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Interior ideals in Γ-semirings

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Abstract

The concepts of an interior ideal, minimal interior ideal and an interior-simple Γ-semiring are defined. Various properties of an interior ideal and minimal interior ideal of a Γ-semiring are studied. Some characterizations of a minimal interior ideal and an interior-simple Γ-semiring are discussed.

Keywords

Interior ideal, minimal interior ideal , interior-simple Γ-semiring, regular Γ-semiring, intra-regular Γ-semiring.

AMS Subject Classification

16Y60, 16Y99.

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Contents

1. Introduction

Γ-semiring is a generalization of a semiring. Rao [7] defined and studied Γ-semiring. Dutta and Sardar [1] studied different types of ideals in a Γ-semiring. Quasi-ideals and bi-ideals in a Γ-semiring were studied by Author [2–4]. Lajos [6] defined the concept of an interior ideal in a semigroup. Interior ideal in a semigroup was studied by Szasz [8, 9]. Interior ideals in ordered semigroups and the interior ideal elements in poe-semigroups were discussed by Kehayopulu [5].

The concepts of an interior ideal and minimal interior ideal in a Γ-semiring are introduced in this paper. Some properties of an interior ideal and minimal interior ideal of a Γ-semiring are proved. Some characterizations of a minimal interior ideal are studied. Also the notion of an interior-simple Γ-semiring is defined. Some properties and characterizations of an interior-simple Γ-semiring are furnished. For the concepts in a Γ-semiring see Dutta and Sardar [1] and Jagatap and Pawar [2, 4].

Now onwards S denotes a Γ-semiring with absorbing zero unless otherwise stated.

2. Interior Ideals

Here we define the notion of an interior ideal of a Γ-semiring S.

Definition 2.1. *A non-empty subset I of a Γ-semiring S is an interior ideal of S if I is an additive subsemigroup of S and $S\Gamma I\Gamma S \subseteq I$.*

Example : Let $S = \{0,1,2,3,4\}$. Define two binary operations $+$ and $\cdot$ on S as follows:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	2
2	2	3	4	2	3
3	3	4	2	3	4
4	4	3	2	4	2

.	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	3	2
3	0	3	3	3	3
4	0	4	2	3	4

For $\Gamma = S$, both S and Γ are additive commutative semigroups. A mapping $S\times\Gamma\times S \to S$ is defined as $a\alpha b =$ usual product of a, α, b; for all $a, b \in S$ and $\alpha \in \Gamma$. Then S forms a Γ-semiring. $\{0\}$, $\{0,3\}$, $\{0,2,3,4\}$ and S are interior ideals of S.

Remark 2.2. *Every ideal is an interior ideal of S but not conversely.*

For this consider the following example.
Example : Let $S=\{a,b,c,d\}$. Two binary operations + and $\cdot$ are defined on S such as

+	a	b	c	d
a	a	c	b	d
b	c	d	d	d
c	b	d	d	d
d	d	d	d	d

.	a	b	c	d
a	a	c	b	d
b	c	d	d	d
c	b	d	d	d
d	d	d	d	d

For $\Gamma=S$, both S and Γ are additive commutative semigroups. A mapping $S\times\Gamma\times S\rightarrow S$ is defined as $x\alpha y=$ usual product of x,α,y; for all $x,y\in S$ and $\alpha\in\Gamma$. Then S forms a Γ-semiring. Here $\{b,d\}$ and $\{c,d\}$ are interior ideals of S. But $\{b,d\}$ and $\{c,d\}$ are neither left ideals nor right ideals of S .

Theorem 2.3. *Let X be any non-empty subset of S. Then $S\Gamma X\Gamma S$ is an interior ideal of S.*

Corollary 2.4. *If $a\in S$, then $S\Gamma a\Gamma S$ is an interior ideal of S.*

Theorem 2.5. *Let X be any non-empty subset of S. Then $(X)_i=N_0X+S\Gamma X\Gamma S$, where N_0 is the set of non-negative integers.*

Proof. Let $T=N_0X+S\Gamma X\Gamma S$. For any $x\in X$, $x=1x+0\in N_0X+S\Gamma X\Gamma S=T$. Therefore $X\subseteq T$. Let $a,b\in T=N_0X+S\Gamma X\Gamma S$. Hence we have $a=a_1+a_2$, $b=b_1+b_2$; $a_1,b_1\in N_0X,a_2,b_2\in S\Gamma X\Gamma S$. Therefore $a_1=\sum_{i=1}^{p}n_ix_i,n_i\in N_0,x_i\in X$ and $b_1=\sum_{j=1}^{q}m_jy_j,m_j\in N_0,y_j\in X$. Hence $a_1+b_1=\sum_{i=1}^{p}n_ix_i+\sum_{j=1}^{q}m_jy_j$. This shows that $a_1+b_1\in N_0X$. Clearly $a_2+b_2\in S\Gamma X\Gamma S$. Now $a+b=(a_1+a_2)+(b_1+b_2)=(a_1+b_1)+(a_2+b_2)\in N_0X+S\Gamma X\Gamma S=T$. Therefore T is an additive subsemigroup of S. Then $S\Gamma T\Gamma S=S\Gamma(N_0X+S\Gamma X\Gamma S)\Gamma S\subseteq N_0(S\Gamma X\Gamma S)+S\Gamma X\Gamma S\subseteq S\Gamma X\Gamma S\subseteq T$. Therefore T is an interior ideal of S. Let M be an interior ideal of S containing X. Then we have $N_0X\subseteq M$, $S\Gamma X\Gamma S\subseteq M$. Therefore $T=N_0X+S\Gamma X\Gamma S\subseteq M$. This shows that T is the smallest ideal of S containing X. Hence $T=(X)_i=N_0X+S\Gamma X\Gamma S$. ☐

Corollary 2.6. *If $a\in S$, then $(a)_i=N_0a+S\Gamma a\Gamma S$.*

Theorem 2.7. *Arbitrary Intersection of interior ideals of S is an interior ideal of S provided it is non-empty.*

Proof. Let $\{A_i\}_{i\in\Delta}$ (Δ denotes any indexing set) be the family of interior ideals of S and $T=\bigcap_{i\in\Delta}A_i$ be a non-empty set. Clearly T is a subsemigroup of (S,+). Therefore $S\Gamma T\Gamma S=S\Gamma\left(\bigcap_{i\in\Delta}A_i\right)\Gamma S\subseteq S\Gamma A_i\Gamma S\subseteq A_i$, for all $i\in\Delta$. Hence $S\Gamma T\Gamma S\subseteq\bigcap_{i\in\Delta}A_i$. Therefore $T=\bigcap_{i\in\Delta}A_i$ is an interior ideal of S. ☐

Corollary 2.8. *The set of all interior ideals of S forms a Moore family.*

Theorem 2.9. *If I is an interior ideal and T is a sub-Γ-semiring of S , then $I\cap T$ is an interior ideal of T.*

Proof. Let I be an interior ideal and T be a sub-Γ-semiring of S. Then clearly $I\cap T$ is a subsemigroup of $(T,+)$. Therefore $T\Gamma(I\cap T)\Gamma T\subseteq T\Gamma I\Gamma T\subseteq I$. Also $T\Gamma(I\cap T)\Gamma T\subseteq T\Gamma T\Gamma T\subseteq T$. Hence $T\Gamma(I\cap T)\Gamma T\subseteq I\cap T$. Hence $I\cap T$ is an interior ideal of T. ☐

Theorem 2.10. *If S is regular, then $I=S\Gamma I\Gamma S$, for every interior ideal I of S.*

Proof. Let S be regular and I be an interior-ideal of S. Take any $a\in I$. Therefore $a\in a\Gamma S\Gamma a$. Hence $a\Gamma S\Gamma a\subseteq(a\Gamma S\Gamma a)\Gamma S\Gamma a\subseteq S\Gamma I\Gamma S$. Thus $I\subseteq S\Gamma I\Gamma S$. But $S\Gamma I\Gamma S\subseteq I$ always. Therefore $I=S\Gamma I\Gamma S$. ☐

Theorem 2.11. *Following statements are equivalent in S.*
1) S is regular.
2) For a quasi ideal Q and an ideal J of S, $Q\cap J=Q\Gamma J\Gamma Q$.
3) For a quasi ideal Q and an interior ideal I of S, $Q\cap I=Q\Gamma I\Gamma Q$.

Proof. (1) ⇒ (2) Let Q be a quasi-ideal and J be an ideal of S. Now $Q\Gamma J\Gamma Q\subseteq Q\Gamma S\Gamma Q\subseteq Q$ and $Q\Gamma J\Gamma Q\subseteq J$. Hence $Q\Gamma J\Gamma Q\subseteq Q\cap J$. Take any $a\in Q\cap J$. Therefore $a\in a\Gamma S\Gamma a$. Hence $a\Gamma S\Gamma a\subseteq(a\Gamma S\Gamma a)\Gamma S\Gamma(a\Gamma S\Gamma a)\subseteq(Q\Gamma S\Gamma Q)\Gamma(S\Gamma J\Gamma S)\Gamma Q\subseteq Q\Gamma J\Gamma Q$. Thus we get $Q\cap J\subseteq Q\Gamma J\Gamma Q$. Therefore $Q\Gamma J\Gamma Q=Q\cap J$.
(2) ⇒ (3) Implication holds, since every ideal is an interior ideal, .
(3) ⇒ (1) Take Q be any quasi-ideal of S. Therefore by (3), $Q\Gamma S\Gamma Q=Q\cap S$. Hence $Q\Gamma S\Gamma Q=Q$. Therefore S is regular (see Theorem 3.2 in [3]). ☐

Theorem 2.12. *Following conditions are equivalent in S.*
1) S is regular.
2) For an interior ideal I and a bi-ideal B of S , $I\cap B=B\Gamma I\Gamma B$.
3) For an interior ideal I and a quasi-ideal Q of S , $I\cap Q=Q\Gamma I\Gamma Q$.

Proof. (1) ⇒ (2) Let B be a bi-ideal and I be an interior ideal of S. Now $B\Gamma I\Gamma B\subseteq B\Gamma S\Gamma B\subseteq B$. Therefore $B\Gamma I\Gamma B\subseteq S\Gamma I\Gamma S\subseteq I$. Hence we get $B\Gamma I\Gamma B\subseteq B\cap I$. Let $a\in B\cap I$. Hence $a\in a\Gamma S\Gamma a$. Therefore $a\Gamma S\Gamma a\subseteq(a\Gamma S\Gamma a)\Gamma S\Gamma(a\Gamma S\Gamma a)\subseteq(B\Gamma S\Gamma B)\Gamma(S\Gamma I\Gamma S)\Gamma B\subseteq B\Gamma I\Gamma B$. Thus $B\cap I\subseteq B\Gamma S\Gamma B$. Hence $B\Gamma S\Gamma B=B\cap I$.
(2) ⇒ (3) As every quasi-ideal is a bi-ideal, implication holds.
(3) ⇒ (1) Let Q be a quasi-ideal of S. By (3), $Q\Gamma S\Gamma Q=Q\cap S$, since S itself is an interior ideal. Hence $Q\Gamma S\Gamma Q=Q$. Therefore S is regular (see Theorem 3.2 in [3]). ☐

Theorem 2.13. *Following statements in S are equivalent.*
1) S is regular.
2) $B\cap I\cap L\subseteq B\Gamma I\Gamma L$, for a bi-ideal B, left ideal L and interior ideal I of S.

3) $Q\cap I\cap L\subseteq Q\Gamma I\Gamma L$, for a quasi-ideal Q, left ideal L and interior ideal I of S,.
4) $B\cap I\cap R\subseteq R\Gamma I\Gamma B$, for a bi-ideal B, right ideal R and interior ideal I of S.
5) $Q\cap I\cap R\subseteq R\Gamma I\Gamma Q$, for a quasi-ideal Q, right ideal R and interior ideal I of S.

Proof. $(1)\Rightarrow(2)$ Take any $a\in B\cap I\cap L$. Therefore $a\in a\Gamma S\Gamma a$. Hence

$a\Gamma S\Gamma a\subseteq(a\Gamma S\Gamma a)\Gamma S\Gamma(a\Gamma S\Gamma a)\subseteq(B\Gamma S\Gamma B)\Gamma(S\Gamma I\Gamma S)\Gamma L\subseteq B\Gamma I\Gamma L$. Thus we get $B\cap I\cap L\subseteq B\Gamma I\Gamma L$.
$(2)\Rightarrow(3)$ Clearly implication holds.
$(3)\Rightarrow(1)$ For a right ideal R and a left ideal L of S, by (3) we have $R\cap S\cap L\subseteq R\Gamma S\Gamma L$. Hence $R\cap L\subseteq R\Gamma S\Gamma L\subseteq R\Gamma L$. But always $R\Gamma L\subseteq R\cap L$ holds. Thus we get $R\Gamma L=R\cap L$. Therefore S is a regular Γ-semiring (see Theorem 3.2 in [3]).
$(1)\Rightarrow(4)$ Let $a\in B\cap I\cap R$. Hence $a\in a\Gamma S\Gamma a$. Therefore $a\Gamma S\Gamma a\subseteq(a\Gamma S\Gamma a)\Gamma S\Gamma(a\Gamma S\Gamma a)\subseteq R\Gamma(S\Gamma I\Gamma S)\Gamma(B\Gamma S\Gamma B)\subseteq R\Gamma I\Gamma B$. Hence $B\cap I\cap R\subseteq R\Gamma I\Gamma B$.
$(4)\Rightarrow(5)$ Clearly implication follows.
$(5)\Rightarrow(1)$ For a right ideal R and a left ideal L of S, by (5) we have $L\cap S\cap R\subseteq R\Gamma S\Gamma L$. Hence $R\cap L\subseteq R\Gamma S\Gamma L\subseteq R\Gamma L$. But always $R\Gamma L\subseteq R\cap L$. Therefore $R\cap L=R\Gamma L$. Hence S is a regular Γ-semiring (see Theorem 3.2 in [3]). □

Theorem 2.14. *In an intra-regular Γ-semiring an ideal and an interior ideal coincide.*

Proof. Let S be an intra-regular Γ-semiring. If a non-empty subset I of S is an ideal of S, then clearly I is an interior ideal of S. Suppose that a non-empty subset I of S is an interior ideal of S. Hence $I\subseteq S\Gamma I\Gamma I\Gamma S$. Therefore $S\Gamma I\subseteq S\Gamma(S\Gamma I\Gamma I\Gamma S)\subseteq S\Gamma I\Gamma S\subseteq I$ and $I\Gamma S\subseteq(S\Gamma I\Gamma I\Gamma S)\Gamma S\subseteq S\Gamma I\Gamma S\subseteq I$. Hence I is an ideal of S. □

Definition 2.15. *An interior ideal I of S is semiprime if for any interior ideal A of S, $A^2=A\Gamma A\subseteq I$ implies $A\subseteq I$.*

Definition 2.16. *An interior ideal I of S is completely semiprime if for any $a\in S$, $a\Gamma a\subseteq I$ implies $a\in I$.*

Theorem 2.17. *In an intra-regular Γ-semiring a proper interior ideal is semiprime.*

Proof. Let S be an intra-regular Γ-semiring and P be a proper interior ideal of S. Take A is any interior ideal of S such that $A\Gamma A\subseteq P$. For any $a\in A$, we have $a\in S\Gamma a\Gamma a\Gamma S$. Hence $S\Gamma a\Gamma a\Gamma S\subseteq S\Gamma A\Gamma A\Gamma S\subseteq S\Gamma P\Gamma S\subseteq P$. Thus $A\subseteq P$. Therefore P is a semiprime interior ideal of S. □

Theorem 2.18. *S is intra-regular if and only if each interior ideal of S is completely semiprime.*

Proof. Suppose that S is intra-regular. Let P be a proper interior ideal of S. For any element a of S, $a\Gamma a\subseteq P$. Then we have $a\in S\Gamma a\Gamma a\Gamma S$. Therefore $S\Gamma a\Gamma a\Gamma S\subseteq S\Gamma P\Gamma S\subseteq P$. Hence $a\in P$. Therefore P is a completely semiprime interior ideal of S. Conversely, assume that each interior ideal of S is completely semiprime. Take any $a\in S$. We have $S\Gamma a\Gamma a\Gamma S$ is an interior ideal of S. Therefore by assumption $S\Gamma a\Gamma a\Gamma S$ is completely semiprime. Hence $(a\Gamma a)\Gamma(a\Gamma a)\subseteq S\Gamma a\Gamma a\Gamma S$ implies $a\Gamma a\subseteq S\Gamma a\Gamma a\Gamma S$. Hence $a\in S\Gamma a\Gamma a\Gamma S$. Hence S is intra-regular. □

Theorem 2.19. *If S is regular, then S is duo if and only if every bi-ideal of S is an ideal of S.*

Theorem 2.20. *If S is regular, then a non-empty subset of S is an ideal if and only if it is an interior ideal.*

Proof. Let S be regular. If a non-empty subset I of S is an ideal of S, then I is an interior ideal of S. Conversely, suppose that a non-empty subset I of S is an interior ideal of S. Hence $I\subseteq I\Gamma S\Gamma I$. Therefore $S\Gamma I\subseteq S\Gamma(I\Gamma S\Gamma I)\subseteq S\Gamma I\Gamma S\subseteq I$ and $I\Gamma S\subseteq(I\Gamma S\Gamma I)\Gamma S\subseteq S\Gamma I\Gamma S\subseteq I$. Therefore I is an ideal of S. □

From Theorems 2.19 and 2.20 we have,

Theorem 2.21. *If S is regular and duo, then a non-empty subset of S is a bi-ideal if and only if it is an interior ideal.*

Corollary 2.22. *If S is regular and duo, then a non-empty subset of S is a quasi-ideal if and only if it is an interior ideal.*

3. Interior-Simple Γ-semiring

Definition 3.1. *S is said to be an interior-simple Γ-semiring if S has no non zero proper interior ideal.*

That is S is an interior-simple Γ-semiring if S and $\{0\}$ are the only interior ideal of S.

Theorem 3.2. *In S following statements are equivalent.*
1) S is an interior-simple Γ-semiring.
2) $S\Gamma a\Gamma S=S$, for all $0\neq a\in S$.
3) $(a)_i=S$, for all $0\neq a\in S$.

Proof. $(1)\Rightarrow(2)$ Suppose that S is an interior-simple Γ-semiring. For any $0\neq a\in S$, $S\Gamma a\Gamma S$ is an interior ideal of S and $S\Gamma a\Gamma S\subseteq S$. Hence $S=S\Gamma a\Gamma S$.
$(2)\Rightarrow(1)$ suppose that $S=S\Gamma a\Gamma S$, for $0\neq a\in S$. Let I be an interior ideal of S. For any $0\neq b\in I$, $S=S\Gamma b\Gamma S$ by (2). Hence $S\Gamma b\Gamma S\subseteq S\Gamma I\Gamma S\subseteq I$. Therefore $S\subseteq I$. Thus $S=I$. Hence S is an interior-simple Γ-semiring.
$(1)\Rightarrow(3)$ Suppose that S be an interior-simple Γ-semiring. For any $0\neq a\in S$, $(a)_i=N_0a+S\Gamma a\Gamma S$. But $S\Gamma a\Gamma S=S$. Therefore $(a)_i=N_0a+S\subseteq S$. By (1), we have $(a)_i=S$.
$(3)\Rightarrow(1)$ let I be an interior ideal of S. Then for any $0\neq a\in I$, $(a)_i=S$ by (3). Hence $S=(a)_i\subseteq I$. Therefore $I=S$. Hence S is an interior-simple Γ-semiring. □

Theorem 3.3. *Let I be an interior ideal and T be a sub-Γ-semiring of S. If T is interior-simple with $T\setminus\{0\}\cap I\neq\phi$, then $T\subseteq I$.*

Proof. Let T be an interior-simple Γ-semiring with $T\setminus\{0\}\cap I\neq\phi$ and $a\in T\setminus\{0\}\cap I$. Hence $T\Gamma a\Gamma T=T$ by Theorem 3.2. Therefore $T=T\Gamma a\Gamma T\subseteq T\Gamma I\Gamma T\subseteq S\Gamma I\Gamma S\subseteq I$. Thus $T\subseteq I$. □

4. Minimal Interior Ideals

Definition 4.1. *An interior ideal I of S is said to be a minimal interior ideal of S if I does not contain any other proper non zero interior ideal of S.*

Theorem 4.2. *If I is an interior ideal of S , then following statements are equivalent.*
(1) I is a minimal interior ideal of S.
(2) $I = S\Gamma a\Gamma S$, for all $0 \neq a \in I$.
(3) $I = (a)_i$, for all $0 \neq a \in I$.

Proof. Let I be an interior ideal of S.
$(1) \Rightarrow (2)$ Let $0 \neq a \in I$. Therefore $S\Gamma a\Gamma S \subseteq S\Gamma I\Gamma S \subseteq I$. But $S\Gamma a\Gamma S$ is an interior ideal of S. Therefore we have, $I = S\Gamma a\Gamma S$.
$(2) \Rightarrow (1)$ Let J be any interior ideal of S contained in I. For any $0 \neq a \in J, I = S\Gamma a\Gamma S$. $I = S\Gamma a\Gamma S \subseteq S\Gamma J\Gamma S \subseteq J$.Therefore we have $I = J$. Hence I is a minimal interior ideal of S.
$(1) \Rightarrow (3)$ Take any $0 \neq a \in I$. Then $(a)_i \subseteq I$. But I is a minimal interior ideal of S. Hence we have $I = (a)_i$.
$(3) \Rightarrow (1)$ Let J be any interior ideal of S contained in I. For any $0 \neq x \in J, I = (x)_i$. $I = (x)_i \subseteq J$. Therefore $I = J$. Hence I is a minimal interior ideal of S. □

Theorem 4.3. *A proper interior ideal of S is minimal if and only if the intersection of any two distinct proper interior ideals is empty.*

Proof. Assume that any proper interior ideal of S is minimal. Let A and B be any two distinct proper interior ideals of S. Suppose that $A \cap B \neq \phi$. Therefore $A \cap B$ is an interior ideal of S. Then we have $A \cap B \subseteq A$ and $A \cap B \subseteq B$. But by hypothesis A and B are minimal interior ideals of S. Therefore $A \cap B = A$ and $A \cap B = B$. Thus we get $A = B$, which is a contradiction. Therefore $A \cap B = \phi$. Conversely, assume that the intersection of any two distinct proper interior ideals is empty. Then no any proper interior ideal of S is contained in any other proper interior ideal. That is each proper interior ideal of S is a minimal interior ideal of S. □

Theorem 4.4. *Let R be a minimal right ideal and L be a minimal left ideal of a duo Γ-semiring S , then $L\Gamma R$ is a minimal interior ideal of S.*

Proof. Let R be a minimal right ideal and L be a minimal left ideal of a duo Γ-semiring S. Take $I = L\Gamma R$. Therefore $S\Gamma(L\Gamma R)\Gamma S \subseteq L\Gamma R$. Hence $I = L\Gamma R$ is an interior ideal of S. Let J be an interior ideal of S such that $J \subseteq I$. Since, $S\Gamma J$ is a left ideal and $J\Gamma S$ is a right ideal of S. Then $S\Gamma J \subseteq S\Gamma I = S\Gamma L\Gamma R \subseteq L$. Similarly we can show that $J\Gamma S \subseteq R$. But L is a minimal left ideal and R is a minimal right ideal of S . Therefore $S\Gamma J = L$ and $J\Gamma S = R$. Hence $I = L\Gamma R = S\Gamma J\Gamma J\Gamma S \subseteq S\Gamma J\Gamma S \subseteq J$. Thus we get $I = J$. Therefore $I = L\Gamma R$ is a minimal interior ideal of S. □

Theorem 4.5. *If I is an interior ideal of S , then I is a minimal interior ideal of S if and only if $(a)_i = (b)_i$, for all $0 \neq a, 0 \neq b \in I$.*

Proof. Assume that I is a minimal interior ideal of S. Take any $0 \neq a, 0 \neq b \in I$. Hence by Theorem 4.2, $I = (a)_i$ and $I = (b)_i$. Therefore $(a)_i = (b)_i$, for all $0 \neq a, 0 \neq b \in I$. Conversely assume that $(a)_i = (b)_i$, for all $0 \neq a, 0 \neq b \in I$. Let J be any interior ideal of S such that $J \subseteq I$. Let $0 \neq x \in J$. For any $0 \neq y \in I$, we have $(x)_i = (y)_i$. Since $y \in (y)_i$ always. Hence $y \in (x)_i \subseteq J$. Therefore $I \subseteq J$. Thus we get $I = J$. Hence I is a minimal interior ideal of S. □

Definition 4.6. *The Green's relation $\mathfrak{L}$ $\mathfrak{R}$, and $\mathfrak{H}$ on S are defined as follows*
(1) $a\mathfrak{L}b$ if and only if $(a)_l = (b)_l$.
(2) $a\mathfrak{R}b$ if and only if $(a)_r = (b)_r$.
(3) $\mathfrak{H} = \mathfrak{L} \cap \mathfrak{R}$.

Definition 4.7. *A Green's relation $\mathfrak{T}$ on S is defined as for any $a, b \in S$, $a\mathfrak{T}b$ if and only if $(a) = (b)$.*

Definition 4.8. *A relation $\mathfrak{I}$ on S is defined as for any $a, b \in S$, $a\mathfrak{I}b$ if and only if $(a)_i = (b)_i$.*

Remark 4.9. $\mathfrak{T} \subseteq \mathfrak{I}$

Theorem 4.10. *If I is an interior ideal of S, then I is a minimal interior ideal of S if and only if I is a $\mathfrak{I}$-class.*

Proof. Let I be an interior ideal of S. Assume that I is a minimal interior ideal of S. Take any $0 \neq a, 0 \neq b \in I$. Hence by Theorem 4.2, $I = (a)_i$ and $I = (b)_i$. Therefore $(a)_i = (b)_i$.This shows that $a\mathfrak{I}b$. Thus I is a $\mathfrak{I}$ class. Conversely assume that I is a $\mathfrak{I}$ class. Then we have, $(a)_i = (b)_i$, for all $a, b \in I$. Therefore $(a)_i = (b)_i$, for all $0 \neq a, 0 \neq b \in I$. Hence by the Theorem 4.5, I is a minimal interior ideal of S. □

Theorem 4.11. *If S is regular, then $\mathfrak{T} = \mathfrak{I}$.*

Proof. Let S be a regular Γ-semiring. For any $a, b \in S, a\mathfrak{I}b$. Therefore $(a)_i = (b)_i$. Hence by Theorem 2.20, $(a) = (b)$. Therefore $a\mathfrak{T}b$. Hence $\mathfrak{I} \subseteq \mathfrak{T}$. But $\mathfrak{T} \subseteq \mathfrak{I}$ always. Thus we get $\mathfrak{T} = \mathfrak{I}$. □

Proof of following theorem follows from proof of Theorem 4.11

Theorem 4.12. *If S is intra-regular, then $\mathfrak{T} = \mathfrak{I}$.*

From Theorem 4.10 and Theorem 4.11 we have

Theorem 4.13. *If S is regular and I is an interior ideal of S, then I is a minimal interior ideal of S if and only if I is a $\mathfrak{T}$ class.*

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