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Intuitionistic fuzzy translation of anti-intuitionistic fuzzy T-idelas of subtraction BCK/BCI-algebras

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Abstract

In this paper, the conception of intuitionistic fuzzy translation to anti-intuitionistic fuzzy T-ideals in Subtraction BCK/BCI-algebras is introduced. The ideas of anti-intuitionistic fuzzy extensions andanti-intuitionistic fuzzy multiplications of anti-intuitionistic fuzzy T-ideals with several related propertiesare investigated. Also the relationships between intuitionistic fuzzy translations, intuitionisticfuzzy extensions and intuitionistic fuzzymultiplications of anti-intuitionistic fuzzy T-ideals are investigated.

Keywords

Subtraction BCK/BCI – algebra, anti-intuitionistic fuzzy ideal, anti-intuitionistic fuzzy T-ideal, anti-intuitionistic fuzzy translation, anti-intuitionistic fuzzy extension, anti-intuitionistic fuzzy multiplication.

AMS Subject Classification

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Contents

1. Introduction

The learning of BCK/BCI-algebra was initiated by Imai and Iseki [1] as a simplification of the commencement of set theoretic distinction and comparative calculi. Huang gave a new designation of fuzzy BCI-algebras and several consequences as regards it. After the introduction of fuzzy sets by Zadeh [2], there contain be a quantity of generalizations of this necessary perception. The suggestion of intuitionistic fuzzy sets accessible by Atanassov [3] is one amongst them. To elaborate the theory of BCK/BCI-algebras the ideal theory plays a most important part. A number of researchers explore properties of fuzzy subalgebra and ideals in BCK/BCI-algebras. In 1999, KhalIid et al. [4] introduced fuzzy H-ideals in BCI-algebras. Also, Senapati et al. contain presented more than a few consequences on BCK/BCI-algebras, BG-algebra and B-algebra [5]. In the authors have studied uncertainty intuitionistic fuzzy sub algebra and anti-intuitionistic fuzzy ideals in BCK/BCI-algebras. Bejet al., Ann. Fuzzy Math. notify In 2003, Zhan and Tan [6] newly uncertainty fuzzy T-ideals in BCK-algebras and in the current long-ago in 2010, Satyanarayana et al. [7] introduced intuitionistic fuzzy T-ideals in BCK-algebras respectively and also more than a few investigate properties of these concepts are calculated. Following [6] and [7], we are going to initiate the conception of anti-intuitionistic fuzzy T-ideals in BCK/BCI-algebras. After an elaborate learn of its properties, we move toward to this ending that in BCK/BCI-algebras, an intuitionistic fuzzy subset is an uncertainty intuitionistic fuzzy H-ideal if and only if the complement of this intuitionistic fuzzy subset is an intuitionistic fuzzy H-ideal. Relatives amongst uncertainty intuitionistic fuzzy ideals and uncertainty intuitionistic fuzzy H-ideals are also in conclusion investigated.

Following the beginning of fuzzy sets by Zadeh, here enclose be a quantity of simplification of this basic formation. The idea of intuitionistic fuzzy sets presented by Atanassov [3] is once amongst them. Fuzzy sets present a degree of members of a component in a certain set; although intuitionistic fuzzy sets present both degrees of members and non-members. Together degrees belong to the period [0; 1] along with their computation must not go above 1.

In 1991 Xi [8] theoretical beginning of fuzzy sets to BCK-algebras. In 1993 Jun and Ahmad [9] worked it to BCI-algebras. Thereafter that Jun, Meng and Liu [10] more than a few researchers investigate auxiliary properties of fuzzy sub algebras and ideals in BCK/BCI-algebras. In 1999, Khalid et al [4] introduced fuzzy H-ideals in BCI - algebras. In [11], Zhan and Tan discussed classification of fuzzy H-ideals and uncertainty fuzzy H-ideals in BCK-algebras. Newly, Satyanarayana [7] initiated intuitionistic fuzzy H-ideals in BCK-algebras.

Schein [12] developed a set of functions closed under the composition "$\circ$" and hence $(X,\circ)$ is a function semi group and the set theoretical subtraction"-" (and hence (X, -) is a subtraction algebra in the sense of Abbot (1969)). Zelinka [13] solved the problem for subtraction algebras of an extraordinary type are called the atomic subtraction algebras. Jun, Kim, and Roh [14] introduced the ideas of ideals in subtraction algebras and discussed classification of ideals. To learn more about subtraction algebras see Ceven [15], Jun and Kim [16].The fuzzifications of ideals in subtraction algebras were discussed in Lee and Park [17]. Ragavan et al. [18] introduced some new results on intuitionistic fuzzy H-ideal in BCI - algebra and discussed several examples.

The conception of fuzzy translations in fuzzy sub algebras and ideals in BCK/BCI-algebras has been discussed correspondingly by Lee and Jun [18].They explored relatives amongst fuzzy translations, fuzzy extensions and fuzzy multiplications. Motivated by this in the authors contain considered fuzzy translations of fuzzy H-ideals in BCK/BCI-algebras. They also extend this learning from fuzzy translations to intuitionistic fuzzy translations in BCK/BCI-algebras.

In this manuscript, anti-intuitionistic fuzzy translations, anti-intuitionistic fuzzy elongation and intuitionistic fuzzy accumulations of anti-intuitionistic fuzzy T-ideals in Subtraction BCK/BCI-algebras are disputed. Relations amongst intuitionistic fuzzy translations, intuitionistic fuzzy elongation and intuitionistic fuzzy accumulations of anti-intuitionistic fuzzy T-ideals in BCK/BCI-algebras are also investigated.

2. Preliminaries

Definition 2.1. *An algebra $X(X,-,0)$ of type $(2,0)$ is called a Subtraction BCK/BCI-Algebra if foe every $x,y,z \in X$ satisfies,*

(BCK-1) $((x-y)-(x-z))-(z-y)=0$;

(BCK-2) $(x-(x-y))-y=0$;

(BCK-3) $x-x=0$;

(BCK-4) $0-x=0$;

(BCK-5) $x-y=0$ *and* $y-x=0$ *imply* $x=y$.

Any subtraction BCK/BCI-Algebra satisfies the following conditions for every $x,y,z \in X$,

(I) $x-0=x$;

(II) $(x-y)-z=(x-z)-y$;

(III) $x \leq y$ *imply* $x-z \leq y-z$ *and* $z-y \leq z-x$;

(IV) $(x-z)-(y-z) \leq x-y$;

where $x \leq y$ if and only if $x-y=0$.

Definition 2.2. *A non-empty subset S of X is sub-algebra of X if $x-y \in S$ for any $x,y \in S$.*

Definition 2.3. *A non-empty subset I of X is said to be a T-ideal of X if it satisfies $(I1)$ and $(I3)(x-y)-z \in I$ and $y \in I$ imply $x-z \in I$ for all $x,y,z \in X$.*

Example 2.4. *Let $X=\{0,a,b,c\}$ be a BCI-algebra with the following Cayley's table and $A=\{0,a,c\}$ is T-ideal of X.*

$-$	0	b	b	c
0	0	c	0	a
a	a	0	a	c
b	b	c	0	a
c	c	a	c	0

Definition 2.5. *A BCI-algebra is associative if $(x-y)-z=x-(y-z)$ for all $x,y,z \in X$.*

Definition 2.6. *A fuzzy set $A=\{<x,\mu_A(x)>: x \in X\}$ in X is called an anti-fuzzy sub algebra of X if it satisfies the inequality $\mu_A(x-y) \leq \max\{\mu_A(x),\mu_A(y)\}$ for all $x,y \in X$.*

Definition 2.7. *A fuzzy set $A=\{<x,\mu_A(x)>: x \in X\}$ in X is an anti-fuzzy ideal of X if it satisfies $(AFI1)\mu_A(0) \leq \mu_A(x)$ and $(AFI2)\mu_A(x) \leq \max\{\mu_A(x-y),\mu_A(y)\}$ for all $x,y \in X$.*

Definition 2.8. *A fuzzy set $A=\{<x,\mu_A(x)>: x \in X\}$ in X is a anti fuzzy T-ideal of X if it satisfies $(AFI1)$ and $(AFI3)\mu_A(x-z) \leq \max\{\mu_A((x-y)-z),\mu_A(y)\}$ for all $x,y,z \in X$.*

Definition 2.9. *An intuitionistic fuzzy set $A=\{<x,\mu_A(x),\nu_A(x)>: x \in X\}$ in X is an anti-intuitionistic fuzzy sub algebra of X if it satisfies the following two conditions:*

(AIFI4) $\mu_A(x-y) \leq \max\{\mu_A(x),\mu_A(y)\}$ *and*

(AIFI5) $\nu_A(x-y) \geq \min\{\nu_A(x),\nu_A(y)\}$ *for all* $x,y \in X$.

Definition 2.10. *An intuitionistic fuzzy set $A=\{<x,\mu_A(x),\nu_A(x)>: x \in X\}$ in X is called an anti-intuitionistic fuzzy ideal of X if it satisfies;*

(AIFI6) $\mu_A(0) \leq \mu_A(x), \nu_A(0) \geq \nu_A(x)$,

(AIFI7) $\mu_A(x) \leq \max\{\mu_A(x-y),\mu_A(y)\}$ *and*

(AIFI8) $\nu_A(x) \geq \min\{\nu_A(x-y),\nu_A(y)\}$ *for all* $x,y \in X$.

Definition 2.11. *An intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$ in X is called an anti-intuitionistic fuzzy T-ideal of X if it satisfies $(AIFI6)$ and $(AIFI9)\mu_A(x-z) \leq \max\{\mu_A((x-y)-z), \mu_A(y)\}(AIFI10)\nu_A(x-z) \geq \min\{\nu_A((x-y)-z), \nu_A(y)\}$ for all $x,y,z \in X$.*

Remark 2.12. *The symbol $A = (\mu_A, \nu_A)$ is used for the intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$. Throughout this chapter, $\mathfrak{T} = \sup\{\nu_A(x)|x \in X\}$ for any anti-intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of X.*

Definition 2.13. *Let $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of X and $\alpha \in [0, \mathfrak{T}]$. An object having the form $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ is called an anti-intuitionistic fuzzy α– translation of A if $(\mu_A)^T_\alpha(x) = \mu_A(x) + \alpha$ and $(\nu_A)^T_\alpha(x) = \nu_A(x) - \alpha$, for all $x \in X$.*

Theorem 2.14. *If $A = (\mu_A, \nu_A)$ is an anti-intuitionistic fuzzy T-ideal of X, then the intuitionistic fuzzy α–translation $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X for all $\alpha \in [0, \mathfrak{T}]$*

Proof. Let $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy T-ideal of X and $\alpha \in [0, \mathfrak{T}]$. Then $(\mu_A)^T_\alpha(0) = \nu_A(0) + \alpha \leq \mu_A(x) + \alpha = (\mu_A)^T_\alpha(x)$ and $(\nu_A)^T_\alpha(0) = \nu_A(0) - \alpha \geq \nu_A(x) - \alpha = (\nu_A)^T_\alpha(x)$ for all $x \in X$. Now,

$$\begin{aligned}(\mu_A)^T_\alpha(x-z) &= \mu_A(x-z) + \alpha \\ &\leq \max\{\mu_A((x-y)-z), \mu_A(y)\} + \alpha \\ &\leq \max\{\mu_A((x-y)-z) + \alpha, \mu_A(y) + \alpha\} \\ &= \max\{(\mu_A)^T_\alpha((x-y)-z), (\mu_A)^T_\alpha(y)\},\end{aligned}$$

and

$$\begin{aligned}(\nu_A)^T_\alpha(x-z) &= \nu_A(x-z) - \alpha \\ &\geq \min\{\nu_A((x-y)-z), \nu_A(y)\} - \alpha \\ &= \min\{\nu_A((x-y)-z) - \alpha, \nu_A(y) - \alpha\} \\ &= \min\{(\nu_A)^T_\alpha((x-y)-z), (\nu_A)^T_\alpha(y)\}\end{aligned}$$

for all $x,y,z \in X$. Hence, an intuitionistic fuzzy α-translation $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X.

□

Theorem 2.15. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of X such that the intuitionistic fuzzy α-translation $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X for some $\alpha \in [0, \mathfrak{T}]$. Then $A = (\mu_A, \nu_A)$ is an anti-intuitionistic fuzzy T-ideal of X.*

Proof. Assume that $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ is an intuitionistic fuzzy T-ideal of X for some $\alpha \in [0, \mathfrak{T}]$. It follows that

$$\begin{aligned}\mu_A(0) + \alpha &= (\mu_A)^T_\alpha(0) \leq (\nu_A)^T_\alpha(x) \\ &= \mu_A(x) + \alpha\end{aligned}$$

and

$$\begin{aligned}\nu_A(0) - \alpha &= (\nu_A)^T_\alpha(0) \geq (\nu_A)^T_\alpha(x) \\ &= \nu_A(x) - \alpha\end{aligned}$$

for all $x \in X$, which implies $\mu_A(0) \leq \mu_A(x)$ and $\nu_A(0) \geq \nu_A(x)$.

It finds that

$$\begin{aligned}\mu_A(x-z) + \alpha &= (\mu_A)^T_\alpha(x-z) \\ &\leq \max\{(\mu_A)^T_\alpha((x-y)-z), (\mu_A)^T_\alpha(y)\} \\ &\leq \max\{\mu_A((x-y)-z) + \alpha, (\mu_A)^T_\alpha(y) + \alpha\} \\ &\leq \max\{\mu_A((x-y)-z), (\mu_A)^T_\alpha(y)\} + \alpha,\end{aligned}$$

and

$$\begin{aligned}\nu_A(x-z) - \alpha &= (\nu_A)^T_\alpha(x-z) \\ &\geq \min\{(\nu_A)^T_\alpha((x-y)-z), (\nu_A)^T_\alpha(y)\} \\ &= \min\{\nu_A((x-y)-z) - \alpha, \nu_A(y) - \alpha\} \\ &= \min\{\nu_A((x-y)-z), \nu_A(y)\} - \alpha,\end{aligned}$$

which implies, $\mu_A(x-z) \leq \max\{\mu_A((x-y)-z), \mu_A(y)\}$ and $\nu_A(x-z) \geq \min\{\nu_A((x-y)-z), \nu_A(y)\}$ for all $x,y,z \in X$. Hence $A = (\mu_A, \nu_A)$ is an anti-intuitionistic fuzzy T-ideal of X. □

Theorem 2.16. *If the intuitionistic fuzzy α-translation $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X for some $\alpha \in [0, \mathfrak{T}]$ then it must be an anti-intuitionistic fuzzy ideal of X.*

Proof. Let the intuitionistic fuzzy α-translation $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X. Then we have $(\mu_A)^T_\alpha(x-z) \leq \max\{(\mu_A)^T_\alpha((x-y)-z), (\mu_A)^T_\alpha(y)\}$ and $(\nu_A)^T_\alpha(x-z) \geq \min\{(\nu_A)^T_\alpha((x-y)-z), (\nu_A)^T_\alpha(y)\}$, for all $x,y,z \in X$. Substituting $z = 0$, it gets that

$$\begin{aligned}(\mu_A)^T_\alpha(x-0) &\leq \max\{(\mu_A)^T_\alpha((x-y)-0), (\mu_A)^T_\alpha(y)\} \\ (\mu_A)^T_\alpha(x) &\leq \max\{(\mu_A)^T_\alpha(x-y), (\mu_A)^T_\alpha(y)\}\end{aligned}$$

and

$$\begin{aligned}(\nu_A)^T_\alpha(x-0) &\geq \min\{(\nu_A)^T_\alpha((x-y)-0), (\nu_A)^T_\alpha(y)\} \\ &\geq \min\{(\nu_A)^T_\alpha(x-y), (\nu_A)^T_\alpha(y)\}\end{aligned}$$

Therefore $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ is an anti-intuitionistic fuzzy ideal of X.

□

Theorem 2.17. *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy subset of X such that the intuitionistic fuzzy α-translation $A^T_\alpha = ((\mu_A)^T_\alpha, (\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X for some $\alpha \in [0, \mathfrak{T}]$. If $(x-b)-a = 0$ for all $x,a,b \in X$, then $(\mu_A)^T_\alpha(x) \leq \max\{(\mu_A)^T_\alpha(a), (\nu_A)^T_\alpha(b)\}$ and $(\nu_A)^T_\alpha(x) \geq \min\{(\nu_A)^T_\alpha(a), (\nu_A)^T_\alpha(b)\}$.*

Proof. Let $x,a,b \in X$ such that $(x-b)-a=0$. Then

$$\begin{aligned}&(\mu_A)^T_\alpha(x)\\&\le \max\{(\mu_A)^T_\alpha(x-a),(\mu_A)^T_\alpha(a)\}\\&\le \max\{\max\{(\mu_A)^T_\alpha((x-b)-a),(\mu_A)^T_\alpha(b)\},(\mu_A)^T_\alpha(a)\}\\&=\max\{\max\{(\mu_A)^T_\alpha(0),(\mu_A)^T_\alpha(b)\},(\mu_A)^T_\alpha(a)\}\\&=\max\{(\mu_A)^T_\alpha(b),(\mu_A)^T_\alpha(a)\}.\end{aligned}$$

Since $(\mu_A)^T_\alpha(0)\le(\mu_A)^T_\alpha(b)$ then it implies that $(\mu_A)^T_\alpha(a)\le \max\{(\mu_A)^T_\alpha(a),(\mu_A)^T_\alpha(b)\}$ and

$$\begin{aligned}&(\nu_A)^T_\alpha(x)\\&\ge \min\{(\nu_A)^T_\alpha(x-a),(\nu_A)^T_\alpha(a)\}\\&\ge \min\{\min\{(\nu_A)^T_\alpha((x-b)-a),(\nu_A)^T_\alpha(b)\},(\mu_A)^T_\alpha(a)\}\\&=\min\{\min\{(\nu_A)^T_\alpha(0),(\nu_A)^T_\alpha(b)\},(\nu_A)^T_\alpha(a)\}\\&=\min\{(\nu_A)^T_\alpha(b),(\nu_A)^T_\alpha(a)\}.\end{aligned}$$

Since $(\nu_A)^T_\alpha(0)\ge(\nu_A)^T_\alpha(b)$ then it becomes that $(\nu_A)^T_\alpha(x)\ge \min\{(\nu_A)^T_\alpha(a),(\nu_A)^T_\alpha(b)\}$.

□

Corollary 2.18. *Let $A=(\mu_A,\nu_A)$be an intuitionistic fuzzy subset of X such that the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X for some $\alpha\in[0,\mathfrak{T}]$. If $(\ldots(x-a_1)-\ldots)-a_n=0$ for all $x,a_1,a_2,a_n\in$, then*

$$(\mu_A)^T_\alpha(x)\le \max\{(\mu_A)^T_\alpha(a_1),(\mu_A)^T_\alpha(a_2),\ldots,(\mu_A)^T_\alpha(a_n)\}$$

and $(\nu_A)^T_\alpha(x)\ge \min\{(\nu_A)^T_\alpha(a_1),(\nu_A)^T_\alpha(a_2),\ldots,(\nu_A)^T_\alpha(a_n)\}$.

3. Intuitionistic fuzzy α-translation of intuitionistic fuzzy T-ideal

Theorem 3.1. *Let $A=(\mu_A,\nu_A)$ is an intuitionistic fuzzy subset of X such that the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T- ideal of X for $\alpha\in[0,\mathfrak{T}]$. If it satisfies the condition $(\mu_A)^T_\alpha(x-y)\le(\mu_A)^T_\alpha(x)$ and $(\nu_A)^T_\alpha(x-y)\ge(\nu_A)^T_\alpha(x)$ for all $x,y\in X$, then the intuitionistic fuzzy α-translation A^α_T of A is an anti-intuitionistic fuzzy T- ideal of X.*

Proof. Let the intuitionistic fuzzy α-translation A^T_α of A is anti-intuitionistic fuzzy T- ideal of X. For any $x,y,z\in X$. it gives that

$$\begin{aligned}&(\mu_A)^T_\alpha(x-z)\\&\le \max\{(\mu_A)^T_\alpha((x-z)-(y-z)),(\mu_A)^T_\alpha(y-z)\}\\&=\max\{(\mu_A)^T_\alpha((x-(y-z))-z),(\mu_A)^T_\alpha(y-z)\}\\&\le \max\{(\mu_A)^T_\alpha((x-y)-z),(\mu_A)^T_\alpha(y)\}\end{aligned}$$

and

$$\begin{aligned}&(\nu_A)^T_\alpha(x-z)\\&\ge \min\{(\nu_A)^T_\alpha((x-z)-(y-z)),(\nu_A)^T_\alpha(y-z)\}\\&=\min\{(\nu_A)^T_\alpha((x-(y-z))-z),(\nu_A)^T_\alpha(y-z)\}\\&\ge \min\{(\nu_A)^T_\alpha((x-y)-z),(\nu_A)^T_\alpha(y)\}\end{aligned}$$

Hence, the intuitionistic fuzzy α-translation A^T_α of A is an anti-intuitionistic fuzzy T- ideal of X for some $\alpha\in[0,\mathfrak{T}]$. □

Theorem 3.2. *If $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy subset of associative BCI-algebra X such that the intuitionistic fuzzy α- translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T- ideal of X for $\alpha\in[0,\mathfrak{T}]$ then the intuitionistic fuzzy α- translation A^T_α of A is an anti-intuitionistic fuzzy T- ideal of X.*

Proof. Let intuitionistic fuzzy α-translation A^T_α of A is an anti-intuitionistic fuzzy T- ideal of X. For any $x,y,z\in X$, it gets that

$$\begin{aligned}(\mu_A)^T_\alpha(x-z)&\ge \min\{(\mu_A)^T_\alpha((x-z)-y),(\mu_A)^T_\alpha(y)\}\\&\ge \min\{(\mu_A)^T_\alpha((x-y)-z),(\mu_A)^T_\alpha(y)\}\end{aligned}$$

and

$$\begin{aligned}(\nu_A)^T_\alpha(x-z)&\le \max\{(\nu_A)^T_\alpha((x-z)-y),(\nu_A)^T_\alpha(y)\}\\&\le \max\{(\nu_A)^T_\alpha((x-y)-z),(\nu_A)^T_\alpha(y)\}.\end{aligned}$$

Hence, the intuitionistic fuzzy α-translation A^T_α of A is an anti-intuitionistic fuzzy T- ideal of X.

□

Theorem 3.3. *If $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy subset of X such that the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X for $\alpha\in[0,\mathfrak{T}]$ then the sets $T_{\mu_A}=\{x|x\in X$ and $(\mu_A)^T_\alpha(x)=(\mu_A)^T_\alpha(0)\}$ and $T_{\nu_A}=\{x|x\in X$ and $(\nu_A)^T_\alpha(x)=(\nu_A)^T_\alpha(0)\}$ are T-ideals of X.*

Proof. Suppose that $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ is an anti-intuitionistic fuzzy ideal of X.

Then $(\mu_A)^T_\alpha$ and $(\nu_A)^T_\alpha$ are an anti-fuzzy T-ideal of X. Obviously $0\in T\mu_A, T_{\nu_A}$.

Let $x,y,z\in X$ be such that $(x-y)-z\in T_{\mu_A}$ and $y\in T_{\nu_A}$. Then $(\mu_A)^T_\alpha((x-y)-z)=(\mu_A)^T_\alpha(0)$ and $(\mu_A)^T_\alpha(y)=(\mu_A)^T_\alpha(0)$ and so

$$\begin{aligned}(\mu_A)^T_\alpha(x-z)&\le \max\{(\mu_A)^T_\alpha((x-y)-z),(\mu_A)^T_\alpha(y)\}\\&=\max\{(\mu_A)^T_\alpha(0),(\mu_A)^T_\alpha(0)\}\\&\le(\mu_A)^T_\alpha(0).\end{aligned}$$

Since $(\mu_A)^T_\alpha$ is an anti-fuzzy T-ideal.

$$(\mu_A)^T_\alpha(0)\le(\mu_A)^T_\alpha(x-z).$$

It concludes that $(\mu_A)^T_\alpha(x-z)=(\mu_A)^T_\alpha(0)$.

This implies, $\mu_A(x-z)+\alpha=\mu_A(0)+\alpha, \mu_A(x-z)=\mu_A(0)$ so that $x-z\in T_{\mu_A}$.

Therefore T_{μ_A} is an anti T-ideal.

Again, let $a,b,c\in X$ be such that $(a-b)-c\in T_{\nu_A}$ and $b\in T_{\nu_A}$.

Then $(\nu_A)^T_\alpha((a-b)-c)=(\nu_A)^T_\alpha(0)$
and $(\nu_A)^T_\alpha(b)=(\nu_A)^T_\alpha(0)$ and so

$$(\nu_A)^T_\alpha(a-c)\geq \min\{(\nu_A)^T_\alpha((a-b)-c),(\nu_A)^T_\alpha(b)\}$$
$$(\nu_A)^T_\alpha(a-c)\geq \min\{(\nu_A)^T_\alpha(0),(\nu_A)^T_\alpha(0)\}$$
$$(\nu_A)^T_\alpha(a-c)\geq (\nu_A)^T_\alpha(0).$$

Since, $(\nu_A)^T_\alpha$ is a fuzzy T-ideal $(\nu_A)^T_\alpha(0)\geq(\nu_A)^T_\alpha(x-z)$. Also, $(\nu_A)^T_\alpha((a-c)=(\nu_A)^T_\alpha(0)$.

This implies, $\nu_A(a-c)-\alpha=\nu_A(0)-\alpha, \nu_A(a-b)=\nu_A(0)$ so that $a-c\in T_{\nu_A}$.

Therefore, T_{ν_A} is an anti T-ideal.

□

Theorem 3.4. *Let the intuitionistic fuzzy α- translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T- ideal of X for $\alpha\in[0,\mathfrak{T}]$. If $x\geq y$, then $(\mu_A)^T_\alpha(x)\leq(\mu_A)^T_\alpha(y)$ and $(\nu_A)^T_\alpha(x)\geq(\nu_A)^T_\alpha(y)[(\mu_A)^T_\alpha$ is order-reversing and $(\nu_A)^T_\alpha$ is order-preserving].*

Proof. Let $x,y,z\in X$ such that $x\geq y$. Then $x-y=0$ and Hence,

$$\begin{aligned}(\mu_A)^T_\alpha(x)&=(\mu_A)^T_\alpha(x-0)\\&\leq\max\{(\mu_A)^T_\alpha((x-y)-0),(\mu_A)^T_\alpha(y)\}\\&=\max\{(\mu_A)^T_\alpha(x-y),(\mu_A)^T_\alpha(y)\}\\&=\max\{(\mu_A)^T_\alpha(0),(\mu_A)^T_\alpha(y)\}\\&=(\mu_A)^T_\alpha(y),\end{aligned}$$

and

$$\begin{aligned}(\nu_A)^T_\alpha(x)&=(\nu_A)^T_\alpha(x-0)\\&\geq\min\{(\nu_A)^T_\alpha((x-y)-0),(\nu_A)^T_\alpha(y)\}\\&=\min\{(\nu_A)^T_\alpha(x-y),(\nu_A)^T_\alpha(y)\}\\&=\min\{(\nu_A)^T_\alpha(0),(\nu_A)^T_\alpha(y)\}\\&=(\nu_A)^T_\alpha(y).\end{aligned}$$

□

Theorem 3.5. *Let $A=(\mu_A,\nu_A)$ is an intuitionistic fuzzy subset of X such that the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T- ideal of X for $(\alpha\in[0,\mathfrak{T}]$. Then the following assertions are equivalent:*

(i) *A^T_α is an anti-intuitionistic fuzzy T- ideal of X.*

(ii) *$(\mu_A)^T_\alpha(x-y)\leq(\mu_A)^T_\alpha((x-0)-y)$ and $(\nu_A)^T_\alpha(x-y)\geq(\nu_A)^T_\alpha((x-0)-y)$ for all $x,y\in X$.*

(iii) *$(\mu_A)^T_\alpha((x-y)-z)\leq(\mu_A)^T_\alpha((\mu_A)^T_\alpha(x-0)-y)$ and*

$$(\nu_A)^T_\alpha((x-y)-z)\geq(\nu_A)^T_\alpha((x-0)-y)$$

for all $x,y,z\in X$.

Proof. Assume that $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ is an anti-intuitionistic fuzzy ideal of X. Then $(\mu_A)^T_\alpha$ and $(\nu_A)^T_\alpha$ are fuzzy T-ideals of X. $(i)\Rightarrow(ii)$

Let A^T_α is an anti-intuitionistic fuzzy T-ideal of X. Then for all $x,y\in X$. It gives that

$$\begin{aligned}(\mu_A)^T_\alpha(x-y)&\leq\max\{(\mu_A)^T_\alpha((x-0)-y),(\mu_A)^T_\alpha(0)\}\\&=(\mu_A)^T_\alpha((x-0)-y),\end{aligned}$$

and

$$\begin{aligned}(\nu_A)^T_\alpha(x-y)&\geq\min\{(\nu_A)^T_\alpha((x-0)-y),(\nu_A)^T_\alpha(0)\}\\&=(\nu_A)^T_\alpha((x-0)-y),\end{aligned}$$

for all $x,y\in X$. Therefore, the inequality (ii) is satisfied.

Thus $(ii)\Rightarrow(iii)$ is got.

Assume that (ii) is satisfied. For all $x,y,z\in X$.

Now,

$$\begin{aligned}&((x-y)-(0-z))((x-y)-z)\\&=((x-y)-((x-y)-z))-(0-z)\\&\leq((y-z)-y)-(0-z)\\&=((y-y)-z)-(0-z)\\&=(0-z)-(0-z)\\&=0.\end{aligned}$$

It follows that

$$(\mu_A)^T_\alpha((x-y)-(0-z))-((x-y)-z)\leq(\mu_A)^T_\alpha(0)$$

and

$$(\nu_A)^T_\alpha((x-y)-(0-z))-((x-y)-z)\geq(\nu_A)^T_\alpha(0)$$

.

Since $(\mu_A)^T_\alpha$ and $(\nu_A)^T_\alpha$ are anti-fuzzy T-ideal of X.

Therefore, it implies that

$$(\mu_A)^T_\alpha(((x-y)-(0-z))-((x-y)-z))=(\mu_A)^T_\alpha(0)$$

and

$$(\nu_A)^T_\alpha(((x-y)-(0-z))-((x-y)-z))=(\nu_A)^T_\alpha(0)$$

. Using (ii), it gets that

$$\begin{aligned}
&(\mu_A)^T_\alpha((x-y)-z)\\
&\leq (\mu_A)^T_\alpha((x-y)-(0-z))\\
&= \max\{(\mu_A)^T_\alpha((x-y)-(0-z))-((x-y)-z),\\
&\quad (\mu_A)^T_\alpha((x-y),z)\}\\
&= \max\{(\mu_A)^T_\alpha(0),(\mu_A)^T_\alpha((x-y)-z)\}\\
&= (\mu_A)^T_\alpha((x-y)-z)\\
&(\nu_A)^T_\alpha((x-y)-z)\\
&\geq (\nu_A)^T_\alpha((x-y)-(0-z))\\
&= \min\{(\nu_A)^T_\alpha((x-y)-(0-z))-((x-y)-z),\\
&\quad (\nu_A)^T_\alpha(x-(y-z))\}\\
&= \min\{(\nu_A)^T_\alpha(0),(\nu_A)^T_\alpha((x-y)-z)\}\\
&= (\nu_A)^T_\alpha((x-y)-z).
\end{aligned}$$

Therefore, inequality (iii) is also satisfied. Then $(iv)\Rightarrow(i)$ is obtained. Assume that (iii) is satisfied.

For all $x,y,z\in X$, it followsthat

$$\begin{aligned}
(\mu_A)^T_\alpha(x-z) &\leq \max\{(\mu_A)^T_\alpha((x-z)-y),(\mu_A)^T_\alpha(y)\}\\
&\leq \max\{(\mu_A)^T_\alpha((x-y)-z),(\mu_A)^T_\alpha(y)\},
\end{aligned}$$

and

$$\begin{aligned}
(\nu_A)^T_\alpha(x-z) &\geq \min\{(\nu_A)^T_\alpha((x-z)-y),(\nu_A)^T_\alpha(y)\}\\
&\geq \min\{(\nu_A)^T_\alpha((x-y)-z),(\nu_A)^T_\alpha(y)\}.
\end{aligned}$$

Therefore, $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ is an anti-intuitionistic fuzzy T- ideal of X.

□

Theorem 3.6. *Let $A=(\mu_A,\nu_A)$ is an intuitionistic fuzzy subset of X such that the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T- ideal of X for $\alpha\in[0,\mathfrak{T}]$. Then the following assertions are equivalent:*

(i) *A^T_α is an anti-intuitionistic fuzzy T- ideal of X.*

(ii)

$$(\mu_A)^T_\alpha((x-z)-y)\leq(\mu_A)^T_\alpha((x-z)-(y-0))$$

and

$$(\nu_A)^T_\alpha((x-z)-y)\geq(\nu_A)^T_\alpha((x-z)-(y-0))$$

for all $x,t\in X$.

(iii)

$$(\mu_A)^T_\alpha(x-y)\leq\min\{(\mu_A)^T_\alpha((x-z)-(y-0)),(\mu_A)^T_\alpha(z)\}$$

and

$$(\nu_A)^T_\alpha(x-y)\geq\max\{(\nu_A)^T_\alpha((x-z)-(y-0)),(\nu_A)^T_\alpha(z)\}$$

for all $x,y,z\in X$.

Proof. $(i)\Rightarrow(ii)$ is the same as the above theorem (3.5).

For $(ii)\Rightarrow(iii)$, assume that (ii) is valid. For all $x,y,z\in X$, it gives that

$$\begin{aligned}
&(\mu_A)^T_\alpha(x-y)\\
&\leq \max\{(\mu_A)^T_\alpha((x-y)-z),(\mu_A)^T_\alpha(z)\}\\
&= \max\{(\mu_A)^T_\alpha((x-z)-y),(\mu_A)^T_\alpha(z)\}\\
&\leq \max\{(\mu_A)^T_\alpha((x-z)-(y-0)),(\mu_A)^T_\alpha(z)\},
\end{aligned}$$

and

$$\begin{aligned}
&(\nu_A)^T_\alpha(x-y)\\
&\geq \min\{(\nu_A)^T_\alpha((x-y)-z),(\nu_A)^T_\alpha(z)\}\\
&= \min\{(\nu_A)^T_\alpha((x-z)-y),(\nu_A)^T_\alpha(z)\}\\
&\geq \min\{(\nu_A)^T_\alpha((x-z)-(y-0)),(\nu_A)^T_\alpha(z)\},
\end{aligned}$$

Therefore, inequality (iii) is also satisfied.

For $(iii)\Rightarrow(i)$, assume that (iii) is satisfied. Therefore, for all $x,y,z\in X$. we have,

$$\begin{aligned}
&(\mu_A)^T_\alpha(x-y)\\
&\leq \max\{(\mu_A)^T_\alpha((x-z)-(y-0)),(\mu_A)^T_\alpha(z)\}
\end{aligned}$$

and

$$\begin{aligned}
&(\nu_A)^T_\alpha(x-y)\\
&\geq \min\{(\nu_A)^T_\alpha((x-z)-(y-0)),(\nu_A)^T_\alpha(z)\}.
\end{aligned}$$

Putting $z=0$, it gets that

$$\begin{aligned}
&(\mu_A)^T_\alpha(x-y)\\
&\leq \max\{(\mu_A)^T_\alpha((x-0)-(y-0)),(\mu_A)^T_\alpha(0)\}\\
&= \max\{(\mu_A)^T_\alpha((x-0)-y),(\mu_A)^T_\alpha(0)\}\\
&= (\mu_A)^T_\alpha((x-0)-y),
\end{aligned}$$

and

$$\begin{aligned}
&(\nu_A)^T_\alpha(x-y)\\
&\geq \min\{(\nu_A)^T_\alpha((x-0)-(y-0)),(\nu_A)^T_\alpha(0)\}\\
&= \min\{(\nu_A)^T_\alpha((x-0)-y),(\nu_A)^T_\alpha(0)\}\\
&= (\nu_A)^T_\alpha((x-0)-y).
\end{aligned}$$

It follows from (3.5) that A^T_α is an anti-intuitionistic fuzzy T-ideal of X.

□

4. Properties on intuitionistic fuzzy T-ideal Extension

Definition 4.1. *Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two intuitionistic fuzzy subset of X. If $A\leq B,\mu_A(x)\leq\mu_B(x)$ and $\nu_A(x)\geq\nu_B(x)$ for all $x\in X$, then B is an anti-intuitionistic fuzzy extension of A.*

Definition 4.2. *Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be two intuitionistic fuzzy subset of X. Then B is an anti-intuitionistic fuzzy T-ideal extension of A if the following assertions are valid:*

(i) B is an anti-intuitionistic fuzzy extension of A.

(ii) If A is an anti-intuitionistic fuzzy T-ideal of X, then B is an anti-intuitionistic fuzzy T-ideal of X. From the definition of intuitionistic fuzzy α-translation, we get $(\mu_A)^T_\alpha(x)=\mu_A(x)+\alpha$ and $(\nu_A)^T_\alpha(x)=\nu_A(x)-\alpha$ for all $x\in X$.

Theorem 4.3. *Let $A(\mu_A,\nu_A)$ is an anti-intuitionistic fuzzy T- ideal of X and $\alpha\in[0,\mathfrak{T}]$. Then the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T- ideal extension of A. An anti-intuitionistic fuzzy T-ideal extension of an anti-intuitionistic fuzzy T-ideal A may not be represented as an intuitionistic fuzzy α-translation of A, that is, the converse of Theorem is not true in general as seen in the following example.*

Example 4.4. *Let $X=\{0,1,2,3,4\}$ is a BCI-algebra with the following Cayley table:*

-	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	1	0

Let $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy subset of X defined by

X	0	1	2	3	4
μ_A	0.27	0.32	0.40	0.55	0.55
ν_A	0.72	0.64	0.55	0.41	0.41

Then $A=(\mu_A,\nu_A)$ be an anti-intuitionistic fuzzy T-ideal of X.

Let $B=(\mu_B,\nu_B)$ be an intuitionistic fuzzy subset of X defined by

X	0	1	2	3	4
μ_B	0.25	0.28	0.37	0.51	0.51
ν_B	0.74	0.70	0.58	0.45	0.45

Then $B=(\mu_B,\nu_B)$ is an anti-intuitionistic fuzzy T-ideal extension of A. But it is not the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A for all $\alpha\in[0,\mathfrak{T}]$.

If $\alpha=0.16$ then $\alpha=0.16>0.13=\beta$ and the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is given as follows.

X	0	1	2	3	4
$(\mu_A)^T_\alpha$	0.27	0.41	0.52	0.41	0.52
$(\nu_A)^T_\alpha$	0.71	0.58	0.50	0.58	0.50

But $(\mu_A)^T_\alpha(x)+(\nu_A)^T_\alpha(x)\nleq 1$. Therefore B is not the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A for all $\alpha\in[0,\mathfrak{T}]$.

Theorem 4.5. *Let $A=(\mu_A,\nu_A)$ is an anti-intuitionistic fuzzy T- ideal of X and $\alpha\in[0,\mathfrak{T}]$. Then the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal extension of A. Then intersection of anti-intuitionistic fuzzy T-ideal extensions of an anti-intuitionistic fuzzy T-ideal A of X is an anti-intuitionistic fuzzy T- ideal extension of A. But the union of anti-intuitionistic fuzzy T-ideal extensions of an anti-intuitionistic fuzzy T-ideal A of X is not an anti-intuitionistic fuzzy T-ideal extension of a as seen in the following example.*

Example 4.6. *Let $X=\{0,1,2,3,4\}$ is a BCI-algebra with the following Cayley table:*

-	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

Let $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy subset of X defined by

X	0	1	2	3	4
μ_A	0.35	0.58	0.58	0.58	0.58
ν_A	0.63	0.41	0.41	0.41	0.41

Then $A=(\mu_A,\nu_A)$ be an anti-intuitionistic fuzzy T-ideal of X.

Let $B=(\mu_B,\nu_B)$ and $C=(\mu_C,\nu_C)$ be an intuitionistic fuzzy subset of X defined by

X	0	1	2	3	4
μ_B	0.22	0.33	0.54	0.54	0.33
ν_B	0.76	0.66	0.43	0.43	0.66

and

X	0	1	2	3	4
μ_C	0.21	0.45	0.41	0.45	0.45
ν_C	0.78	0.51	0.57	0.51	0.51

Then B and C are anti-intuitionistic fuzzy T-ideal extensions of A.

(1) The intersection $B\cap C$ is an anti-intuitionistic fuzzy extension of A, Since, $\mu_{B\cap C}(3-0)=\mu_{B\cap C}(3)=0.43\leq 0.43=\max\{\mu_{B\cap C}((3-2)-0),\mu_{B\cap C}(2)\}$ and $\nu_{B\cap C}(3-0)=\nu_{B\cap C}(3)=0.54\leq 0.54=\min\{\nu_{B\cap C}(1),\nu_{B\cap C}(2)\}=\min\{\nu_{B\cap C}((3-2)-0),\nu_{B\cap C}(2)\}$.

(2) The union $B\cup C$ is an anti-intuitionistic fuzzy extension of A but it is not an anti-intuitionistic fuzzy ideal extension of A. Since, $\mu_{B\cup C}(3-0)=\mu_{B\cup C}(3)=0.51\nleq 0.57=\max\{\mu_{B\cup C}((3-2)-0),\mu_{B\cup C}(2)\}$ and $\nu_{B\cup C}(3-0)=\nu_{B\cup C}(3)=0.45\ngeq 0.41=\min\{\nu_{B\cup C}(1),\nu_{B\cup C}(2)\}=\min\{\nu_{B\cup C}((3-2)-0),\nu_{B\cup C}(2)\}$.

Definition 4.7. *For an intuitionistic fuzzy subset $A=(\mu_A,\nu_A)$ of X, $\alpha\in[0,\mathfrak{T}]$ and $t,s\in[0,1]$ with $t\geq\alpha$, let $U_\alpha(\mu_A;t)=\{x|x\in X$ and $\mu_A(x)\geq t-\alpha\}$ and $L_\alpha(\nu_A;s)=\{x|x\in X$ and $\nu_A(x)\geq s+\alpha\}$.*

Theorem 4.8. *If A is an anti-intuitionistic fuzzy T-ideal of X, then it is clear that $U_\alpha(\mu_A;t)$ and $L_\alpha(\nu_A;s)$ are T-ideals of X for all $t\in Im(\mu_A)$ with $t\geq\alpha$. But if we do not give a condition that A is an anti-intuitionistic fuzzy T-ideal of X, then $U_\alpha(\mu_A;t)$ and $L_\alpha(\nu_A;s)$ are not T-ideals of X as seen in the following example.*

Example 4.9. *Let $X=\{0,1,2,3,4\}$ be a BCI-algebra in example 2 and $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy subset of X defined by*

X	0	1	2	3	4
μ_A	0.66	0.49	0.31	0.49	0.49
ν_A	0.33	0.48	0.67	0.48	0.48

Since

$$\begin{aligned}&\mu_A(3-1)=\mu_A(2)=0.31\nleq 0.49\\&=\max\{\mu_A(1),\mu_A(0)\}\\&=\min\{\mu_A((3-0)-1),\mu_A(1),\mu_A(0)\},\end{aligned}$$

and

$$\begin{aligned}&\nu_A(3-1)=\nu_A(2)=0.67\ngeq 0.48\\&=\min\{\nu_A(1),\nu_{\nu A}(0)\}\\&=\max\{\nu_A((3-0)-1),\nu_A(0)\}.\end{aligned}$$

Therefore, $A=(\mu_A,\nu_A)$ is not an anti-intuitionistic fuzzy T-ideal of X.

For $\alpha=0.16,t=0.60$ and $s=0.45$, we obtain

$$U_\alpha(\mu_A;t)=L_\alpha(\nu_A;s)=\{0,1,2,3,4\}$$

which are not T-ideals of X.

Since $((3-0)-1)=2\in\{0,1,3,4\}$ but $3-1=2\in\{0,1,3,4\}$.

5. Level sets in anti-intuitionistic fuzzy α-translation

Theorem 5.1. *Let $A=(\mu_A,\nu_A)$ is an intuitionistic fuzzy subset of X and $\alpha\in[0,\mathfrak{T}]$. Then the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is an anti-intuitionistic fuzzy T-ideal of X if only if $U_\alpha(\mu_A;t)$ and $L_\alpha(\nu_A;s)$ are T-ideals of X fot $t\in Im(\mu_A)$ and $s\in Im(\nu_A)$ with $t\geq\alpha$.*

Proof. Suppose that $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ is an anti-intuitionistic fuzzy T-ideal of X.

Then $(\mu_A)^T_\alpha$ and $(\nu_A)^T_\alpha$ are anti-fuzzy T-ideal of X.

Let $t\in Im(\mu_A)$ and $s\in Im(\nu_A)$ with $t\geq\alpha$. Since

$$(\mu_A)^T_\alpha(0)\leq(\mu_A)^T_\alpha(x)$$

for $x\in X$, it follows that

$$\begin{aligned}&\mu_A(0)+\alpha\\&=(\mu_A)^T_\alpha(0)\leq(\mu_A)^T_\alpha(x)\\&=\mu_A(x)+\alpha\leq t\end{aligned}$$

for $x\in U_\alpha(\mu_A;t)$. Hence $0\in U_\alpha(\mu_A;t)$.

Let $x,y,z\in X$ such that $(x-y)-z,y\in U_\alpha(\mu_A;t)$. Then $\mu_A((x-y)-z)\leq t-\alpha$ and $\mu_A(y)\leq t-\alpha$.

i.e., $(\mu_A)^T_\alpha((x-y)-z)=\mu_A((x-y)-z)+\alpha\leq t$ and $\mu_A(y)+\alpha$.

Since, $(\mu_A)^T_\alpha$ is a fuzzy T-ideal, therefore, we have

$$\begin{aligned}&\mu_A(x-z)+\alpha\\&=(\mu_A)^T_\alpha(x-z)\\&\leq\max\{(\mu_A)^T_\alpha((x-y)-z),(\mu_A)^T_\alpha(y)\}\\&=\min\{t,t\}\leq t,\end{aligned}$$

i.e., $\mu_A((x-y)-z)\geq t-\alpha$ so that $x-z\in U_\alpha(\mu_A;t)$.

Therefore, $U_\alpha(\mu_A;t)$ is a T-ideal of X.

Again, since $(\nu_A)^T_\alpha(0)\geq(\nu_A)^T_\alpha(x)$ for $x\in X$, it becomes that

$$\begin{aligned}(\nu_A)(0)-\alpha&=(\nu_A)^T_\alpha(0)\\&\geq(\nu_A)^T_\alpha(x)\\&=(\nu_A)(x)-\alpha.\end{aligned}$$

For $x\in L_\alpha(\nu_A;s)$. Hence, $0\in L_\alpha(\nu_A;s)$.

Let $x,y,z\in X$ such that $(x-y)-z$, and $y\in L_\alpha(\nu_A;s)$.

Then $\mu_A((x-y)-z)\geq s+\alpha$ and $\nu_A(y)\geq s+\alpha$.

So $(\nu_A)^T_\alpha((x-y)-z)=\nu_A((x-y)-z)-\alpha\geq s$ and $(\nu_A)^T_\alpha(y)=\nu_A(y)-\alpha\geq s$. Since $(\nu_A)^T_\alpha$ is a fuzzy T-ideal, therefore it gives that

$$\begin{aligned}&\nu_A(x-z)-\alpha\\&=(\nu_A)^T_\alpha(x-z)\\&\geq\min\{(\nu_A)^T_\alpha((x-y)-z),(\nu_A)^T_\alpha(y)\}\geq s.\end{aligned}$$

i.e.,$\nu_A((x-y)-z)\geq s+\alpha$ so that $x-z\in L_\alpha(\nu_A;s)$.

Therefore $L_\alpha(\nu_A;s)$ is a T-ideal of X.

Conversely, suppose that $U_\alpha(\mu_A;t)$ and $L_\alpha(\nu_A;s)$ are T-ideals of X for $t\in Im(\mu_A)$ and $s\in Im(\nu_A)$ with $t\geq\alpha$.

If there exists $u\in X$ s.t $(\mu_A)^T_\alpha(0)<(\mu_A)^T_\alpha\psi\leq(\mu_A)^T_\alpha(u)$ then $\mu_A(u)\geq\psi-\alpha$ but $\mu_A(0)<\psi-\alpha$.

This shows that $u\in U_\alpha(\mu_A;t)$ and $0\notin U_\alpha(\mu_A;t)$. This is a contradiction, and $(\mu_A)^T_\alpha(0)\leq(\mu_A)^T_\alpha(x)$ for $x\in X$.

Again, if there exists $\nu\in X$ such that $(\nu_A)^T_\alpha(0)>k\geq(\nu_A)^T_\alpha(\nu)$, then $\nu_A(\nu)\leq k+\alpha$. But $\nu_A(0)>k+\alpha$.

This shows that $\nu \in L_\alpha(\nu_A;s)$ and $0 \notin L_\alpha(\nu_A;s)$.

This is a contradiction, and $(\nu_A)^T_\alpha(0) \geq (\nu_A)^T_\alpha(x)$ for $x \in X$.

Now we assume that there exists $a,b,c \in X$ such that

$$(\mu_A)^T_\alpha(a-c) < \zeta \leq \min\{(\mu_A)^T_\alpha(a-(b-c)),(\mu_a)^T_\alpha(b)\}.$$

Then $\mu_A((a-b)-c) \geq \zeta-\alpha$ and $\mu_A(b) \geq \zeta-\alpha$ but $\mu_A(a*c) < \zeta-\alpha$.

Hence, $(a-b)-c \in U_\alpha(\mu_A;t)$ and $b \in U_\alpha(\mu_A;t)$.

But, $a-c \notin U_\alpha(\mu_A;t)$. This is a contradiction.

Therefore,

$$(\mu_A)^T_\alpha(x-z) \leq \max\{(\mu_A)^T_\alpha((x-y)-z),(\mu_A)^T_\alpha(y)\}$$

for all $x,y,z \in X$.

Again, assume that there exists $d,e,f \in X$ such that

$$(\nu_A)^T_\alpha(d-f) > \max\{(\nu_A)^T_\alpha((d-e)-f),(\nu_A)^T_\alpha(e)\},$$

then $\nu_A((d-e)-f) \geq \eta+\alpha$ and $\nu_A(e) \geq \eta+\alpha$ but $\nu_A(d-f) < \eta+\alpha$.

Hence, $((d-e)-f) \in L_\alpha(\nu_A;s)$. and $e \in L_\alpha(\nu_A;s)$.

But $d*f \notin L_\alpha(\nu_A;s)$. Which is a contradiction.

Therefore,

$$(\nu_A)^T_\alpha(x-z) \geq \max\{(\nu_a)^T_\alpha(x-(y-z)),(\nu_A)^T_\alpha(y)\}$$

for $x,y,z \in X$.

Consequently, A^T_α is an anti-intuitionistic fuzzy T-ideal of X. □

Remark 5.2. *Let $A=(\mu_A,\nu_A)$ is an anti-intuitionistic fuzzy T-ideal of X and $\alpha,\beta \in [0,\mathfrak{T}]$. If $\alpha \geq \beta$, then the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of X is an anti-anti-intuitionistic fuzzy T-ideal extension of the intuitionistic fuzzy β-translation $A^T_\alpha=((\mu_A)^T_\beta,(\nu_A)^T_\alpha)$ of A.*

For every anti-intuitionistic fuzzy T-ideal $A=(\mu_A,\nu_A)$ of X and $\beta \in [0,\mathfrak{T}]$ the intuitionistic fuzzy β-translation $A^T_\beta=(\mu_A)^T_\beta,(\nu_A)^T_\beta)$ of A is an anti-intuitionistic fuzzy T-ideal of X. If $B=(\mu_B,\nu_B)$ is an anti-intuitionistic fuzzy T-ideal extension of A^T_β, then there exists $\alpha \in [0,\mathfrak{T}]$ such that $\alpha \geq \beta$ and $B \geq A^T_\alpha$ that is $\mu_B(x) \leq (\mu_A)^T_\alpha$ and $\nu_B(x) \geq (\nu_A)^T_\alpha$ for all $x \in X$. Hence, the following theorem is obtained.

Theorem 5.3. *Let $A=(\mu_A,\nu_A)$ be an anti-intuitionistic fuzzy T-ideal of X and let $\beta \in [0,\mathfrak{T}]$. For every anti-intuitionistic fuzzy T-ideal extension $B=(\mu_B,\nu_B)$ of the intuitionistic fuzzy β-translation $A^T_\beta=((\mu_A)^T_\beta,(\nu_A)^T_\beta)$ of A, there exists $\alpha \in [0,\mathfrak{T}]$ such that $\alpha \geq \beta$ and B is an anti-intuitionistic fuzzy T-ideal extension of the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A.*

Let us illustrate using the following example.

Example 5.4. *Let $X=\{0,1,2,3,4\}$ is a BCI-algebra with the following Cayley table:*

-	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	2	4	0

(1) *Let $A=(\mu_A,\nu_A)$ be an intuitionistic fuzzy subset of X defined by*

X	0	1	2	3	4
μ_A	0.43	0.57	0.66	0.57	0.66
ν_A	0.55	0.42	0.34	0.42	0.34

Then $A=(\mu_A,\nu_A)$ be an anti-intuitionistic fuzzy T-ideal of X and $\mathfrak{T}=0.43$. If we take $\beta=0.13$, then the intuitionistic fuzzy β-translation of $A^T_\beta=((\mu_A)^T_\beta,(\nu_A)^T_\beta)$ of A is given by

X	0	1	2	3	4
$(\mu_A)^T_\beta$	0.43	0.44	0.53	0.44	0.53
$(\nu_A)^T_\beta$	0.68	0.55	0.47	0.55	0.47

Let $B=(\mu_B,\nu_B)$ is an intuitionistic fuzzy subset of X is defined by

X	0	1	2	3	4
μ_B	0.26	0.36	0.48	0.36	0.48
ν_B	0.76	0.63	0.52	0.63	0.52

Then B is clearly an intuitionistic fuzzy T-ideal of X which an intuitionistic fuzzy T-ideal extension of the intuitionistic fuzzy β-translation $A^T_\beta$of A.

(2) *If we take $\alpha=0.16$ then $\alpha=0.16>0.13=\beta$ and the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is given as follows.*

X	0	1	2	3	4
$(\mu_A)^T_\alpha$	0.27	0.41	0.52	0.41	0.52
$(\nu_A)^T_\alpha$	0.71	0.58	0.50	0.58	0.50

But, $(\mu_A)^T_\alpha(x)+(\nu_A)^T_\alpha(x) \nleq 1$.

Therefore, B is not an intuitionistic fuzzy α-translation of A for all $\alpha \in [0,\mathfrak{T}]$.

(3) *If we take $\alpha=0.16$ then $\alpha=0.16>0.13=\beta$ and the intuitionistic fuzzy α-translation $A^T_\alpha=((\mu_A)^T_\alpha,(\nu_A)^T_\alpha)$ of A is given as follows.*

X	0	1	2	3	4
$(\mu_A)^T_\alpha$	0.71	0.58	0.50	0.58	0.50
$(\nu_A)^T_\alpha$	0.27	0.41	0.52	0.41	0.52

Note that $B(x) \leq A^T_\alpha(x)$ that is $\mu_B(x) \leq (\mu_A)^T_\alpha$ and $\nu_B(x) \geq (\nu_A)^T_\alpha$ for all $x \in X$.

Hence, B is an anti-intuitionistic fuzzy T-ideal extension of the intuitionistic fuzzy α-translation A^T_α of A.

6. Conclusion

In this manuscript, intuitionistic fuzzy translation of anti-intuitionistic fuzzy T-ideals in subtraction BCK/BCI-algebras are introduced and examine some of their applicable properties. The association between intuitionistic fuzzy translations and intuitionistic fuzzy elongation of anti-intuitionistic fuzzy T-ideals have been constructed. In our future, we will execute direct product of intuitionistic fuzzy soft theory of subtraction algebras.

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