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Reverse Zagreb indices of corona product of graphs

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Abstract

In this paper, some exact expressions for the first reverse Zagreb alpha index, first reverse Zagreb beta index and second reverse Zagreb index of corona product of two simple graphs are determined. Using this results we obtained these indices for $t-$thorny graph and bottleneck graph.

Keywords

Reverse Zagreb index, Corona product, $t-$thorny graph, bottleneck graph.

AMS Subject Classification

05C12, 05C76.

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Contents

1. Introduction

Let $G=(V(G),E(G))$ be graph with vertex set $V(G)$ and edge set $E(G)$. All the graphs considered in this paper are simple and connected. Graph theory has successfully provided chemists with a variety of useful tools [1, 8, 10, 11], among which are the topological indices. In theoretical chemistry, assigning a numerical value to the molecular structure that will closely correlate with the physical quantities and activities. Molecular structure descriptors (also called topological indices) are used for modeling physicochemical, pharmacologic, toxicologic, biological and other properties of chemical compounds. Zagreb indices were introduced more than forty years ago by Gutman and Trinajestic [9]. The Zagreb indices are found to have applications in QSPR and QSAR studies as well, see [3]. One of the recently introduced indices called *Reverse Zagreb indices* by Ediz [5] and he obtained the maximum and minimum graphs with respect to the first reverse Zagreb alpha index and minimum graphs with respect to the first reverse Zagreb beta index and the second reverse Zagreb index. Kulli [15, 16] defined first and second reverse hyper-Zagreb indices of a graph and obtained first two reverse Zagreb indices, the first two reverse hyper Zagreb indices and their polynomials of rhombus silicate networks, moreover he introduced the product connectivity reverse index of a graph G and obtained it for silicate networks and hexagonal networks.

2. Preliminaries

In this section, we recall some definitions and basic results of Zagreb index of graph which will be used throughout the paper.

Definition 2.1. *The first Zagreb index is defined as* $M_1(G)=\sum\limits_{u\in V(G)} d_G(u)^2$, *where* $d_G(u)$ *denote the degree of the vertex* u *in* G. *In fact, one can rewrite the first Zagreb index as* $M_1(G)=\sum\limits_{uv\in E(G)}(d_G(u)+d_G(v))$.

Definition 2.2. *The second Zagreb index is defined as* $M_2(G)=\sum\limits_{uv\in E(G)} d_G(u)d_G(v)$, *where* $d_G(u)$, $d_G(v)$ *denotes the degree of the vertex* u,v *respectively in* G.

Definition 2.3. *Let* G *be a simple connected graph. Then the total reverse vertex degree of* G *defined as* $TR(G)=\sum\limits_{a\in V(G)} c_a$, *where* $c_a(G)=\Delta_G-d_G(a)+1$, *known as reverse vertex degree of* $a\in V(G)$, *in shortly* $c_a(G)=c_a$ *and* Δ_G *denote the largest of all degrees of vertices of* G.

Definition 2.4. *Let* G *be a simple connected graph. Then the first reverse Zagreb alpha index of* G *defined as* $CM_1^{\alpha}(G)=\sum\limits_{a\in V(G)} c_a^2$.

Definition 2.5. *Let G be a simple connected graph. Then the first reverse Zagreb beta index of G defined as $CM_1^{\beta}(G) = \sum_{ab \in E(G)} (c_a + c_b)$.*

Definition 2.6. *Let G be a simple connected graph. Then the second reverse Zagreb index of G defined as $CM_2(G) = \sum_{ab \in E(G)} c_a c_b$.*

In [4], the chemical applications of these new indices have been investigated. In [6], Ediz and Cancan obtained the reverse Zagreb indices for Cartesian product of two simple connected graphs. In this extend we obtain the value of this index for corona product of graphs.

Graph operations play an important role in the study of graph decompositions into isomorphic subgraphs. The *corona* of two graphs was first introduced by Frucht and Harary in [7]. Let G and H be two simple graphs. The *corona product* $G \circ H$, is obtained by taking one copy of G and $|V(G)|$ copies of H; and by joining each vertex of the i-th copy of H to the i-th vertex of G, where $1 \leq i \leq |V(G)|$, see Figure 1. Let the vertex set of G and H denoted as $\{v_1, v_2, \cdots, v_{n_1}\}$ and $\{u_1, u_2, \cdots, u_{n_2}\}$ respectively. For $1 \leq i \leq n_1$, denote H^i the i^{th} copy of H joined to the vertex v_i and let the vertex set of H^i is $\{x_{i1}, x_{i2}, \cdots, x_{in_2}\}$, denoted by $V_2 = V(H^i)$ and let $V_1 = V(G')$, be the vertex set of G', where G' is a copy of G which contained in $G \circ H$. Coronas sometimes appear in chemical literature as plerographs of the usual hydrogen-suppressed molecular graphs known as kenographs, see [13] for more information. Different topological indices such as Wiener-type indices [2], Szeged, vertex PI, first and second Zagreb indices [18], weighted PI index [14], etc. of the corona product of two graphs have already been studied. For convenience, we partition the edge set of $G \circ H$ into three sets as follow, $E_1 = \{v_i v_j \,|\, v_i, v_j \in V(G')\}$, $E_2 = \{x_{ir} x_{is} \,|\, x_{ir}, x_{is} \in V(H^i)\}$ and $E_3 = \{v_i x_{ir} \,|\, v_i \in V(G'), x_{ir} \in V(H^i)\}$, where $1 \leq i, j \leq n_1$ and $1 \leq r, s \leq n_2$. From the structure of the corona product G and H (see Fig.1), one can easily observe the following lemma and corollary.

Lemma 2.7. *Let G and H be two connected graphs with n_1 and n_2 vertices, respectively. Then*
(a) $|V(G \circ H)| = |V(G')| + |V(H^i)| = n_1 + n_1 n_2$
(b) $|E(G \circ H)| = |E(G')| + |V(G)||E(H)| + |V(G)||V(H)| = E_1 + n_1|E(H)| + n_1 n_2$
(c) $d_{G \circ H}(a) = \begin{cases} d_G(a) + n_2, & \text{if } a \in V_1 \\ d_H(a) + 1, & \text{if } a \in V_2. \end{cases}$

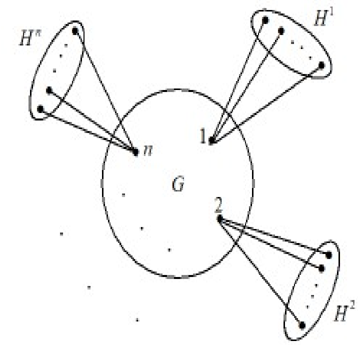

Fig. 1 The Corona product of G and H

Corollary 2.8. *Let G and H be two connected graphs with n_1 and n_2 vertices, respectively. Then the $\Delta_{G \circ H} = \Delta_G + O(H) = \Delta_G + n_2$.*

Proof. Let Δ_G and Δ_H be the largest degree of the respective graph. Since $\Delta_H < n_2$, then one can easily observed from the structure of corona product of $G \circ H$ and from lemma 2.7 that $\Delta_{G \circ H} = \Delta_G + n_2$. □

3. Main Results

In this section, we have obtain the reverse zagreb index of corona two connected graphs using it we have deduced for some standard graphs .

Proposition 3.1. *Let G and H be two connected graphs with n_1 and n_2 vertices, respectively. Then*

$$c_{G \circ H}(a) = \begin{cases} c_a(G) & \text{if } a \in V_1 \\ \Delta(G \circ H) - d_H(a) + 1 & \text{if } a \in V_2 \end{cases} \tag{3.1}$$

Proof. By using lemma 2.7 and corollary 2.8, we have

$$\begin{aligned} c_{G \circ H}(a) &= \Delta_{G \circ H} - d_{G \circ H}(a) + 1 \\ &= \begin{cases} \Delta_G + n_2 - (d_G(a) + n_2) + 1 & \text{if } a \in V_1 \\ \Delta_G + n_2 - (d_H(a) + 1) + 1 & \text{if } a \in V_2 \end{cases} \\ &= \begin{cases} \Delta_G - d_G(a) + 1 & \text{if } a \in V_1 \\ \Delta_G + n_2 - d_H(a) & \text{if } a \in V_2 \end{cases} \\ &= \begin{cases} c_a(G) & \text{if } a \in V_1 \\ \Delta(G \circ H) - d_H(a) + 1 & \text{if } a \in V_2 \end{cases} \end{aligned}$$

. □

Theorem 3.2. *Let G and H be two connected graphs with n_1,n_2 vertices, respectively. Then $CM_1^{\alpha}(G \circ H) = CM_1^{\alpha}(G) + n_1 \Big(n_2 \Delta_{G \circ H}^2 - 4\Delta_{G \circ H} E(H) + M_1(H) \Big)$.*

Proof. Using Proposition 2.3, we have

$$CM_1^{\alpha}(G \circ H) \quad = \sum_{a \in V(G \circ H)} c_a^2$$

$$
\begin{aligned}
&= \sum_{a\in V_1} c_a^2 + \sum_{a\in V_2} c_a^2 \\
&= \sum_{a\in V_1}\Big(\Delta_{G\circ H} - d_{G'}(a)+1\Big)^2 + \sum_{a\in V_2}\Big(\Delta_{G\circ H} - d_{H^i}(a)+1\Big)^2 \\
&= \sum_{a\in V(G)}\Big((\Delta_G+n_2)-(d_G(a)+n_2)+1\Big)^2 \\
&\quad + n_1\sum_{a\in V(H)}\Big((\Delta_G+n_2)-(d_H(a)+1)+1\Big)^2 \\
&= \sum_{a\in V(G)}\Big(\Delta_G - d_G(a)+1\Big)^2 \qquad (3.2)\\
&\quad + n_1\sum_{a\in V(H)}\Big((\Delta_G+n_2)-d_H(a)\Big)^2 \\
&= \sum_{a\in V(G)} {c_a}^2 + n_1\Big(\sum_{a\in V(H)}(\Delta_G+n_2)^2 \\
&\quad -2\sum_{a\in V(H)}(\Delta_G+n_2)d_H(a) + \sum_{a\in V(H)} d_H^2(a)\Big) \\
&= CM_1^{\alpha}(G) + n_1\Big(n_2\Delta_{G\circ H}^2 - 2\Delta_{G\circ H}(2E(H)) + M_1(H)\Big) \\
&= CM_1^{\alpha}(G) + n_1\Big(n_2\Delta_{G\circ H}^2 - 4\Delta_{G\circ H}E(H) + M_1(H)\Big).
\end{aligned}
$$

. ◻

Proposition 3.3. *Let P_n be a path and $\overline{K}_n$ be a n independent vertices, with $n \geq 3$. Then*
(a) $CM_1^{\alpha}(P_n) = n+6$, $CM_1^{\alpha}(C_n) = n$ and $CM_1^{\alpha}(\overline{K}_n) = 0$
(b) $CM_1^{\beta}(P_n) = 2n$, $CM_1^{\beta}(C_n) = 2n$ and $CM_1^{\beta}(\overline{K}_n) = 0$
(c) $CM_2(P_n) = n+1$, $CM_2(C_n) = n$ and $CM_2(\overline{K}_n) = 0$.

Lemma 3.4. *Let P_n be a path and C_n be a cycle with $n \geq 3$. Then $TR(P_n) = n+2$ and $TR(C_n) = n$.*

The $t-$*thorny graph* of a given graph G is obtained as $G\circ\overline{K}_t$, where $\overline{K}_t$ denotes the empty graph on t vertices [12]. Using Theorem 3.2 and Proposition 3.3, for the $t-$thorny graph of a graph G, we have the following corollaries.

Corollary 3.5. *Let G be a simple connected graph of order n and size $E(G)$. Then $CM_1^{\alpha}(G\circ\overline{K}_t) = CM_1^{\alpha}(G) + n_1t(\Delta_G + t)^2$.*

Now by using Corollary 3.5 and Proposition 3.3, we present two formulas for the reverse Zagreb indices of the $t-$*thorny path* $P_n\circ\overline{K}_t$ and the $t-$*thorny cycle* $C_n\circ\overline{K}_t$.

Corollary 3.6. *Let G be a simple connected graph of order n and size $E(G)$. Then $CM_1^{\alpha}(G\circ\overline{K}_t) = CM_1^{\alpha}(G) + n_1t(\Delta_G + t)^2$.*

Now by using Corollary3.6 and Proposition3.3, we present two formulas for the reverse Zagreb indices of the $t-$*thorny path* $P_n\circ\overline{K}_t$ and the $t-$*thorny cycle* $C_n\circ\overline{K}_t$.

Corollary 3.7. *For $n \geq 3$, if $G = P_n$ and $H = \overline{K}_t$, we have $CM_1^{\alpha}(P_n\circ\overline{K}_t) = (n+6) + nt(2+t)^2$.*

Corollary 3.8. *For $n \geq 3$, if $G = C_n$ and $H = \overline{K}_t$, we have $CM_1^{\alpha}(C_n\circ\overline{K}_t) = n(t(2+t)^2+1)$.*

Interesting classes of graphs can also be obtained by specializing the first component in the corona product. For example, for a graph G, the graph $G\circ K_2$ is called its *bottleneck graph*. For the bottleneck graph of a graph G, we obtain the following corollaries.

Corollary 3.9. *Let G be a simple connected graph of order $n \geq 3$. Then $CM_1^{\alpha}(G\circ K_2) = CM_1^{\alpha}(G) + 18n$.*

Using Corollary 3.9 and Proposition 3.3, the formulas for the bottleneck graph of a cycle $C_n\circ K_2$ and the bottleneck graph of a path $P_n\circ K_2$ are easily obtained.

Corollary 3.10. *For $n \geq 3$, if $G = C_n$, we have $CM_1^{\alpha}(C_n\circ K_2) = 19n$.*

Corollary 3.11. *For $n \geq 3$, if $G = P_n$, we have $CM_1^{\alpha}(P_n\circ K_2) = 19n+6$.*

Theorem 3.12. *For a connected graph G and H with n_1,n_2 vertices respectively, then $CM_1^{\beta}(G\circ H) = CM_1^{\beta}(G)$*
$+n_1\Big(2\Delta_{G\circ H}E(H) - M_1(H)\Big) + n_1n_2\Delta_{G\circ H} - 2n_1E(H) + n_2TR(G).$

Proof. Using Proposition 2.3, we have $CM_1^{\beta}(G\circ H)$ $= \sum_{ab\in E_1}(c_a+c_b) + \sum_{ab\in E_2}(c_a+c_b) + \sum_{ab\in E_3}(c_a+c_b)$. Let us calculate each summation separately, for the edge set E_1, we have $\sum_{ab\in E_1}(c_a+c_b)$

$$
\begin{aligned}
&= \sum_{ab\in E_1}\Big((\Delta_{G\circ H} - d_{G'}(a)+1) + (\Delta_{G\circ H} - d_{G'}(b)+1)\Big) \\
&= \sum_{ab\in E(G)}\binom{(\Delta_G+n_2-(d_G(a)+n_2)+1)}{+(\Delta_G+n_2-(d_G(b)+n_2)+1)} \\
&= \sum_{ab\in E(G)}\Big((\Delta_G - d_G(a)+1) + (\Delta_G - d_G(b)+1)\Big) \\
&= \sum_{ab\in E(G)}(c_a+c_b) \\
&= CM_1^{\beta}(G).
\end{aligned}
$$

Now for the edge set E_2, we have $\sum_{ab\in E_2}(c_a+c_b)$

$$
\begin{aligned}
&= \sum_{ab\in E_2}\Big((\Delta_{G\circ H} - d_{H^i}(a)+1) + (\Delta_{G\circ H} - d_{H^i}(b)+1)\Big) \\
&= n_1\sum_{ab\in E(H)}\binom{(\Delta_G+n_2-(d_H(a)+1)+1)}{+(\Delta_G+n_2-(d_H(b)+1)+1)} \\
&= n_1\sum_{ab\in E(H)}\Big(2(\Delta_{G\circ H}) - (d_H(a)+d_H(b))\Big) \\
&= n_1\Big(2\Delta_{G\circ H}E(H) - M_1(H)\Big).
\end{aligned}
$$

Finally for the edge set E_3, we have $\sum_{ab\in E_3}(c_a+c_b)$

$$
= \sum_{\substack{ab\in E_3\\ a\ \in H^i,\ b\in G'}}\Big((\Delta_{G\circ H} - d_{H^i}(a)+1) + (\Delta_{G\circ H} - d_{G'}(b)+1)\Big)
$$

$$
\begin{aligned}
&= \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}} \binom{(\Delta_G+n_2-(d_H(a)+1)+1)}{+(\Delta_G+n_2-(d_G(b)+n_2)+1)}\\
&= \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}} \Big((\Delta_G+n_2-d_H(a))+(\Delta_G-d_G(b)+1)\Big)
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}} (\Delta_G+n_2-d_H(a)) \quad + \\
&\quad \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}} (\Delta_G-d_G(b)+1)\\
&= \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}} (\Delta_G+n_2)-\\
&\quad \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}} d_H(a)+n_2\sum_{\substack{ab\in E_3\\ b\in V(G)}}(\Delta_G-d_G(b)+1)\\
&= n_1n_2\Delta_{G\circ H}-n_1(2E(H))+n_2\sum_{b\in V(G)} c_b\\
&= n_1n_2\Delta_{G\circ H}-2n_1E(H)+n_2TR(G).
\end{aligned}
$$

Hence, we have $CM_1^{\beta}(G\circ H)=CM_1^{\beta}(G)+n_1\Big(2\Delta_{G\circ H}E(H)-M_1(H)\Big)+n_1n_2\Delta_{G\circ H}-2n_1E(H)+n_2TR(G)$. □

Using Theorem 3.12, Proposition 3.3 and Lemma 3.4, we have the following corollaries

Corollary 3.13. *For $n\geq 3$, if $G=P_n$ and $H=K_2$, we have $CM_1^{\beta}(P_n\circ K_2)=4(4n+1)$.*

Corollary 3.14. *For $n\geq 3$, if $G=P_n$ and $H=\overline{K}_t$, we have $CM_1^{\beta}(P_n\circ \overline{K}_t)=n(t^2+3t+2)+2t$.*

Corollary 3.15. *For $n\geq 3$, if $G=C_n$ and $H=\overline{K}_t$, we have $CM_1^{\beta}(C_n\circ \overline{K}_t)=n(t^2+3t+2)$.*

Theorem 3.16. *For a connected graph G and H with n_1,n_2 vertices respectively, then $CM_2(G\circ H)=CM_2(G)+n_1\Big(\Delta^2_{G\circ H}E(H)-\Delta_{G\circ H}M_1(H)+M_2(H)\Big)+\Big(n_2\Delta_{G\circ H}-2E(H)\Big)TR(G)$.*

Proof. Using Proposition 3.1, we have

$$
\begin{aligned}
CM_2(G\circ H) &= \sum_{ab\in E_{G\circ H}}(c_ac_b)\\
&= \sum_{ab\in E_1}(c_ac_b)+\sum_{ab\in E_2}(c_ac_b)+\sum_{ab\in E_3}(c_ac_b).
\end{aligned}
$$

Let us calculate each summation separately, for the edge set E_1 we have $\sum_{ab\in E_1}(c_ac_b)$

$$
\begin{aligned}
&= \sum_{ab\in E_1}\Big((\Delta_{G\circ H}-d_{G'}(a)+1)(\Delta_{G\circ H}-d_{G'}(b)+1)\Big)\\
&= \sum_{ab\in E(G)}\binom{(\Delta_G+n_2-(d_G(a)+n_2)+1)}{(\Delta_G+n_2-(d_G(b)+n_2)+1)}\\
&= \sum_{ab\in E(G)}\Big((\Delta_G-d_G(a)+1)(\Delta_G-d_G(b)+1)\Big)\\
&= \sum_{ab\in E(G)}(c_ac_b)\\
&= CM_2(G).
\end{aligned}
$$

Now for the edge set E_2, we have $\sum_{ab\in E_2}(c_ac_b)$

$$
\begin{aligned}
&= \sum_{ab\in E_2}\Big((\Delta_{G\circ H}-d_{H^i}(a)+1)(\Delta_{G\circ H}-d_{H^i}(b)+1)\Big)\\
&= n_1\sum_{ab\in E(H)}\binom{(\Delta_G+n_2-(d_H(a)+1)+1)}{(\Delta_G+n_2-(d_H(b)+1)+1)}\\
&= n_1\sum_{ab\in E(H)}\Big((\Delta_{G\circ H}-d_H(a))(\Delta_{G\circ H})-d_H(b))\Big)\\
&= n_1\Big(\sum_{ab\in E(H)}(\Delta^2_{G\circ H}-\Delta_{G\circ H}(d_H(a)+d_H(b))+d_H(a)d_H(b))\Big)\\
&= n_1\Big(\sum_{ab\in E(H)}\Delta^2_{G\circ H}-\Delta_{G\circ H}\sum_{ab\in E(H)}(d_H(a)+d_H(b))+\\
&\quad \sum_{ab\in E(H)}d_H(a)d_H(b)\Big)\\
&= n_1\Big(\Delta^2_{G\circ H}E(H)-\Delta_{G\circ H}M_1(H)+M_2(H)\Big)
\end{aligned}
$$

.

Finally for the edge set E_3, we have $\sum_{ab\in E_3}(c_ac_b)$

$$
\begin{aligned}
&= \sum_{\substack{ab\in E_3\\ a\in H^i,\ b\in G'}}\Big((\Delta_{G\circ H}-d_{H^i}(a)+1)(\Delta_{G\circ H}-d_{G'}(b)+1)\Big)\\
&= \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}}\binom{(\Delta_G+n_2-(d_H(a)+1)+1)}{(\Delta_G+n_2-(d_G(b)+n_2)+1)}\\
&= \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}}\Big((\Delta_G+n_2-d_H(a))(\Delta_G-d_G(b)+1)\Big)\\
&= \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}}\Big((\Delta_G+n_2-d_H(a))(c_b)\Big)\\
&= \sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}}c_b\Delta_{G\circ H}-\sum_{\substack{ab\in E_3\\ a\in V(H),\ b\in V(G)}}d_H(a)c_b\\
&= n_2\Delta_{G\circ H}\sum_{b\in V(G)}c_b-\sum_{a\in V(H)}d_H(a)\sum_{b\in V(G)}c_b\\
&= \Big(n_2\Delta_{G\circ H}-\sum_{a\in V(H)}d_H(a)\Big)\sum_{b\in V(G)}c_b\\
&= \Big(n_2\Delta_{G\circ H}-2E(H)\Big)TR(G).
\end{aligned}
$$

Hence, we have $CM_2(G \circ H) = CM_2(G) + n_1\Big(\Delta^2_{G\circ H}E(H) - \Delta_{G\circ H}M_1(H) + M_2(H)\Big) + \Big(n_2\Delta_{G\circ H} - 2E(H)\Big)TR(G)$. $\square$

As a direct consequence of Theorem 3.16 and using Proposition 3.3. we have the following corollaries

Corollary 3.17. *For $n \geq 3$, if $G = P_n$ and $H = K_2$, we have $CM_2(P_n \circ K_2) = 16n + 3$.*

Corollary 3.18. *For $n \geq 3$, if $G = P_n$ and $H = \overline{K}_t$, we have $CM_2(P_n \circ \overline{K}_t) = (n+1) + (n+2)(t+2)t$.*

Corollary 3.19. *For $n \geq 3$, if $G = C_n$ and $H = \overline{K}_t$, we have $CM_2(C_n \circ \overline{K}_t) = n(t+1)^2$.*

4. Conclusion

In this paper, we have obtained the exact value of the first reverse Zagreb alpha index, first reverse Zagreb beta index and second reverse Zagreb index of corona product of two simple graphs and we have derived these indices for t-thorny graph and bottleneck graph of path and cycle. It would be interesting to study mathematical properties of these modified indices and report their chemical relevance and formulas for some important graph classes. In particular, some exact expressions for the first reverse Zagreb alpha index, first reverse Zagreb beta index and second reverse Zagreb index of other graph operations (such as the composition, join, disjunction and symmetric difference of graphs, bridge graphs and Kronecker product of graphs) can be derived similarly.

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