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Weak synchronization of fuzzy automata

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Abstract

The purpose of this paper is to study the structural characterizations of weak synchronization of fuzzy automata. We introduce weak reducubilty, weak stability relation, weak synchronization of fuzzy automata. We prove weak stability relation is an equivalence relation, algorithm is given to find weak synchronized word for fuzzy automata using weak stability relation.

Keywords

Weak reducibility, Weak stability, Weak synchronization.

AMS Subject Classification

03D05, 20M35, 18B20, 68Q45, 68Q70, 94A45.

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Contents

1. Introduction

Automata theory is the study of abstract computing devices or machines. Finite automata are useful models for many kind of software and used in software for designing digital circuits, Pattern matching, File searching program and so on. A finite automaton consists of finite set of states and set of transitions from state to state that occur on input symbols chosen from a finite set of elements called alphabet. Any system that is at each moment in one of finite number of discrete states and moves among the states in response to individual input signals can be modeled by a finite automaton. Automata are basically language acceptors. The family of languages accepted by any finite automata is called the family of regular languages.

Fuzzy concept is introduced whenever uncertainty occurs. Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets were introduced by Lotfi A. Zadeh in 1965 [9] as an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition - an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval $[0,1]$. Fuzzy sets generalize classical sets, since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. In fuzzy set theory, classical bivalent sets are usually called crisp sets. The fuzzy set theory can be used in a wide range of domains in which information is incomplete or imprecise.

L. A. Zadeh (1965) [9] introduced the notion of fuzzy subset of a set as a tool for representing uncertainty. His ideas have been applied to wide range of scientific areas. W. Z. Wee (1967) [8] applied the ideas of Zadeh in automata theory and language theory. E. S. Santos (1968) [5] proposed fuzzy automata as a model of pattern recognition and control systems. Friedrich Steimann and Klaus-Peter Adlassnig (1994) dealt with applications of fuzzy automata in the field of Clinical Monitoring [7]. J. N. Mordeson and D. S. Malik gave a detailed account of fuzzy automata and languages in their book (2002) [4].

Synchronizable automata also known as, cofinal and reset automata, are a significant type of automata with very interesting algebraic properties and important applications in various branches of Computer science [synchronization in binary messages, verification software, etc.].

Synchronization is an important concept in the theory of automata. Synchronization has a lot of applications in various fields. Synchronization allows simple error recovery in finite automata. If an error is detected, a synchronizing word can be used to reset the automata into a known state. This is useful in any system in any field which changes states on receiving discrete inputs. Another application is the leader identification in the process of networks.

Formally an automaton is called synchronized if there exists a word (a string of input symbol) w which takes each state of an automaton to a single state. Synchronization problem has a lot to do with the famous Road Coloring problem in graph theory. Adler, Goodwyn and Weiss showed that aperiodicity was necessary for such an instruction to exist [1].

Synchronization of a fuzzy automaton were introduced by Rm. Somasundaram and M. Rajasekar [6]. It means that there exists a word that brings each state of a fuzzy automaton to a single state with some membership value. consequently, this concept extended to γ-synchronized fuzzy automata and strong γ-synchronized fuzzy automata [2, 3]. The strong γ-synchronized fuzzy automata used to find minimal weight in synchronization of fuzzy automata. The synchronizing word does not exist in any fuzzy automaton. If it exists it is called a synchronized fuzzy automaton. But in a deterministic, strongly connected and aperiodic fuzzy automaton, synchronizing word exist. Initially it may not exist, after changing some labeling w get synchronized word. In this paper we study the structural characterizations of weak synchronization of fuzzy automata. We introduce weak reducubilty, weak stability relation, weak synchronization of fuzzy automata. We prove weak stability relation is an equivalence relation, algorithm is given to find weak synchronized word for fuzzy automata using weak stability relation.

2. Preliminaries

Definition 2.1. *[4] A fuzzy automaton $S = (D, I, \psi)$, where,*

D - set of states $\{d_0, d_1, d_2, ..., d_n\}$,

I - alphabets (or) input symbols,

ψ - function from $D \times I \times D \to [0,1]$,

The set of all words of I is denoted by I^*. The empty word is denoted by λ, and the length of each $t \in I^*$ is denoted by $|t|$.

Definition 2.2. *[4] Let $S = (D, I, \psi)$ be a fuzzy automaton. Define fuzzy set ψ^* in $D \times I^* \times D$ by*

$$\psi^*(d_i, \lambda, d_j) = \begin{cases} 1 & \text{if } d_i = d_j \\ 0 & \text{if } d_i \neq d_j \end{cases}$$

$\psi^*(d_i, tt', d_j) > 0 = \vee_{q_r \in D}\{\psi^*(d_i, t, d_r) \wedge \psi(d_r, t', d_j)\} > 0, t \in I^*, t' \in I$.

Definition 2.3. *[2] Let $S = (D, I, \psi)$ be a fuzzy automaton. Let $\pi = \{P_1, P_2 ... P_t\}$ be a partition of the states set Q such that if $\psi(q_i, t, q_j) > 0$, for some $t \in I$, then $d_i \in P_r$ and $d_j \in P_{r+1}$. Then π will be called periodic partition of order $y \geq 2$. A fuzzy automaton S is periodic of period $y \geq 2$ if and only if $y = Max\{card(\pi)\}$ where this maximum is taken over all periodic partitions π of S. If S has no periodic partition, then S is called aperiodic fuzzy automaton.*

Definition 2.4. *[2] Let $S = (D, I, \psi)$ be a fuzzy automaton. We say that a fuzzy automaton is γ-synchronized at the state d_j if there exist a real number γ with $0 < \gamma \leq 1$, and a word $t \in I^*$ that takes each state d_i of S into d_j such that $\psi^*(d_i, t, d_j) \geq \gamma$.*

Definition 2.5. *[4] A relation R on a set D is said to be equivalence relation if it is reflexive, symmetric and transitive.*

Definition 2.6. *[3] Let $S = (D, I, \psi)$ be a fuzzy automaton. An equivalence relation R on D in S is called congruence relation if $\forall d_i, d_j \in D$ and $t \in I, d_i R d_j$ implies that, then there exists $d_l, d_k \in D$ such that $\psi(d_i, a, d_l) > 0, \psi(d_j, a, d_k) > 0$ and $d_l R d_k$.*

Remark 2.7. *In this paper we consider only deterministic, strongly connected, and aperiodic fuzzy automaton and shortly denoted as DSA.*

3. Weak Synchronization of Fuzzy Automata

Definition 3.1. *Let $S = (D, I, \psi)$ be fuzzy automaton. We say that two states $d_i, d_j \in D$ are weak reducible relation and denoted by $d_i \sim d_j$, if there exist a word $t \in I^*$ and a state $d_k \in D$ such that $\{\psi^*(d_i, t, d_k) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_j, t, d_k) < \gamma\} > 0$, $\gamma =$ Fixed value in $(0,1]$.*

Example 3.2. *Let $S = (D, I, \psi)$ be fuzzy automaton, where $D = \{d_1, d_2, d_3, d_4\}$,*
$I = \{t, z\}, \gamma = 0.8$ and ψ are defined as below.
$\psi(d_1, t, d_4) = 0.6, \psi(d_1, z, d_2) = 0.7$
$\psi(d_2, t, d_3) = 0.5, \psi(d_2, z, d_4) = 0.4$
$\psi(d_3, t, d_2) = 0.3, \psi(d_3, z, d_4) = 0.6$
$\psi(d_4, t, d_1) = 0.6, \psi(d_4, z, d_3) = 0.2$
The states d_2 and d_3 are weak reducibility relation, since $\{\psi^*(d_2, tz, d_4) < \gamma\} > 0 \Leftrightarrow \psi^*(d_3, tz, d_4) < \gamma\} > 0$.

Example 3.3. *Let $S = (D, I, \psi)$ be fuzzy automaton, where $D = \{d_1, d_2, d_3, d_4\}$,*

$I = \{t,z\}, \gamma = 0.8$ and ψ are defined as below.
$\psi(d_1,t,d_3) = 0.6, \psi(d_1,z,d_1) = 0.5$
$\psi(d_2,t,d_1) = 0.4, \psi(d_2,z,d_1) = 0.3$
$\psi(d_3,t,d_4) = 0.3, \psi(d_3,z,d_4) = 0.6$
$\psi(d_4,t,d_2) = 0.6, \psi(d_4,z,d_4) = 0.2$
Now, there exists a word $zz \in I^*$, such that
$\{\psi^*(d_1,zz,d_1) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_2,zz,d_1) < \gamma\} > 0.$
Thus d_1,d_2 are weak reducibility related. Also there exists a string $tz \in I^*$ such that
$\{\psi^*(d_2,tz,d_l) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_4,tz,d_l) < \gamma\} > 0.$
Thus d_2,d_4 are weak reducibility relation but d_1 and d_4 are not weak reducibility related for any word $v \in I^*$. Hence weak reducibility relation is not transitive.

Definition 3.4. *Let $S = (D,\ I,\ \psi)$ be fuzzy automaton. $d_i,d_j \in D$ are said to be weak stability relation and denoted by $d_i \equiv d_j$, if for any word $t \in I^*$, there exists a word $t' \in I^*$ and $d_k \in D$ such that $\{\psi^*(d_i,tt',d_k) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_i,tt',d_k) < \gamma\} > 0.$*

Example 3.5. *Let $S = (D,\ I,\ \psi)$ be fuzzy automaton, where $D = \{d_1,d_2,d_3,d_4\}$,*
$I = \{t,z\}, \gamma = 0.8$ and ψ are defined as below.
$\psi(d_1,t,d_4) = 0.6, \psi(d_1,z,d_2) = 0.7$
$\psi(d_2,t,d_3) = 0.5, \psi(d_2,z,d_4) = 0.4$
$\psi(d_3,t,d_2) = 0.3, \psi(d_3,z,d_4) = 0.6$
$\psi(d_4,t,d_1) = 0.6, \psi(d_4,z,d_3) = 0.2$
for any word $t' \in I^*$, there exists a word $tzz \in I^*$ such that
$\{\psi^*(d_1,t'tzz,d_k) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_4,t'tzz,d_k) < \gamma\} > 0$ *and*
$\{\psi^*(d_2,t'tzz,d_l) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_3,t'tzz,d_l) < \gamma\} > 0.$
The states d_1,d_4 and d_2,d_3 are weak stability relation.

Remark 3.6. *(i) Weak reducibility relation is not an equivalence relation since transitive does not exist.*

Definition 3.7. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. We say that a fuzzy automaton is weak synchronized at the state d_j if there exist a word $t \in I^*$ that takes each state d_i of D into d_j such that $\{\psi^*(d_i,t,d_j) < \gamma\} > 0.$*

4. Properties of Weak Synchronization of Fuzzy Automata

Theorem 4.1. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Then weak stability relation is an equivalence relation.*

Proof. Clearly weak stability relation is reflexive and symmetric. We prove only transitive relation.
Let $d_i \equiv d_j$ and $d_j \equiv d_k$. To prove $d_i \equiv d_k$ we need to show that for any word t, there exist a word t' and $d_l \in D$ such that $\{\psi^*(d_i,tt',d_l) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_k,tt',d_l) < \gamma\} > 0$. Since $d_i \equiv d_j$, for any word t, there exists a word t'' and $d_m \in D$ such that
$\{\psi^*(d_i,tt'',d_m) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_j,tt'',d_m) < \gamma\} > 0.$
Since $d_j \equiv d_k$, for any word $tt'' \in I^*$, there exists a word t''' and $d_l \in D$ such that
$\{\psi^*(d_j,tt''t''',d_l) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_k,tt''t''',d_l) < \gamma\} > 0.$
$\{\psi^*(d_j,tt''t''',d_l) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_i,tt''t''',d_l) < \gamma\} > 0$ [since $d_i \equiv d_j$]. Therefore,
$\{\psi^*(d_i,tt''t''',d_l) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_k,tt''t''',d_l) < \gamma\} > 0.$
We can choose $t''t''' = t'$, for any word $t \in I^*$, there exist $t' = t''t''' \in I^*$ and $d_l \in Q$ such that
$\{\psi^*(d_i,tt',d_l) < \gamma\} > 0 \Leftrightarrow \{\psi^*(d_k,tt',d_l) < \gamma\} > 0.$
Hence, $d_i \equiv d_k$. □

Theorem 4.2. *Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Then weak stability relation is congruence relation.*

Proof. Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. Weak stability relation is an equivalence relation. Construct the equivalence classes using weak stability relation. Let $d_i \equiv d_j$, $d_i,d_j \in [D_i], i \in N$. Since S is DSA fuzzy automaton $\exists y \in I, d_l,d_k \in D$ such that $\{\psi^*(d_i,y,d_l) < \gamma\} > 0, \{\psi^*(d_j,y,d_k) < \gamma\} > 0$, $d_l,d_k \in [D_j]$. Hence weak stability relation is congruence relation. □

4.1 Algorithm for Finding the Weak Synchronized word for Fuzzy Automata Using Stability Relation

(i) Find the equivalence classes of the states of D in S using weak stability relation.
(ii) Construct the quotient fuzzy automaton $[D]$ by considering each equivalence class as a state.
(iii) Relabel the quotient fuzzy automaton D into D', preserving the weak stability classes.
(iv) Obtain S_1 from D' which is relabeling of S.
(v) S_1 will give the weak synchronized word.

4.2 Weak Synchronization Degree of a Fuzzy Automaton

Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton and let $D_1 \subseteq D$. The weak synchronization degree is defined as
$\theta_S = \wedge_{t \in I^*}\{Card(D_1) | \wedge \{\psi^*(D_1,t,D) < \gamma, d_1 \in D_1, d \in D\}\}$.
a fuzzy automaton is weak synchronized if and only if θ_S is equal to 1.

4.3 Procedure to Find Weak Synchronized Words

Let $S = (D,\ I,\ \psi)$ be a fuzzy automaton. We define another fuzzy automaton S_A as follows:
$S_A = (2^D, I, \psi_A^*, D_1, F \subseteq D)$, where,
D is called initial state on S_A,
F is called set of all final states on S_A,
ψ_{S_A} is the transition function on S_A. The transition function ψ_{S_A} is defined by,
$\psi_{S_A}(D_1,a,H) = \wedge\{\psi(d,a,h), d \in D_1, h \in H\}$, $D_1,\ H\ \in\ 2^D$ for $a \in I$.
Clearly, S_A is a deterministic fuzzy automaton and more over a word t is weak synchronized in S if and only if there exists a singleton subsets $B \in 2^D$ such that $\psi^*_{S_A}(D,t,B) = \gamma_1 < \gamma$.

5. Conclusion

The purpose of this paper is to study the structural characterizations of weak synchronization of fuzzy automata. We introduce weak reducubilty, weak stability relation, weak synchronization of fuzzy automata. We prove weak stability relation is an equivalence relation and congruence relation. Consequently, algorithm is given to find weak synchronized word for fuzzy automata using weak stability relation.

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