Malaya Journal of Matematik, Vol. 9, No. 1, 64-70, 2021
https://doi.org/10.26637/MJM0901/0010

Maximal flow problem in fuzzy environment

K. K. Mishra[1*]

Abstract
In this paper the existing algorithm is modified to find fuzzy number maximal flow between source and sink by representing all the parameters as trapezoidal fuzzy number. The trapezoidal fuzzy numbers are defuzzified by using linear ranking function proposed by Maleki [20]. The modified algorithms is illustrated by a suitable example and the obtained results are compared with the existing results.

Keywords
Fuzzy number maximal flow, Linear ranking function, Trapezoidal fuzzy number.

AMS Subject Classification
94D05, 06D72.

[1] *Department of Mathematics, Gandhi Institute for Technological Advancement (GITA), Bhubaneswar-752054, Odisha, India.*
[*]**Corresponding author**: [1]kkm.math1973@gmail.com
Article History: Received **01** November **2020**; Accepted **24** December **2020**

Contents

1. Introduction

The maximal flow problem was originally proposed by Fulkerson and Dantzig [21] and solved by specializing the simplex method for the linear programming. Ford and Fulkerson [19] solved the maximal flow problem by augmenting path algorithm. There are different algorithms to solve the crisp maximal flow problems [1,2]. In real life situation, it may not be possible to get the parameters of the maximal flow problem like cost, capacities and demand as certain quantity. Considering the uncertainty of the relevant parameters of the maximal flow problem, these parameters can be represented by fuzzy numbers [37]. Hence the maximal flow problems with fuzzy parameters are known as fuzzy maximal flow problems.

Here the proposed algorithm is the direct extension of the existing algorithm [34] to find fuzzy maximal flow between source and sink. The parameters are represented by trapezoidal fuzzy number and linear ranking function is used to compare the fuzzy numbers. The paper is organised as follows. In section 2, Some fundamental concept on fuzzy number and ranking function are given. In section 3, Fuzzy number maximal flow algorithm is formulated. In section 4 the proposed algorithm is illustrated with a suitable example. The results are discussed in section 5 and the paper is concluded in section 6.

2. Fundamental of fuzzy set theory

Definition 2.1. *(Fuzzy Sets) Let X is a collections of objects denoted generically by X, then a fuzzy set $\underline{A}$ in X is a set of ordered pairs*

$$\underline{A} = \left\{ (x, \mu_{\underline{A}}(x)) \,|x \in X, \mu_{\underline{A}}(x) \in [0,1] \right\}$$

where $\mu_{\underline{A}}(x)$ is called the membership function.

Definition 2.2. *(Support) The support of a fuzzy set $\underline{A}$ is the crisps set defined by*

$$\underline{A} = \{x \in X | \mu_{\underline{A}}(x) > 0\}$$

Definition 2.3. *(core) The core of a fuzzy set $\underline{A}$ is the crisp set of points $x \in X$ with $\mu_{\underline{A}}(x) = 1$.*

Definition 2.4. *(Boundary) The boundary of a fuzzy set $\underline{A}$ are defined set of points $x \in X$ such that $0 < \mu_{\underline{A}}(x) < 1$. It is evident that the boundary is defined as the region of the universal set containing elements that have non-zero membership but not complete membership. The **Fig.1** illustrates the region.*

Definition 2.5. *(Normality) A fuzzy set $\underline{A}$ is normal if and only there exists $x_i \in X$ such that $\mu_{\underline{A}}(x_i) = 1$.*

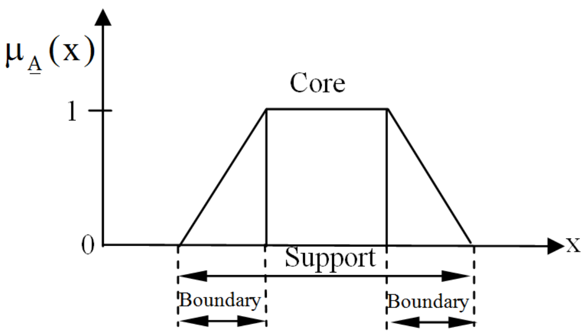

Figure 1

Definition 2.6. *(Sub-normality) A fuzzy set $\underline{A}$ is sub normal if $\mu_{\underline{A}}(x) < 1$.*

Definition 2.7. *(α-cut and strong α-cut) The α-cut of a fuzzy set $\underline{A}$ denoted by $[\underline{A}]_\alpha$ and is defined by $[\underline{A}]_\alpha = \{x \in X | \mu_{\underline{A}}(x) \geq \alpha\}$. If $\mu_{\underline{A}}(x) > \alpha$, then $[\underline{A}]_\alpha$ is called strong α-cut. It is clear that α-cut (strong α-cut) is a crisp set.*

Definition 2.8. *(Convexity) A fuzzy set $\underline{A}$ on X is convex if for any $x_1, x_2 \in X$ and $\lambda \in [0,1]$*

$$\mu_{\underline{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\underline{A}}(x_1), \mu_{\underline{A}}(x_2)\}.$$

It is to be noted that a fuzzy set is convex if and only if its α–cut is convex.

Definition 2.9. *(Fuzzy number) A fuzzy number is a fuzzy subset in universal set X which is both convex and normal.*

Definition 2.10. *A fuzzy number $\underline{A} = \{a,b,c,d\}$ is said to be a trapezoidal fuzzy number if its membership function is given by*

$$\mu_{\underline{A}}(x) \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x < b \\ 1 & b \leq x \leq c \\ \frac{(x-d)}{(c-d)}, & c < x \leq d \\ 0 & otherwise \end{cases}$$

Definition 2.11. *(Trapezoidal Fuzzy number) Let $\underline{A} = (a^L, a^U, \alpha, \beta)$ be the TrFN, where $(a^L - \alpha, a^U + \beta)$ is the support $\underline{A}$ and $[a^L, a^U]$ is the core of $\underline{A}$.*

Arithmetic on Trapezoidal Fuzzy Numbers
Let $F(R)$ be set of all trapezoidal fuzzy numbers over the real line R. The arithmetic operations on trapezoidal fuzzy numbers are defined as follows:
Let $\underline{a} = (a^L, a^U, \alpha, \beta)$ and $\underline{b} = (b^L, b^U, \gamma, \delta)$ $\left(\frac{\pi}{2} - \theta\right)$ be two trapezoidal fuzzy numbers and $x \in R$. We define

$$\begin{aligned} x > 0, x \in R; x\underline{a} &= (xa^L, xa^U, x\alpha, x\beta) \\ x < 0, x \in R; x\underline{a} &= (xa^U, xa^L, -x\beta, -x\alpha) \\ \underline{a} + \underline{b} &= (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \delta) \\ \underline{a} - \underline{b} &= (a^L - b^L, a^U - b^U, \alpha + \delta, \beta + \gamma) \end{aligned}$$

Ranking Function
Ranking is one of the effective method for ordering fuzzy numbers. Various types of ranking function have been introduced and some have been used for solving linear programming problems with fuzzy parameters. An effective approach for ordering the element of $F(R)$ is to define a ranking function. Let $\mathfrak{R} : F(R) \rightarrow (R)$. We define order on $F(R)$ as follows:

1. $\underset{\mathfrak{R}}{\underline{a} \geq \underline{b}}$ iff $\mathfrak{R}(\underline{a}) \geq \mathfrak{R}(\underline{b})$
2. $\underset{\mathfrak{R}}{\underline{a} \geq \underline{b}}$ iff $\mathfrak{R}(\underline{a}) > \mathfrak{R}(\underline{b})$
3. $\underset{\mathfrak{R}}{\underline{a} = \underline{b}}$ iff $\mathfrak{R}(\underline{a}) = \mathfrak{R}(\underline{b})$
4. $\underset{\mathfrak{R}}{\underline{a} \leq \underline{b}}$ iff $\underline{b}) \leq (\underline{a})$

Here $\mathfrak{R}$ is the ranking functions, such that

$$\mathfrak{R}(k\underline{a} + \underline{b}) = k\mathfrak{R}(\underline{a}) + \mathfrak{R}(\underline{b}).$$

Here, we introduce a linear ranking function that is similar to the ranking function adopted by Maleki [20]. For a trapezoidal fuzzy number $\underline{a} = (a^L, a^U, \alpha, \beta)$, we use ranking function as follows:

$$\mathscr{R}(\underline{a}) = \int_0^1 (\inf \underline{a}_\alpha + \sup \underline{a}_\alpha) d\alpha$$

$$\mathscr{R}(\underline{a}) = (a^L + a^U) + \frac{1}{2}(\beta - \alpha)$$

For any trapezoidal fuzzy numbers $\underline{a} = (a^L, a^U, \alpha, \beta)$ and $\underline{b} = (b^L, b^U, \gamma, \delta)$.
We have $\underset{\mathfrak{R}}{\underline{a} \geq \underline{b}}$ if and only if $a^L + a^U + \frac{1}{2}(\beta - \alpha) \geq b^L + b^U + \frac{1}{2}(\delta - \gamma)$

3. Fuzzy maximal flow algorithm

Consider arc (i,j) with initial fuzzy capacities $\left(\tilde{C}_{ij}, \tilde{C}_{ji}\right)$. As portions of these fuzzy capacities are committed to the flow in the arc, the fuzzy residuals or remaining fuzzy capacities of the arc are updated. We use the notation $\left(\underline{C}_{ij}, \underline{C}_{ji}\right)$ to represent these fuzzy residuals.
For a node j that receives flow from node i, we assign label $[a_i, i]$ where $\underline{a}_i$, is the fuzzy flow from node i to node j. The steps of the algorithm are summarized as follows.
Step 1: For all arcs (i,j). Set the residual fuzzy capacity equal to the initial fuzzy capacity i.e, $\left(\underline{c_{ij}}, \underline{c_{ji}}\right) = \left(\tilde{C}_{ij}, \tilde{C}_{ji}\right)$. Let $\underline{a}_1 = (\infty, \infty, \infty, \infty)$ and label source node 1 with $[(\infty, \infty, \infty, \infty), -]$ set $i = 1$ and go to step 2.
Step 2: Determine S_i as the set of unlabeled nodes j that can be reached directly from node i by arcs with positive residuals (that is $c_{ij} > 0$ for all $j \in S_i$). If $S_i \neq \phi$, go to step 3. Otherwise, go to step 4.

Step 3: Determine $k \in S_i$ such that $\Re(c_{ik}) = \max_{j \in S_i}\{\Re c_{ij}\}$. Set $\underline{a}_k = \underline{c}_k$ and label node k with $[a_k, i]$. If $k = n$, the sink node has been labeled, and a breakthrough path is found, go to step 5. Otherwise, set $i = k$, and go to step 2.
Step 4: (Backtracking). If $i = 1$, no further breakthroughs are possible; go to step 6. Otherwise, let r be the node that has been labeled immediately before the current node i and remove i from the set of nodes that are adjacent to r. Set $i = r$, and go to step 2.
Step 5: (Determination of Residue Network). Let $N_p = (1, k_1, k_2, \ldots, n)$ define the nodes of the p^{th} breakthrough path from source node 1 to sink node n. Then the maximum flow along the path is computed as

$$\underline{f}_p = \min\{\underline{a}_1, \underline{a}_{k_1}, \underline{a}_{k_2}, \ldots, \underline{a}_n\}$$

The residual capacity of each arc along the breakthrough path is decreased by f_p in the direction of the flow and increased by $\underline{f}_p$ in the reverse direction-that is, for nodes i and j on the path, the residual flow is changed from the current $(c_{ij,}, c_{ij})$ to

(a) $(c_{ij} - f_p, c_{ji} + f_p)$ if the flow is from i to j

(b) $\left(\underline{c_{ij}} + \underline{f_p}, \underline{c_{ji}} - \underline{f_p}\right)$ if the flow is from j to i.

Reinstate any nodes that were removed in step 4. Set $i = 1$, and return to step 2 to attempt a new breakthrough path.
Step 6: (Solution)

(a) Given that m breakthrough paths have been determined, the fuzzy maximal flow in the network is $\underline{F} = \underline{f}_1 + \underline{f}_2 + \ldots + \underline{f}_m$. Where m is the fuzzy number of iteration to get no breakthrough.

(b) Using the initial and final fuzzy residuals of arc (i, j) are given by $(\tilde{C}_{ij}, \tilde{C}_{ji})$ and $(\underline{c}_j, c_{ji})$, respectively, the fuzzy optimal flow in arc (i, j) is computed as follows: Let $(\underline{\alpha}, \underline{\beta}) = (\tilde{C}_{ij} - \underline{c}_i, \tilde{C}_{ji} - c_{ji})$. If $\Re(\underline{\alpha}) > 0$, the fuzzy optimal flow from i to j is $\underline{\alpha}$. Otherwise, if $\Re(\beta) > 0$, the fuzzy optimal flow from j to is β. (It is impossible to have both $\Re(\underline{\alpha}) > 0$ and $\Re(\underline{\alpha}) > 0$).

4. Numerical examples

Example 4.1. *Determine the fuzzy maximal flow in this network between source node 1 and sink node 5.*

Solution:

Iteration-1 Set the initial fuzzy residuals $(\underline{c}_{ij}, \underline{c}_{ji})$ equal to the initial fuzzy capacities $(\tilde{C}_{ij}, \tilde{C}_{ji})$.

Step 1: Set $\underline{a}_1 = (\infty, \infty, \infty, \infty)$ and label 1 with $[(\infty, \infty, \infty, \infty) -]$ set $i = 1$.
Step 2: $S_1 = \{2, 3, 4\}, (\neq \phi)$
Step 3: $K = 3$, because $\max \Re(\underline{c}_2)\Re(\underline{c}_3)\Re(\underline{c}_{44}) = \Re(\underline{c}_3)$.
Set $\underline{a}_3 = \underline{c}_{13} = (10, 20, 30, 30)$

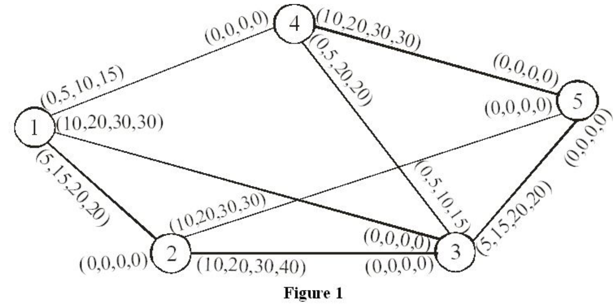

Figure 2

and label node 3 with [(10,20,30,30),1]
Set $i = 3$ and repeat step 2.
Step 4: $S_3 = (4, 5)$
Step 5: $K = 5$, because $\max\{\Re(\underline{c}_{34}), \Re(\underline{c}_{35})\} = \Re(c_{35})$.
Set $\underline{a}_5 = \underline{c_3} = (5, 15, 20, 20)$
and label node 5 with [(5,15,20,20),3].
Break through is achieved. Go to step 5.
Sep 6: The breakthrough path is determined from the labels starting at node 5 end moving backward ending at node 1.
i.e. (5)$\rightarrow [(5, 15, 20, 20), 3] \rightarrow (3) \rightarrow [(10, 0, 30, 40), 1] \rightarrow (1)$
i.e. $1 \rightarrow 3 \rightarrow 5$.
Thus $N_1 = \{1, 2, 5\}$ and

$$\begin{aligned}\underline{f}_1 &= \min\{\underline{a}_1, \underline{a}_3, \underline{a}_5\} \\ &= \min\{(\infty, \infty, \infty, \infty), (10, 20, 30, 40), (5, 15, 20, 20)\} \\ &= (5, 15, 20, 20)\end{aligned}$$

The fuzzy residual capacities along path N_1 are:

$$\begin{aligned}(\underline{c}_1, \underline{c}_{31}) &= [(10, 20, 30, 30) - (5, 15, 20, 20), (0, 0, 0, 0) \\ &\quad + (5, 15, 20, 20)] \\ &= ((-5, 15, 50, 50), (5, 15, 20, 20)) \\ (\underline{c}_{35}, \underline{c}_{53}) &= ((5, 15, 20, 20) - (5, 15, 20, 20), (0, 0, 0, 0) \\ &\quad + (5, 15, 20, 20)) \\ &= ((-10, 10, 40, 40), (5, 15, 20, 20)) \\ f_1 &= (5, 15, 20, 20)\end{aligned}$$

Iteration 2 :

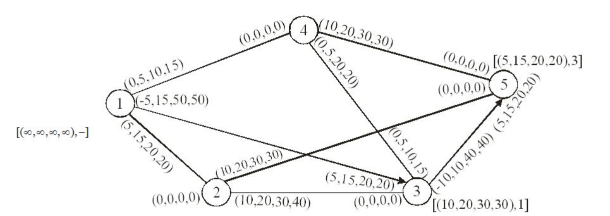

Figure 3

Step 1: Set $\underline{a}_1 = [\infty, \infty, \infty, \infty]$ and label node 1 with $[(\infty, \infty, \infty, \infty), -]$.
Set i = 1.
Step 2: $S_1 = \{2, 3, 4\}$

Step 3: K = 2 because $\max\{\Re(c_{12}),\Re(c_{13}),\Re(c_{44})\} = \Re(c_{12})$. Set $\underline{a}_2 = \underline{c}_{12} = (5,15,20,20)$ and label node 2 with [(5,15,20,20), 1]. Set $i = 2$ and repeat step 2.
Step 4: $S_2 = \{3,5\}$
Step 5: K = 3, because $\max\{\Re(c_3)\Re(c_{25})\} = \Re(c_{23})$.
Set $\underline{a}_3 = \underline{c}_{22} = (10,20,30,40)$
and label node 3 with [(10,20,30,40), 2]
Set $i = 3$ and repeat step 2
Step 6: $S_3 = \{4\}$
$(\because \Re(\underline{c}_{35}) = 0.$ Hence node 5 cannot be included in $S_3)$
Step 7: K = 4, and set $\underline{a}_4 = \underline{c}_{34} = (0,5,10,15)$ and label node 4 with [(0,5,10,15), 3] Set $i = 4$ and repeat step 2.
Step 8: $S_4 = \{5\}$
($\because$ nodes 1 and 3 are already labeled. Hence, they cannot be included in S_4)
Step 9: K = 5 and set $\underline{c}_5 = \underline{c}_{45} = (10,20,30,30)$ and label node 5 with $[(10,20,30,30),4]$. Breakthrough has been achieved. Go to step 5.
Step 10: The obtained breakthrough path is $1 \to 2 \to 3 \to 4 \to 5$. Thus $N_2 = \{1,2,3,4,5\}$ and

$$\begin{aligned}\underline{f}_2 =& \min\{\underline{a}_1,\underline{a}_2,\underline{a}_3,\underline{a}_4,\underline{a}_5\}\\ =& \min\{(\infty,\infty,\infty,\infty),(5,15,20,20),(10,20,30,40),\\ & (0,5,10,15)\}\\ =&(0,5,10,15)\end{aligned}$$

The fuzzy residual capacities along path N_2 are

$$\begin{aligned}(\underline{c}_2,c_{21}) &= ((5,15,20,20)-(0,5,10,15),(0,0,0,0)\\ &\quad +(0,5,10,15))\\ &= ((0,15,35,30),(0,5,10,15))\\ (c_{23},\underline{c}_{32}) &= ((10,20,30,40)-(0,5,10,15),(0,0,0,0)\\ &\quad +(0,5,10,15))\\ &= ((5,20,45,50),(0,5,10,15))\\ (\underline{c}_{34},\underline{c}_{43}) &= ((0,5,10,15),(0,5,10,15),(0,5,20,20)\\ &\quad +(0,5,10,15))\\ &= ((-5,5,25,25),(0,10,30,35))\\ (\underline{c}_{45},\underline{c}_{54}) &= ((10,20,30,30)-(0,5,10,1),(0,0,0,0)\\ &\quad +(0,5,10,15))\\ &= ((5,20,45,40),(0,5,10,1))\\ \underline{f}_2 &= (0,5,10,15)\end{aligned}$$

Iteration 3: Step 1: Se $\underline{a}_1 = (\infty,\infty,\infty,\infty,)$
and label node 1 with $[(\infty,\infty,\infty,\infty,),-]$ set $i = 1$.
Step 2: $S_1 = \{2,3,4\}$
Step 3: K = 2 because $\max\{\Re(c_{12}),\Re(c_{13}),\Re(c_{14})\}\cdot = \Re(c_{12})$
Set $\underline{a}_2 = \underline{c}_{12} = (0,15,35,30)$
and label node 2 with [(0,15,35,30), 1].
Set $i = 2$, and repeat step 2.
Step 4: $S_2 = \{3,5\}$
Step 5: K = 3, because $\max\{\Re(C_{23}),\Re(C_{25}\} = \Re(C_{25})$
Set $\underline{a}_5 = \underline{c}_{25} = (10,20,30,30)$

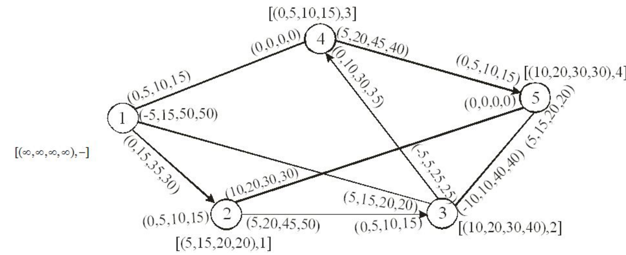

Figure 4

and label node 5 with $[(10,20,30,30),2]$. Breakthrough has been achieved: go to step 5.
Step 6: The obtained breakthrough path is $1 \to 2 \to 5$ then $N_3 = \{1,2,5\}$ and

$$\begin{aligned}\underline{f}_3 =& \min\{\underline{a}_1,\underline{a}_2,\underline{a}_5\}\\ =& \min\{(\infty,\infty,\infty,\infty),(0,15,35,30),(10,20,30,30)\}\\ =&(0,15,35,30)\end{aligned}$$

The fuzzy residuals along the path of N_3 are:

$$\begin{aligned}(\underline{c_{12},c_{21}}) =&((0,15,35,30)-(0,15,35,30),(0,5,10,15)\\ &+(0,15,35,30))\\ =&((-15,15,65,65),(0,20,45,45))\\ (c_{25},c_{52}) =&((10,20,30,30)-(0,15,35,30),(0,0,0,0)\\ &+(0,15,35,30))\\ =&((-5,20,60,65),(0,15,35,30)\end{aligned}$$

Iteration 4:

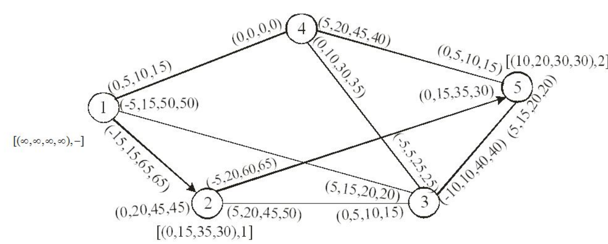

Figure 5

Step 1: Set $\underline{a}_1 = (\infty,\infty,\infty,\infty)$
and label node 1 with $[(\infty,\infty,\infty,\infty),-]$ set $i = 1$.
Step 2: $S_1 = \{2,3,4\}$
Step 3: K = 3, because $\max\{\Re(c_{12}),\Re(c_{13}),\Re(\underline{c}_{14})\} = \Re(c_{13})$.
Set $\underline{a}_3 = \underline{c}_{13} = (-5,15,50,50)$.
Label node 3 with [(-5,15,50,50), 1] Set $i = 3$ and repeat step 2.
Step 2: $S_3 = \{2,4,5\}$
Step 3: K = 2, because $\max\{\Re(\underline{c}_{32}),\Re(c_{34}),\Re(\underline{c}_{35})\} = \Re(c_{32})$.
Set $\underline{a}_2 = \underline{C}_{32} = (0,5,10,15)$.
Label node 2 with [(0,5,10,15), 3] Set $i = 2$ and repeat step 2.
Step 2: $S_2 = \{5\}$ and $\underline{a}_5 = \underline{C}_{25} = (-5,20,60,65)$
Label node 5 with [(-5,20,60,65), 2] Breakthrough has been

achieved. Go to step 5.
Step 5: The obtained breakthrough
Path: $1 \to 3 \to 2 \to 5$. Then

$$
\begin{aligned}
N_4 &= \{1,3,2,5\} \\
\underline{f}_4 &= \min\{\underline{a}_1, \underline{a}_3, \underline{a}_2, \underline{a}_5\} \\
&= \min\{(\infty,\infty,\infty,\infty), (-5,15,50,50), (0,5,10,15), \\
&\quad (-5,20,60,65)\} \\
&= (0,5,10,15)
\end{aligned}
$$

The fuzzy residuals along the path N_4 are:

$$
\begin{aligned}
(\underline{c_1}3, \underline{c_{31}}) &= ((-5,25,50,50) - (0,5,10,15), (5,15,20,20) \\
&\quad + (0,5,10,15)) \\
&= ((-10,15,65,60), (5,20,30,35)) \\
(c_{32}, c_{23}) &= ((0,5,10,15) - (0,5,10,15), (5,20,45,50) \\
&\quad + (0,5,10,15)) \\
&= ((-5,5,25,15), (5,25,55,65)) \\
(\underline{c}_{25}, \underline{c}_{52}) &= ((-5,20,60,65) - (0,5,10,15), (0,15,35,30) \\
&\quad + (0,5,10,15)) \\
&= ((-10,20,75,75), (0,20,45,45))
\end{aligned}
$$

Iteration 5 :

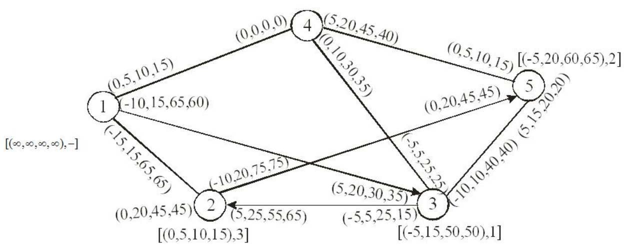

Figure 6

Step 1: Set $\underline{a}_1 = (\infty,\infty,\infty,\infty)$
and label node 1 with $[(\infty,\infty,\infty,\infty),-]$ set $i = 1$.
Step 2: $S_1 = \{2,3,4\}$
Step 3: $K = 4$.
Because $\max\{(c_{12}), \Re(c_{13}), \Re(c_{14})\} = \Re(c_{44})$.
Set $\underline{a}_4 = \underline{c}_4 = (0,5,10,15)$ Label node 4 with [(0,5,10,15), 1] set $i = 4$ and repeat step 2.
Step 4: $S_4 = \{3,5\}$
Step 5: $K = 5$. Because $\max\{\Re(\underline{c}_{43}), \Re(\underline{c}_{45})\} = \Re(\underline{c}_{45})$
Set $\underline{a}_5 = \underline{c}_{45} = (5,20,45,40)$ Label node 5 with [(5,20,45,40), 4] Breakthrough has been achieved. Go to step 5.
Step 6: The obtained breakthrough path is $1 \to 4 \to 5$ Thus

$$
\begin{aligned}
N_4 &= \{1,4,5\} \\
\underline{f_5} &= \min\{\underline{a}_1, \underline{a}_4, \underline{a}_5\} = \min\{(\infty,\infty,\infty,\infty), (0,5,10,15), \\
&\quad (5,20,45,40)\} \\
&= (0,5,10,15)
\end{aligned}
$$

The fuzzy residuals along the path of N_5 are:

$$
\begin{aligned}
(\underline{c_1}, \underline{c}_{41}) &= ((0,5,10,15) - (0,5,10,15), (0,0,0,0) \\
&\quad + (0,5,10,15)) \\
&= ((-5,5,25,25), (0,5,10,15)) \\
(\underline{c}_{45}, \underline{c}_{54}) &= ((5,20,45,40) - (0,5,10,15), (0,5,10,15) \\
&\quad + (0,5,10,15)) \\
&= ((0,20,60,50), (0,10,20,30))
\end{aligned}
$$

Iteration 6: All the arcs out of node 1 have zero fuzzy

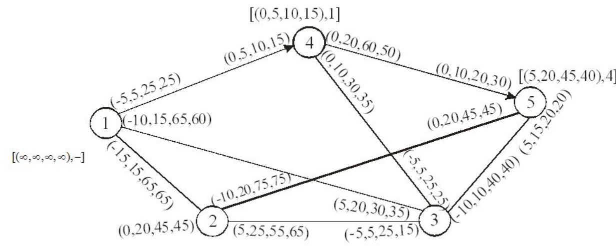

Figure 7

residuals. Hence, no further breakthrough are possible. Go to step 6 to determine fuzzy maximal flow.
Step 1: Fuzzy maximal flow in the network is

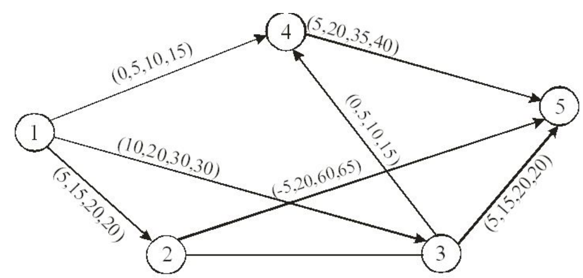

Figure 8

$$
\begin{aligned}
\underline{F} &= \underline{f}_1 + \underline{f}_2 + \underline{f}_3 + \underline{f}_4 + \underline{f}_5 \\
&= (5,15,20,20) + (0,5,10,15) + (0,5,35,30) \\
&\quad + (0,5,10,15) + (0,5,10,15) \\
&= (5,45,85,95)
\end{aligned}
$$

The fuzzy maximal flow in the different arcs as shown in table 1.

Table 1. Fuzzv optimal flow in different arcs

Arc	Fuzzy flow amount	Direction
(1,2)	(5,15,20,20)	$1 \to 2$
(1,3)	(10,20,30,30)	$1 \to 3$
(1,4)	(0,5,10,15)	$1 \to 4$
(2,3)	(0,0,0,0)	-
(2,5)	(-5,20,60,65)	$2 \to 5$
(3,4)	(0,5,10,15)	$3 \to 4$
(3,5)	(5,15,20,20)	$3 \to 5$
(4,5)	(5,20,45,40)	$4 \to 5$

5. Result and discussions

The fuzzy optimal flow is $\underline{F} = (5, 45, 85, 95)$. The obtained result can be explained as follows:

1. According to decision maker the amount of flow between source and sink is greater than 5 and less than 95.
2. The overall level of satisfaction of the decision maker about the statement that the fuzzy maximal flow will be 45 to 85 units is 100%.
3. The overall level of satisfaction of the decision maker for the remaining amount of flow can be obtained as follows:

Let x represents the amount of flow, then the overall level of satisfaction of the decision maker of $x = \mu_{\underline{F}}(x) \times 100$.

$$\mu_{\underline{F}}(x) = \begin{cases} \frac{x-5}{40} & 5 < x < 45 \\ 1 & 45 \leq x \leq 85 \\ \frac{x-95}{-10} & 85 < x \leq 95 \\ 0 & \text{otherwise} \end{cases}$$

6. Conclusion

The proposed fuzzy maximal flow algorithm can be extended for solving real life situation. This algorithm is quite general in nature and can be extended to solve the other network flow problems like shortest path problems.

References

[1] R. K. Ahuja, T. L. Magnanti and J. B. Orlin, *Network Flows, Theory, Algorithms and applications*, Prentice Hall, New Jersey, 1993.

[2] M. S. Bazarra, J.J. Jarvis and H. D. Sherali, *Linear Programming and Network Flows*, 2nd Edition, Wiley, New York, 1990.

[3] L. Campos and A. Gonzalez Munoz, A subjective approach for ranking fuzzy number, *Fuzzy Sets and Systems*, 29(2000), 145-153.

[4] L. Campos and J. L. Verdegay, Linear programming problems and ranking of fuzzy numbers, *Fuzzy Sets and Systems*, 32(1989), 1-11.

[5] S. Chanas, M. Delgado, J. L. Verdegay and M. Vila, Fuzzy optimal flow on imprecise structures, *European Journal of Operational Research*, 83(1995), 568-580.

[6] S. Chanas and W. Kolodziejczyk, Maximum flow in a network with fuzzy arc capacities, *Fuzzy Sets and Systems*, 8(1982), 165-173.

[7] S. Chanas and W. Kolodziejczyk, Real-valued flows in a network with fuzzy arc capacities, *Fuzzy Sets and Systems*, 13(1984), 139-151.

[8] S. Chanas and W. Kolodziejczyk, Integer flows in network with fuzzy capacity constraints, *Networks*, 16(1986), 17-31.

[9] P. T. Chang and E. S. Lee, Ranking of fuzzy Sets based on the concept of existence, *Computers and Mathematics with Applications*, 27(1994), l-21.

[10] S. H. Chen, Operations on fuzzy numbers with function principal, *Tamkang Journal of Management Sciences*, 6(1985), 13-25.

[11] S. J. Chen and S. M. Chen, A new method for handling multicriteria fuzzy decision making problems using FN-IOWA operators, *Cybernetics and Systems,* 34(2003), 109-137.

[12] S. J. Chen and S. M. Chen, Fuzzy risk analysis on the ranking of generalized trapezoidal fuzzy numbers, *Applied Intelligence*, 26(2007), 1-11.

[13] S. M. Chen and J. H. Chen, Fuzzy risk analysis based on the ranking generalized fuzzy numbers with different heights and different spreads, *Expert Systems with Applications,* 36(2009), 6833-6842.

[14] S. H. Chen and G. C. Li, Representation, ranking and distance of fuzzy number with exponential membership function using graded mean integration method, *Tamsui Oxford Journal of Mathematical Sciences*, 16(2000), 125-131.

[15] S. M. Chen and C. H. Wang, Fuzzy risk analysis based on ranking fuzzy numbers using alpha-cuts, belief features and signal/noise ratios, *Expert Systems with Applications,* 36(2009), 5576-5581.

[16] C. H. Cheng, A new approach for ranking fuzzy numbers by distance method, *Fuzzy Sets and Systems*, 95(1998), 307-317.

[17] T. C. Chu and C. T. Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, *Computers and Mathematics with Applications*, 43(2002), 111-117.

[18] A. Diamond, A fuzzy max-flow min-cut theorem, *Fuzzy Sets and Systems*, 119(2001), 139-148.

[19] L. R. Ford and D. R. Fulkerson, Maximal flow through a network, *Canadian Journal of Mathematics*, 8(1956), 399-404.

[20] P. Fortemps and M. Roubens, Ranking and defuzzification methods based on area compensation, *Fuzzy Sets and Systems,* 82(1996), 319-330.

[21] D. R. Fulkerson and G. B. Dantzig, Computation of maximum flow in network, *Naval Research Logisics Quarterly,* 2(1955), 277-283.

[22] F. S Hernandes, M. T. Lamata, M. T. Takahashi, A. Yamakami and J. L. Verdegay, An algorithm for the fuzzy maximum flow problem, *In: Proceeding of IEEE International Fuzzy Systems Conference,* (2007), 1-6.

[23] C. H. Hsieh and S. H. Chen, Similarity of generalized fuzzy numbers with graded mean integration representation, *In: Proceedings of the Eighth International Fuzzy System Association World Congress,* Taipei, Taiwan, Republic of China, 2(1999), 551-555.

[24] R. Jain, Decision-making in the presence of fuzzy variables, *IEEE Transactions on Systems, Man and Cybernet-*

ics, 6(1976), 698-703.

[25] X. Ji, L. Yang and Z. Shao, Chance constrained maximum flow problem with arc capacities, *Lecture Notes in Computer Science,* Springer-Verlag, 4114(2006), 11-19.

[26] A. Kaufmann and M. M. Gupta, *Introduction to Fuzzy Arithmetic: Theory and Applications, Van Nostrand Reinhold*, New York, 1985.

[27] K. Kim and F. Roush, Fuzzy flows on network, *Fuzzy Sets and Systems*, 8(1982), 35-38.

[28] A. Kumar, N. Bhatia and M. Kaur, A new approach for solving fuzzy maximal flow problems, *Lecture Notes in Computer Science,* Springer-Verlag, 5908(2009), 278-286.

[29] A. Kumar, P. Singh, A. Kaur and P. Kaur, RM approach for ranking of generalized trapezoidal fuzzy numbers, *Fuzzy Information and Engineering*, 1(2010), 37-47.

[30] A. Kumar, S. P. Yadav and S. Kumar, Fuzzy system reliability using different types of vague sets, *International Journal of Applied Science and Engineering*, 6(2008), 71-83.

[31] T. S. Liou and M. J. Wang, Ranking fuzzy numbers with integral value, *Fuzzy Sets and Systems*, 50(1992), 247-255.

[32] S. T. Liu and C. Kao, Network flow problems with fuzzy are lengths, *IEEE Transactions on Systems, Man and Cybernetics*, 34(2004), 765-769.

[33] G. S. Mahapatra and T. K. Roy, Fuzzy multi-objective mathematical programming on reliability optimization model, *Applied Mathematics and Computation,* 174(2006), 643-659.

[34] H. A. Taha, *Operational Research: An Introduction*, Prentice-Hall, New Jersey, 2003.

[35] Y. J. Wang and H. S. Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points, *Computers and Mathematics with Applications,* 55(2008), 2033-2042.

[36] D. Yong, S. Wenkang, D. Feng and L. Qi, A new similarity measure of generalized fuzzy numbers and its application to pattern recognition, *Pattern Recognition Letters,* 25(2004), 875-883.

[37] L. A. Zadeh, Fuzzy sets, *Information and Control*, 8(1965), 338-353.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆