[\delta]}$. Therefore, $T_{\tilde{M}[\delta]}$ is an a-ideal of X. □

Theorem 5.5. *Let $\tilde{M}[\delta]$ be an IFSS of X such that the IFAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSID of X. Then the following statement are equivalent:*
(i) $\tilde{M}[\delta]^T_\alpha$ is an IFSAID of X,
(ii) $\tilde{M}[\delta]^T_\alpha((x*z)*(0*y)) \le \tilde{M}[\delta]^T_\alpha(y*(x*z))$,
(iii) $\tilde{M}[\delta]^T_\alpha(x*(0*y)) \le \tilde{M}[\delta]^T_\alpha(y*x)$.

Proof. (i)⇒(ii) Let $\tilde{M}[\delta]^T_\alpha$ is an IFSAID of X. Then for every $x,y \in X$ *and* $\delta \in A$, we have $(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*(x*z)) \ge (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*0)*(0*y)) \wedge \sigma_{\tilde{M}[\delta]}(0)$.
$= (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*0)*(0*y))$
$= (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y))$
and
$(\tau_{\tilde{M}[\delta]})^T_\alpha(y*(x*z)) \le (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*0)*(0*y)) \vee \tau_{\tilde{M}[\delta]}(0)$.
$= (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*0)*(0*y))$
$= (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y))$
(ii)⇒(iii): Taking $z=0$ in (ii) includes (iii).
(iii)⇒(i): Assume that $x,y,z \in X$ *and* $\delta \in A$, we have $(x*(0*y))*((x*z)*(0*y))*(0*y) \le x*(x*z) \le z$. So $(\sigma_{\tilde{M}[\delta]})^T_\alpha(x*(0*y)) \ge (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z)$.
Now, by part (iii), we have $(\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) \ge (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z)$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha(x*(0*y)) \le (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(z)$.
Now, by part (iii), we have $(\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) \le (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(z)$.
Hence $\tilde{M}[\delta]^T_\alpha$ is an IFSAID of X. □

Theorem 5.6. *Let $\tilde{M}[\delta]$ be an IFSS of X such that the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSID of X . If for every $x,y,z \in X$ and $\delta \in A$, $(\sigma_{\tilde{M}[\delta]})^T_\alpha((x*y)*z) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(x*(y*z))$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha((x*y)*z) = (\tau_{\tilde{M}[\delta]})^T_\alpha(x*(y*z))$, then $(\tilde{M}[\delta])^T_\alpha$ is an IFSAID of X.*

Proof. For all $x, \in X$ *and* $\delta \in A$, $(\sigma_{\tilde{M}[\delta]})^T_\alpha(0*x)) = (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*x)*x) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(x*(x*x)) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(x)$, i.e, $(\sigma_{\tilde{M}[\delta]})^T_\alpha(0*x) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(x)$. Now for every $x,y, \in X$ *and* $\delta \in A$, we have
$(\sigma_{\tilde{M}[\delta]})^T_\alpha(x*(0*y) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(x*y) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(0*(x*y))$
$= (\sigma_{\tilde{M}[\delta]})^T_\alpha((0*x)*y) = (\sigma_{\tilde{M}[\delta]})^T_\alpha((0*y)*x)$
$= (\sigma_{\tilde{M}[\delta]})^T_\alpha(0*(y*x))$
$(\sigma_{\tilde{M}[\delta]})^T_\alpha(x*(0*y) = (\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x)$ and
$(\tau_{\tilde{M}[\delta]})^T_\alpha(x*(0*y) = (\tau_{\tilde{M}[\delta]})^T_\alpha(x*y) = (\tau_{\tilde{M}[\delta]})^T_\alpha(0*(x*y)) =$
$(\tau_{\tilde{M}[\delta]})^T_\alpha((0*x)*y) = (\tau_{\tilde{M}[\delta]})^T_\alpha((0*y)*x)$
$=(\tau_{\tilde{M}[\delta]})^T_\alpha(0*(y*x))$
$(\tau_{\tilde{M}[\delta]})^T_\alpha(x*(0*y) = (\tau_{\tilde{M}[\delta]})^T_\alpha(y*x)$.
Hence by Theorem 5.5 (iii), $\tilde{M}[\delta]^T_\alpha$ is an IFSAID of X. □

Definition 5.7. *Let $\tilde{M}[\delta]$ and $\tilde{N}[\delta]$ be IFSS of X. Then $\tilde{N}[\delta]$ is called an IFSAIDE of $\tilde{M}[\delta]$ if the following statements are valid:*
(i) $\tilde{N}[\delta]$ is an IFSE of $\tilde{M}[\delta]$,
(ii) If $\tilde{M}[\delta]$ is an IFSAID of X, then $\tilde{N}[\delta]$ is an IFSAID of X.

Theorem 5.8. *Let $\tilde{M}[\delta]$ be an IFSAID of X and $\alpha \in [0,\Im]$. Then the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$ is an IFSAIDE of $\tilde{M}[\delta]$.*

An IFSAIDE of an IFSAID $\tilde{M}[\delta]$ may not be represented as an IFSAT of $\tilde{M}[\delta]$, i.e., the reverse of Theorem 5.8 is not correct by the following example.

Example 5.9. Consider a B-algebra $X=\{0,1,2\}$ with the Cayley table:

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Let $\tilde{M}[\delta]$ be an IFSS in X by Then, $\tilde{M}[\delta]$ be an IFSAID

X	0	1	2
$\tilde{M}[\delta]$	[0.7,0.2]	[0.5,0.4]	[0.5,0.4]

of X, because
$\sigma_{\tilde{M}[\delta]}(2*1) \ge \sigma_{\tilde{M}[\delta]}((1*0)*(0*2)) \wedge \sigma_{\tilde{M}[\delta]}(0)$
$\sigma_{\tilde{M}[\delta]}(1) = \sigma_{\tilde{M}[\delta]}(0) \wedge \sigma_{\tilde{M}[\delta]}(0)$
$0.5 = 0.7 \wedge 0.7$
$0.5 \not\ge 0.7$
and
$\tau_{\tilde{M}[\delta]}(2*1) \le \tau_{\tilde{M}[\delta]}((1*0)*(0*2)) \vee \tau_{\tilde{M}[\delta]}(0)$
$\tau_{\tilde{M}[\delta]}(1) \le \tau_{\tilde{M}[\delta]}(0) \vee \tau_{\tilde{M}[\delta]}(0)$
$0.4 = 0.2 \vee 0.2$
$0.4 \not\le 0.2.$
Now, for $\alpha=0.2, t=0.62$ and $s=0.43$, we have $U_\alpha(\sigma_{\tilde{M}[\delta]};t) = \{0\}$ and $L_\alpha(\tau_{\tilde{M}[\delta]};s) = \{1\}$ which is not an a-ideal of X, because $(1*0)*(0*2) = 0 \in \{0\}$, but $2*1 = 1 \notin \{0\}$.

Example 5.10. Consider a B-algebra $X=\{0,a,b,c\}$ with the Cayley table:

*	0	1	2	3
0	0	2	1	3
1	1	0	3	2
2	2	3	1	0
3	3	2	1	0

Let $\tilde{M}[\delta]$ be an IFSS in X by

X	0	1	2	3
$\tilde{M}[\delta]$	[0.7,0.1]	[0.7,0.1]	[0.5,0.3]	[0.5,0.3]

Then $\tilde{M}[\delta]$ be an IFSAID of X.Let $\tilde{N}[\delta]$ be an IFSS in X by

X	0	1
$\tilde{N}[\delta]$	[0.73,0.13]	[0.73,0.13]

X	2	3
$\tilde{N}[\delta]$	[0.51,0.31]	[0.51,0.31]

Then $\tilde{N}[\delta]$ is an IFSAIDE of $\tilde{M}[\delta]$. But it is not the IFSAT $\tilde{M}[\delta]^T_\alpha$ of $\tilde{M}[\delta]$, for all $\alpha \in [0,\Im]$.

For an IFSS $\tilde{M}[\delta]$ of X, $\alpha \in [0,\Im]$ and $t \in [0,1]$ with $t \geq \alpha$, let $U_\alpha(\sigma_{\tilde{M}[\delta]};t) := \{x \in : \sigma_{\tilde{M}[\delta](x)} \geq t-\alpha\}$ and $L_\alpha(\tau_{\tilde{M}[\delta]};s) := \{x \in : \tau_{\tilde{M}[\delta](x)} \leq s+\alpha\}$. It is easy to check that, if $\tilde{M}[\delta]$ is an IFSAID of X, then for all $t \in Im(\sigma_{\tilde{M}[\delta]})$, $s \in Im(\tau_{\tilde{M}[\delta]})$ with $t \geq \alpha$, $U_\alpha(\sigma_{\tilde{M}[\delta]};t)$ *and* $L_\alpha(\tau_{\tilde{M}[\delta]};s)$ are an a-ideal of X.

Theorem 5.11. *Let $\tilde{M}[\delta]$ be an IFSS of X and $\alpha \in [0,\Im]$. Then, the IFSAT of $\tilde{M}[\delta]$ is an IFSAID of X if and only if $U_\alpha(\sigma_{\tilde{M}[\delta]};t)$ and $L_\alpha(\tau_{\tilde{M}[\delta]};s)$ are an a-ideal of X for $t \in Im(\sigma_{\tilde{M}[\delta]})$, $s \in Im(\tau_{\tilde{M}[\delta]})$ with $t \geq \alpha$.*

Proof. Let $\tilde{M}[\delta]^T_\alpha = ((\sigma_{\tilde{M}[\delta]})^T_\alpha, (\tau_{\tilde{M}[\delta]})^T_\alpha)$ be an IFSAID of X. Then $(\sigma_{\tilde{M}[\delta]})^T_\alpha$ and $(\tau_{\tilde{M}[\delta]})^T_\alpha$ are FSAID of X. Let $t \in Im(\sigma_{\tilde{M}[\delta]})$, $s \in Im(\tau_{\tilde{M}[\delta]})$ with $t \geq \alpha$. Since $(\sigma_{\tilde{M}[\delta]})^T_\alpha(0) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha(x)$ for all $x \in X$ *and* $\delta \in A$, $\sigma_{\tilde{M}[\delta]}(0)+\alpha = (\sigma_{\tilde{M}[\delta]})^T_\alpha(0) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha(x) = \sigma_{\tilde{M}[\delta]}(x)+\alpha$ for $x \in U_\alpha(\sigma_{\tilde{M}[\delta]};t)$. Hence, $0 \in U_\alpha(\sigma_{\tilde{M}[\delta]};t)$.

Let $x,y,z \in X$ and $\delta \in A$ such that $(x*z)*(0*y), z \in U_\alpha(\sigma_{\tilde{M}[\delta]};t)$. Then $\sigma_{\tilde{M}[\delta]}((x*z)*(0*y)) \geq t-\alpha$ and $\sigma_{\tilde{M}[\delta]}(z) \geq t-\alpha$, i.e, $(\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) = \sigma_{\tilde{M}[\delta]}((x*z)*(0*y))+\alpha \geq t$ and $(\sigma_{\tilde{M}[\delta]})^T_\alpha(z) = \sigma_{\tilde{M}[\delta]}(z)+\alpha \geq t$. Since $(\sigma_{\tilde{M}[\delta]})^T_\alpha$ is a fuzzy soft a-ideal X, $\sigma_{\tilde{M}[\delta]}(y*x)+\alpha = (\sigma_{\tilde{M}[\delta]})^T_\alpha(y*x) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z) \geq t$, i.e.,$\sigma_{\tilde{M}[\delta]}(y*x) \geq t-\alpha$ so that $y*x \in U_\alpha(\sigma_{\tilde{M}[\delta]};t)$. Therefore $U_\alpha(\sigma_{\tilde{M}[\delta]};t)$ is a FSAID of X.

Again, since $(\tau_{\tilde{M}[\delta]})^T_\alpha(0) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha(x)$ for all $x \in X$ *and* $\delta \in A$, we have $\tau_{\tilde{M}[\delta]}(0)-\alpha = (\tau_{\tilde{M}[\delta]})^T_\alpha(0) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha(x) = \tau_{\tilde{M}[\delta]}(x)-\alpha$ for $x \in L_\alpha(\tau_{\tilde{M}[\delta]};s)$. Hence, $0 \in L_\alpha(\tau_{\tilde{M}[\delta]};s)$. Let $x,y,z \in X$ and $\delta \in A$ such that $(x*z)*(0*y), z \in L_\alpha(\tau_{\tilde{M}[\delta]};s)$. Then $\tau_{\tilde{M}[\delta]}((x*z)*(0*y)) \leq s+\alpha$ and $\tau_{\tilde{M}[\delta]}(z) \leq t+\alpha$, i.e, $(\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y))-\alpha \leq s$ and $\tau_{\tilde{M}[\delta]}(z)-\alpha \leq s$. Since $(\tau_{\tilde{M}[\delta]})^T_\alpha$ is a FSAID of X, $\tau_{\tilde{M}[\delta]}(y*x)-\alpha = (\tau_{\tilde{M}[\delta]})^T_\alpha(y*x) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\tau_{\tilde{M}[\delta]})^T_\alpha(z) \leq s$, i.e.,$\tau_{\tilde{M}[\delta]}(y*x) \leq s+\alpha$ so that $y*x \in L_\alpha(\tau_{\tilde{M}[\delta]};s)$. Hence $L_\alpha(\tau_{\tilde{M}[\delta]};s)$ is a FSAID of X.

Conversely, assume that $U_\alpha(\sigma_{\tilde{M}[\delta]};t)$ and $L_\alpha(\tau_{\tilde{M}[\delta]};s)$ are a-ideal of X for $t \in Im(\sigma_{\tilde{M}[\delta]}), s \in Im(\tau_{\tilde{M}[\delta]})$ with $t \geq \alpha$. If there exists $i \in X$ such that $(\sigma_{\tilde{M}[\delta]})^T_\alpha(0) < \lambda \leq (\sigma_{\tilde{M}[\delta]})^T_\alpha(i)$, then $\sigma_{\tilde{M}[\delta]}(i) \geq \lambda-\alpha$ but $\sigma_{\tilde{M}[\delta]}(0) < \lambda-\alpha$. This shows that $i \in U_\alpha(\sigma_{\tilde{M}[\delta]};t)$ and $0 \notin U_\alpha(\sigma_{\tilde{M}[\delta]};t)$. This is a contradiction, and $(\sigma_{\tilde{M}[\delta]})^T_\alpha(0) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha(x)$ for every $x \in X$ and $\delta \in A$.

Again, if there exists $j \in X$ such that $(\tau_{\tilde{M}[\delta]})^T_\alpha(0) > \varphi \geq (\tau_{\tilde{M}[\delta]})^T_\alpha(j)$, then $\tau_{\tilde{M}[\delta]}(i) \leq \varphi+\alpha$ but $\tau_{\tilde{M}[\delta]}(0) > \varphi+\alpha$. This shows that $j \in L_\alpha(\tau_{\tilde{M}[\delta]};s)$ and $0 \notin L_\alpha(\tau_{\tilde{M}[\delta]};s)$. This is a contradiction, and $(\tau_{\tilde{M}[\delta]})^T_\alpha(0) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha(x)$ for all $x \in X$ and $\delta \in A$.

□

Theorem 5.12. *If $\tilde{M}[\delta]$ is an IFSAID of X, then the intuitinistic fuzzy soft ρ-multiplication of $\tilde{M}$ is an IFSAID of X for every $\rho \in [0,1]$.*
Proof Straight forward

Theorem 5.13. *If $\tilde{M}[\delta]$ is any IFSS of X, then the following equations are equivalent:*
(i) $\tilde{M}[\delta]$ is an IFSAID of X
(ii) for any $\rho \in (0,1]$, $\tilde{M}[\delta]^m_\rho$ is an IFSAID of X.

Proof. Necessary follows from Theorem 5.12. For sufficient part let $\rho \in (0,1]$ such that $\tilde{M}[\delta]^m_\rho = ((\sigma_{\tilde{M}[\delta]})^m_\rho, (\tau_{\tilde{M}[\delta]})^m_\rho)$ is an IFSAID of X. Then for every $x,y,z \in X$ and $\delta \in A$, $\sigma_{\tilde{M}[\delta]}(y*x)\bullet\rho = (\sigma_{\tilde{M}[\delta]})^m_\rho(y*x) \geq (\sigma_{\tilde{M}[\delta]})^m_\rho((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^m_\rho(z) = (\sigma_{\tilde{M}[\delta]}((x*z)*(0*y))\bullet\rho) \wedge (\sigma_{\tilde{M}[\delta]}(z)\bullet\rho) = (\sigma_{\tilde{M}[\delta]}((x*z)*(0*y)) \wedge \sigma_{\tilde{M}[\delta]}(z))\bullet\rho$ and $\tau_{\tilde{M}[\delta]}(y*x)\bullet\rho = (\tau_{\tilde{M}[\delta]})^m_\rho(y*x) \leq (\tau_{\tilde{M}[\delta]})^m_\rho((x*z)*(0*y)) \vee (\tau_{\tilde{M}[\delta]})^m_\rho(z) = (\tau_{\tilde{M}[\delta]}((x*z)*(0*y))\bullet\rho) \vee (\tau_{\tilde{M}[\delta]}(z)\bullet\rho) = (\tau_{\tilde{M}[\delta]}((x*z)*(0*y)) \vee \tau_{\tilde{M}[\delta]}(z)) \bullet\rho$. Therefore, $\sigma_{\tilde{M}[\delta]}(y*x)\bullet\rho \geq \sigma_{\tilde{M}[\delta]}((x*z)*(0*y)) \wedge \sigma_{\tilde{M}[\delta]}(z)$ and $\tau_{\tilde{M}[\delta]}(y*x)\bullet\rho \geq \tau_{\tilde{M}[\delta]}((x*z)*(0*y)) \vee \tau_{\tilde{M}[\delta]}(z)$ for every $x,y,z \in X$ and $\delta \in A$ since $\rho \neq 0$. Hence, $\tilde{M}$ is an IFSAID of X.

Let $k,l,m \in X$ and $\delta \in A$ such that $(\sigma_{\tilde{M}[\delta]})^T_\alpha(k) < \beta \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha(k*l) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(l)$. Then $\sigma_{\tilde{M}[\delta]}(k*l) \geq \beta-\alpha$ *and* $\sigma_{\tilde{M}[\delta]}(l) \geq \beta-\alpha$, but $\sigma_{\tilde{M}[\delta]}(k*l) < \beta-\alpha$. This shows that $k*l \in U_\alpha(\sigma_{\tilde{M}[\delta]};t)$ *and* $l \in U_\alpha(\sigma_{\tilde{M}[\delta]};t)$, but $k \notin U_\alpha(\sigma_{\tilde{M}[\delta]};t)$. This is a contradiction and $(\sigma_{\tilde{M}[\delta]})^T_\alpha(x*y) \geq (\sigma_{\tilde{M}[\delta]})^T_\alpha((x*z)*(0*y)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z)$ for every $x,y,z \in X$ and $\delta \in A$.

Again, suppose that there exists $n,o,p \in X$ and $\delta \in A$ such that $(\tau_{\tilde{M}[\delta]})^T_\alpha(o*n) > \gamma \leq (\tau_{\tilde{M}[\delta]})^T_\alpha((n*p)*(0*o)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(p)$. Then $(\tau_{\tilde{M}[\delta]})^T_\alpha((n*p)*(0*o)) \leq \gamma+\alpha$ *and* $\tau_{\tilde{M}[\delta]}(o) \leq \gamma+\alpha$, but $\tau_{\tilde{M}[\delta]}(o*n) > \gamma+\alpha$. Hence, $((n*p)*(0*o)) \in L_\alpha(\tau_{\tilde{M}[\delta]};s)$ *and* $p \in L_\alpha(\tau_{\tilde{M}[\delta]};s)$, but $(o*n) \notin L_\alpha(\tau_{\tilde{M}[\delta]};s)$. This is not possible and therefore, $(\tau_{\tilde{M}[\delta]})^T_\alpha(o*n) \leq (\tau_{\tilde{M}[\delta]})^T_\alpha((n*p)*(0*o)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(p)$ for every $x,y,z \in X$ and $\delta \in A$. Consequently, $\tilde{M}[\delta]^T_\alpha = ((\sigma_{\tilde{M}[\delta]})^T_\alpha, (\tau_{\tilde{M}[\delta]})^T_\alpha)$ is an IFSAID of X. □

Theorem 5.14. *Let $\tilde{M}[\delta]$ and $\tilde{N}[\eta]$ be two IFSAID of X and for a particular $\alpha \in [0,\Im]$, $\tilde{M}[\delta]^T_\alpha$ and $\tilde{N}[\eta]^T_\alpha$ be the IFSAT $\tilde{M}[\delta]^T_\alpha$ and $\tilde{N}[\eta]^T_\alpha$ of $\tilde{M}[\delta]$ and $\tilde{N}[\eta]$, respectively. Then $\tilde{M}[\delta]^T_\alpha \times \tilde{N}[\eta]^T_\alpha$ is an IFSAID of $X \times X$ for a particular $\alpha \in [0,\Im]$.*

Proof. For every $\bar{x} = (x_1, x_2) \in X \times X$ and $(\delta, \eta) \in A \times B$, we obtain

$$\begin{aligned}(\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{0}) &= (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(0,0)\\ &= (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(0) \wedge (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(0)\\ &\geq (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(x_1) \wedge (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(x_2)\\ &= (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(x_1,x_2)\\ (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{0}) &= (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{x})\end{aligned}$$

and

$$\begin{aligned}&(\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{0}) = (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(0,0)\\ &= (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(0) \vee (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(0)\\ &\leq (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(x_1) \vee (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(x_2)\\ &= (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(x_1,x_2)\\ &(\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{0}) = (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{x}).\end{aligned}$$

Also, for every $\bar{x} = (x_1, x_2), \bar{y} = (y_1, y_2), \bar{z} = (z_1, z_2) \in X \times X$ and $(\delta, \eta) \in A \times B$, we have

$$\begin{aligned}&(\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{y}*\bar{x}) \ = (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(y_1*x_1, y_2*x_2)\\ &= (\sigma_{\tilde{M}[\delta]})^T_\alpha(y_1*x_1) \wedge (\sigma_{\tilde{N}[\eta]})^T_\alpha(y_2*x_2)\\ &\geq [(\sigma_{\tilde{M}[\delta]})^T_\alpha((x_1*z_1)*(0*y_1)) \wedge (\sigma_{\tilde{M}[\delta]})^T_\alpha(z_1)] \wedge\\ &[(\sigma_{\tilde{N}[\eta]})^T_\alpha((x_2*z_2)*(0*y_2)) \wedge (\sigma_{\tilde{N}[\eta]})^T_\alpha(z_2)]\\ &= [(\sigma_{\tilde{M}[\delta]})^T_\alpha((x_1*z_1)*(0*y_1)) \wedge (\sigma_{\tilde{N}[\eta]})^T_\alpha((x_2*z_2)*(0*y_2))]\\ &\wedge [(\sigma_{\tilde{M}[\delta]})^T_\alpha(z_1) \wedge (\sigma_{\tilde{N}[\eta]})^T_\alpha(z_2)]\\ &= (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha((x_1,x_2)*(z_1,z_2))*((0,0)*(y_1,y_2))\\ &\wedge (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(z_1,z_2)\\ &(\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{y}*\bar{x}) \ \geq (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha((\bar{x}*\bar{z})*(\bar{0}*\bar{y}))\\ &\wedge (\sigma_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{z})\end{aligned}$$

and

$$\begin{aligned}&(\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{y}*\bar{x}) \ = (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(y_1*x_1, y_2*x_2)\\ &= (\tau_{\tilde{M}[\delta]})^T_\alpha(y_1*x_1) \vee (\tau_{\tilde{N}[\eta]})^T_\alpha(y_2*x_2)\\ &\geq [(\tau_{\tilde{M}[\delta]})^T_\alpha((x_1*z_1)*(0*y_1)) \vee (\tau_{\tilde{M}[\delta]})^T_\alpha(z_1)] \vee\\ &[(\tau_{\tilde{N}[\eta]})^T_\alpha((x_2*z_2)*(0*y_2)) \vee (\tau_{\tilde{N}[\eta]})^T_\alpha(z_2)]\\ &= [(\tau_{\tilde{M}[\delta]})^T_\alpha((x_1*z_1)*(0*y_1)) \vee (\tau_{\tilde{N}[\eta]})^T_\alpha((x_2*z_2)*(0*y_2))]\\ &\vee [(\tau_{\tilde{M}[\delta]})^T_\alpha(z_1) \vee (\tau_{\tilde{N}[\eta]})^T_\alpha(z_2)]\\ &= (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(((x_1,x_2)*(z_1,z_2))*((0,0)*(y_1,y_2)))\\ &\vee (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(z_1,z_2)\\ &(\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{y}*\bar{x}) \ \leq (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha((\bar{x}*\bar{z})*(\bar{0}*\bar{y}))\\ &\vee (\tau_{\tilde{M}[\delta]\times\tilde{N}[\eta]})^T_\alpha(\bar{z})\end{aligned}$$

Therefore, $\tilde{M}[\delta] \times \tilde{N}[\eta]$ is an IFSAID of X. □

6. Conclusion

In this paper, IF soft translations of IFSSUs, IFSIDs, and IFSAIDs of B-algebras are introduced to a few of their useful properties investigated. But the converse is not true with examples are given. IF soft translations, and IF soft extension of IFSSUs, IFSIDs, and IFSAIDs have been constructed. Also, IF soft translations of IF soft ideals are a constant function. Finally, proved that the product of IF soft translations of IF soft a-ideals is an IF soft a-ideals. For future works, we can work on some other IFSIDs in BG-algebras.

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