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β-Weak local necks of fuzzy automata

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Abstract
In this paper we introduce β-weak local necks, β-weak monogenically directable, β-weak monogenically strongly directable, β-weak monogenically trap directable, β-weak uniformly monogenically directable, β-weak uniformly monogenically strongly directable, β-weak uniformly monogenically trap-directable fuzzy automata. consequently we have shown that β-weak local necks of fuzzy automaton exists then it is β-weak subautomaton. Further we prove a some equivalent conditions on fuzzy automaton.

Keywords
β-Weak local necks, β-Weak monogenically directable, β-Weak monogenically trap-directable, β-Weak uniformly monogenically trap-directable.

AMS Subject Classification
03D05, 20M35, 18B20, 68Q45, 68Q70, 94A45.

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Contents

1. Introduction

Fuzzy set was introduced by Zadeh in 1965 [12] whenever uncertainity occurs. Fuzzy sets are sets whose elements have degree of membership. Fuzzy set is an extension of classical notion set. Fuzzy set generalize the classical set, since the indicator functions of classical sets are special cases of the membership function of fuzzy set, if the later only take values 0 or 1. The fuzzy set theory can be used in a wide range of domains in which information is incomplete or imprecise.

Automata are the prime example of general computational systems over discrete spaces. The incorporation of fuzzy logic into automata theory resulted in fuzzy automata which can handle continuous spaces. Moreover, they able to model uncertainty which inherent in many applications. Fuzzy set ideas have been applied to wide range of scientific areas. W. Z. Wee [11] applied the ideas of fuzzy in automata and language theory. E.S. Santos [9] proposed fuzzy automata as a model of pattern recognition and control systems.

K. S. Fu and R. W. McLaren (1965) worked in applications of stochastic automata as a model of learning systems [4]. The syntactic approach to pattern recognition was examined by K. S. Fu (1982) using formal deterministic and stochastic languages [5]. Friedrich Steimann and Klaus-Peter Adlassnig (1994) dealt with applications of fuzzy automata in the field of Clinical Monitoring [10]. J. N. Mordeson and D. S. Malik gave a detailed account of fuzzy automata and languages in their book (2002) [8].

T. Petkovic et al. [1] discussed directable automata, monogenically directable, generalized directable using necks. T. Petkovic et al.[3] introduce and studied trap-directable, trapped automata and other related automata. Also, we refer the survey paper Directable automata and their generalizations were investigated by S. Bogdanovic et al [2]. Further the necks and local necks of fuzzy automata were studied and discussed in [6, 7]. In this paper we introduce β-weak local necks, β-weak monogenically directable, β-weak monogenically strongly directable, β-weak monogenically trap directable, β-weak uniformly monogenically directable, β-weak uniformly monogenically strongly directable, β-weak uniformly monogenically trap-directable fuzzy automata. We have shown that β-weak local necks of fuzzy automata exists then it is β-weak subautomata. Further we prove a some equivalent conditions on fuzzy automaton.

2. Preliminaries

Definition 2.1. *[8] A fuzzy automaton $S = (D,\ I,\ \psi)$, where,*

D - set of states $\{d_0,\ d_1,\ d_2, \ldots,\ d_n\}$,
I - alphabets (or) input symbols,
ψ - function from $D \times I \times D \rightarrow [0,1]$,

The set of all words of I is denoted by I^*. The empty word is denoted by λ, and the length of each $t \in I^*$ is denoted by $|t|$.

Definition 2.2. *[8] Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. The extended transition function is defined by $\psi^* : D \times I^* \times D \rightarrow [0,1]$ and is given by*

$$\psi^*(d_i,\ \lambda,\ d_j) = \begin{cases} 1 & \text{if } d_i = d_j \\ 0 & \text{if } d_i \neq d_j \end{cases}$$

$\psi^*(d_i,tt',d_j) = \vee_{q_r \in D}\{\psi^*(d_i,t,d_r) \wedge \psi(d_r,t',d_j)\}, t \in I^*, t' \in I$.

Definition 2.3. *[6] Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Let $D' \subseteq D$. Let ψ' is the restriction of ψ and let $S' = (D',\ I,\ \psi')$. The fuzzy automaton S' is called a subautomaton of S if*

(i) $\psi' : D' \times I \times D' \rightarrow [0,1]$ and
(ii) For any $d_i \in D'$ and $\psi'(d_i,t,d_j) > 0$ for some $t \in I^*$, then $d_j \in D'$.

Definition 2.4. *[8] Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is said to be strongly connected if for every $d_i,\ d_j \in D$, there exists $t \in I^*$ such that $\psi^*(d_i,\ t,\ d_j) > 0$. Equivalently, S is strongly connected if it has no proper subautomaton.*

Definition 2.5. *[6] Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_j \in D$ is called a neck of S if there exists $t \in I^*$ such that $\psi^*(d_i,\ t,\ d_j) > 0$ for every $d_i \in D$.*
In that case d_j is also called t-neck of S and the word t is called a directing word of S.
If S has a directing word, then we say that S is a directable fuzzy automaton.

Definition 2.6. *[7] Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If $d_i \in Q$ is called local neck of S, if it is neck of some directable subautomaton of S. The set of all local necks of S is denoted by LN(S).*

3. β-Weak Local Necks of Fuzzy Automata

Definition 3.1. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If S is said to be β-weak fuzzy automaton then $\{\psi(d_i,t',d_j) < \beta\} > 0, \forall t' \in I, \beta =$ Fixed value in [0,1].*

Definition 3.2. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton and let $d_i \in D$. The β-weak subautomaton of S generated by d_i is denoted by $\langle d_i \rangle$. It is given by $\langle d_i \rangle = \{\{\ d_j \mid \psi^*(d_i,\ t,\ d_j) < \beta\} > 0,\ t \in I^*,$ $\beta = Fixedvaluein[0,1]\}$. If it exists, then it is called the β-weak least subautomaton of S containing d_i.*

Definition 3.3. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. For any non-empty $D' \subseteq D$, the β-Weak subautomaton of S generated by D' is denoted by $\langle D' \rangle$ and is given by $\langle D' \rangle = \{\{\ d_j \mid \psi^*(d_i,\ t,\ d_j) < \beta\} > 0,\ d_i \in D',\ t \in I^*\}$. It is called the β-weak least subautomaton of S containing D'. The β-weak least subautomaton of a fuzzy automaton S if it exists is called the β-weak kernel of S.*

Definition 3.4. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. A state $d_i \in D$ is called β-weak local neck of S if it is β-weak neck of some β-weak directable subautomaton of S. The set of all β-weak local necks of S is denoted by $\beta WLN(S)$.*

Remark 3.5. *1) The set of all β-weak necks of a fuzzy automaton S is denoted by $\beta WN(S)$.*
2) The set of all β-weak directing words of a fuzzy automaton S is denoted by $\beta WDW(S)$.
3) A fuzzy automaton S is called strongly β-weak directable if $D = \beta WN(S)$.

Definition 3.6. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is called β-weak monogenically directable if every monogenic subautomaton of S is β-weak directable.*

Definition 3.7. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is called β-weak monogenically strongly directable if every monogenic subautomaton of M is β-weak strongly directable.*

Definition 3.8. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is called β-weak monogenically trap-directable if every monogenic subautomaton of S has a single β-weak neck.*

Definition 3.9. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If $t \in I^*$ is β-weak common directing word of S if t is a β-weak directing word of every monogenic subautomaton of S. The set all β-weak common directing words of S will be denoted by $\beta WCDW(S)$. In other words, $\beta WCDW(S) = \cap_{d_i \in D} \beta WDW(\langle d_i \rangle)$.*

Definition 3.10. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is called β-weak uniformly monogenically directable fuzzy automaton if every monogenic subautomaton of S is β-weak directable and have atleast one β-weak common directing word.*

Definition 3.11. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is called β-weak uniformly monogenically strongly directable fuzzy automaton if every monogenic subautomaton of S is strongly β-weak directable and have atleast one β-weak common directing word.*

Definition 3.12. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. S is called β-weak uniformly monogenically trap directable fuzzy automaton if every monogenic subautomaton of S has a single β-weak neck and have atleast one β-weak common directing word.*

4. Properties of β-Weak Local Necks of Fuzzy Automata

Theorem 4.1. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton and $d_i \in D$. Then the following conditions are equivalent:*
(i) d_i is a β-weak local neck;
(ii)$\langle d_i \rangle$ is a strongly β-weak directable fuzzy automaton;
(iii) for every $t' \in I^*$, there exists $t \in I^*$ such that $\{\psi^*(d_i,\ t't,\ t_i) < \beta\} > 0$.

Proof. $(i) \Rightarrow (ii)$
Let d_i be a β-weak local neck of S. Then there exists a β-weak directable subautomaton S' of S such that $d_i \in \beta WN(S')$. Thus $\beta WN(S')$ is a strongly β-weak directable fuzzy automaton. Also, $\langle d_i \rangle \subseteq \beta WN(S')$, and $\beta WN(S')$ is strongly connected, then $\langle d_i \rangle = \beta WN(S')$. Therefore, $\langle d_i \rangle$ is a strongly β-weak directable fuzzy automaton.
$(ii) \Rightarrow (iii)$
Let $\langle d_i \rangle$ be a β-weak strongly directable fuzzy automaton. Then d_i is a t-β-weak neck of $\langle d_i \rangle$ for some $t \in I^*$. Since $\langle d_i \rangle$ is β-weak strongly directable, for every $t' \in I^*$, there exists some $d_l \in \langle d_i \rangle$ such that $\{\psi^*(d_i,\ t',\ d_l) < \beta\} > 0$. Now,
$\psi^*(d_i,\ t't,\ d_i) = \wedge_{d_l\ \in\ D}\{\{\psi^*(d_i,\ t',\ d_l), \psi^*(d_l,\ t,\ d_i)\} < \ \beta\} > 0$.
$(iii) \Rightarrow (i)$
(iii) clearly shows that d_i is a $t-\ \beta$-weak neck of $\langle d_i \rangle$, and hence, it is a β-weak local neck of S. □

Theorem 4.2. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. If $\beta WLN(S) \neq \phi$, then $\beta WLN(S)$ is a β-weak subautomaton of S.*

Proof. Let $d_i \in \beta WLN(S)$ and $e \in I$. Then, the monogenic β-weak subautomaton $\langle d_i \rangle$ of S is strongly β-weak directable. Now, $\langle d_i \rangle \subseteq \langle d_l \rangle$, for some $d_l \in \langle d_i \rangle$. Since $\langle d_i \rangle$ is strongly connected, $\langle d_i \rangle = \langle d_l \rangle$. Therefore, d_l is also a β-weak local neck of S, i.e., $d_l \in \beta WLN(S)$. Hence, $\beta WLN(S)$ is a β-weak subautomaton of S. □

Theorem 4.3. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Then the following conditions are equivalent:*
(i) Every state of D in S is a β-weak local neck;
(ii) S is β-weak monogenically strongly directable;
(iii) S is β-weak monogenically directable and β-weak reversible;
(iv)S is a direct sum of β-weak strongly directable fuzzy automata;
(v) $(\forall\, d_i \in D)(\exists\, t \in I^*)(\forall\, t' \in I^*)$ such that $\{\psi^*(d_i,\ t't,\ d_i) < \beta\} > 0$.

Proof. $(i) \Rightarrow (ii)$
If every state $d_i \in D$ is a β-weak local neck of S. Then we have that for every $d_i \in D$ the β-weak monogenic subautomaton $\langle d_i \rangle$ of D in S is β-weak strongly directable. Hence, S is β-weak monogenically strongly directable.

$(ii) \Rightarrow (iii)$
If S is β-weak monogenically strongly directable, then it is β-weak monogenically directable. Now, every β-weak monogenic subautomaton of S is strongly connected, hence S is β-weak reversible.
$(iii) \Rightarrow (iv)$
If S is β-weak reversible, then it is a direct sum of β-weak strongly connected fuzzy automata S_α, $\alpha \in Y$. Let $\alpha \in Y$ and $d_i \in D_\alpha$. Then $\langle d_i \rangle = S_\alpha$. Since S_α is strongly connected, and by the β-weak monogenic directability of S we have that $S_\alpha = \langle d_i \rangle$ is β-weak directable. Therefore, S_α is β-weak strongly directable, for any $\alpha \in Y$.
$(iv) \Rightarrow (i)$
Let S be a direct sum of β-weak strongly directable fuzzy automata S_α, $\alpha \in Y$. Then for each state $d_i \in D$, there exists $\alpha \in Y$ such that $d_i \in D_\alpha$, that is, $d_i \in S_\alpha = \beta WN(S_\alpha)$, so d_i is a β-weak local neck of S.
$(i) \Rightarrow (v)$
Since, every state of S is a β-weak local neck, for any $d_i \in D$, $\langle d_i \rangle$ is β-weak monogenically strongly directable. Hence, $\langle d_i \rangle$ is β-weak reversible.
$(v) \Rightarrow (i)$
This is an immediate consequence of proof of the Theorem 4.1.

□

5. Conclusion

In this paper we introduce β-weak local necks, β-weak monogenically directable, β-weak monogenically strongly directable, β-weak monogenically trap directable, β-weak uniformly monogenically directable, β-weak uniformly monogenically strongly directable, β-weak uniformly monogenically trap-directable fuzzy automata. We have shown that β-weak local necks of fuzzy automata exists then it is β-weak subautomata. Further we prove a some equivalent conditions on fuzzy automaton.

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