Malaya Journal of Matematik, Vol. 9, No. 1, 89-94, 2021

https://doi.org/10.26637/MJM0901/0014

# Some new open sets in $\mu_N$ topological space

G. Hari Siva Annam[1]* and N. Raksha Ben [2]

**Abstract**

In this article we lay a stress on the open sets which have a enormous impact on the $\mu_N$ topological space. Several types of $\mu_N$ -Open sets were contrived and their roles and natures were enunciated. Also Continuous functions on the $\mu_N$ topological spaces were asseverated.

**Keywords**

$\mu_N$-Semi open sets, $\mu_N$- Pre Open Sets, $\mu_N$-$\alpha$ Open sets, $\mu_N$-$\beta$ open Sets, $\mu_N$-Continuous.

**AMS Subject Classification**

03E72.

[1] *PG and Research Department of Mathematics, Kamaraj College , Thoothukudi-628003, Tamil Nadu, India.*
[2] *Research Scholar [Reg no.19212102092010], PG and Research Department of Mathematics, Kamaraj College, Thoothukudi-628003, Tamil Nadu, India.*
[1,2] Affliated to Manonmaniam Sundaranar University, Abishekapatti , Tirunelveli-627012, Tamil Nadu, India.
***Corresponding author**: [1] hsannam@yahoo.com; [2]rakshaarun218@gmail.com
**Article History**: Received **14** October **2020**; Accepted **29** December **2020**


## Contents



## 1. Introduction

In 1965, Zadeh[15] found out fuzzy set theory which plays a vital role in real life application in order to cope up with uncertainty. In 1968, Chang[3] invented fuzzy topology which gives an utterance in the field of topology. Keeping these two as aspirations, In 1983, K.Attanassov [1] explored intuitionistic fuzzy sets by giving attention to both membership and non-membership of the elements. Several notions of fuzzy sets and fuzzy topology were explored after the existence of intuitionistic fuzzy sets .In 1997, by marking Attanassov's work as inspiration ,Coker[5] worked with the Intuitionististic fuzzy sets by applying the concepts of fuzziness and got Intuitionistic fuzzy topological space which helped Attanossov to discover the interval valued intuitionistic fuzzy set on a universe $X$ as an object $A=\{<x,\mu_A(x),\sigma_A(x),\gamma_A(x)>: x\in X\}$. F.Smarandache[6] foccused his views towards the degree of indeterminancy and which led into neutrosophic sets. Later on , A.A.Salama and S.A .Albowi[11] introduced the neutrosophic topological spaces with the help of neutrosophic sets and proceeding this A.A. Salama, F.Smarandache and Valeri Kromov[13] introduced the continuous functions in neutrosophic topological spaces. By setting all these works together as inspiration, In 2020 we[10] contrived $\mu_N$ Topological Space and their basic properties were discussed.In this discourse,we explore our thoughts towards various open sets in $\mu_N$ Topological Space which can be developed later and some of their basic properties were discussed. Also, $\mu_N$ continuous functions were introduced and also their features were contemplated.

## 2. Preliminaries

The concepts given here are used to brush up our memories regarding the basic concepts of $\mu_N$ Topological Space.

**Definition 2.1.** *[11] Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form* $A=\{<x,\mu_A(x),\sigma_A(x),\gamma_A(x)>: x\in X\}$ *where* $\mu_A(x),\sigma_A(x)$ *and* $\gamma_A(x)$ *which represents the degree of membership function , the degree of indeterminacy and the degree of non-membership function respectively of each element* $x\in X$ *to the set A.*

**Remark 2.2.** *[11] A neutrosophic set* $A=\{<x,\mu_A(x),\sigma_A(x),\gamma_A(x)>: x\in X\}$ *can be identified to an ordered triple* $A=\{<\mu_A(x),\sigma_A(x),\gamma_A(x)>\}$ *in* $]^-0,1^+[$ *on X.*

**Remark 2.3.** *[11] For the sake of simplicity, we shall use the symbol $A = \{< \mu(x), \sigma(x), \gamma(x) >\}$ for the neutrosophic set $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$*

**Remark 2.4.** *Every intuitionistic fuzzy set A is a non empty set in X is obviously on Neutrosophic sets having the form $A = \{< \mu_A(x), 1-\mu_A(x)+\sigma_A(x), \gamma_A(x) >: x \in X\}$.In order to construct the tools for developing Neutrosophic Set and Neutrosophic topology, here we introduce the neutrosophic sets $0_N$ and $1_N$ in X as follows:*
*$0_N$ may be defined as follows*
$(0_1)0_N = \{< x, 0, 0, 1 >: x \in X\}$
$(0_2)0_N = \{< x, 0, 1, 1 >: x \in X\}$
$(0_3)0_N = \{< x, 0, 1, 0 >: x \in X\}$
$(0_1)0_N = \{< x, 0, 0, 0 >: x \in X\}$
*$1_N$ may be defined as follows*
$(1_1)1_N = \{< x, 1, 0, 0 >: x \in X\}$
$(1_2)1_N = \{< x, 1, 0, 1 >: x \in X\}$
$(1_3)1_N = \{< x, 1, 1, 0 >: x \in X\}$
$(1_4)1_N = \{< x, 1, 1, 1 >: x \in X\}$

**Definition 2.5.** *[11] Let $A = \{< \mu_A, \sigma_A, \gamma_A >\}$ be a NS on X, then the complement of the set A [ $C(A)$ for short] may be defined in three ways as follows:*
$(C_1)C(A) = A = \{< x, 1-\mu_A(x), 1-\sigma_A(x), 1-\gamma_A(x) >: x \in X\}$
$(C_2)C(A) = A = \{< x, \gamma_A(x), \sigma_A(x), \mu_A(x) >: x \in X\}$
$(C_3)C(A) = A = \{< x, \gamma_A(x), 1-\sigma_A(x), \mu_A(x) >: x \in X\}$

**Definition 2.6.** *[11] Let X be a non-empty set and neutrosophic sets A and B in the form*
$A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$ *and*
$B = \{< x, \mu_B(x), \sigma_B(x), \gamma_B(x) >: x \in X\}$ *. Then we may consider two possibilities for definitions for subsets($A \subseteq B$).*
*$A \subseteq B$ may be defined as :*
$(A \subseteq B)$
$\iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x), \forall x \in X$
$(A \subseteq B)$
$\iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x), \forall x \in X$

**Proposition 2.7.** *[11] For any neutrosophic set A, the following conditions holds: $0_N \subseteq A, 0_N \subseteq 0_N$*
$A \subseteq 1_N, 1_N \subseteq 1_N$

**Definition 2.8.** *[11] Let X be a non empty set and $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$,*
$B = \{< x, \mu_B(x), \sigma_B(x), \gamma_B(x) >: x \in X\}$ *are NSs.*
*Then $A \cap B$ may be defined as :*
$(I_1)A \cap B =< x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$
$(I_2)A \cap B =< x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$
*$A \cup B$ may be defined as :*
$(I_1)A \cup B =< x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$
$(I_2)A \cup B =< x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$

**Definition 2.9.** *[10] A $\mu_N$ topology is a non - empty set X is a family of neutrosophic subsets in X satisfying the following axioms:*
$(\mu_{N_1})0_N \in \mu_N$
$(\mu_{N_2})G_1 \cup G_2 \in \mu_N$ *for any* $G_1, G_2 \in \mu_N$.

**Remark 2.10.** *[10] The elements of $\mu_N$ are $\mu_N$-open sets and their complement is called $\mu_N$ closed sets.*

**Definition 2.11.** *[10] Let $(X, \mu_N)$ be a $\mu_N$ TS and $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >\}$ be a neutrosophic set in X. Then the $\mu_N$- Closure is the intersection of all $\mu_N$ closed sets containing A.*

**Definition 2.12.** *[10] Let $(X, \mu_N)$ be a $\mu_N$ TS and $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >\}$ be a neutrosophic set in X. Then the $\mu_N$- Interior is the union of all $\mu_N$ open sets contained in A.*

**Definition 2.13.** *[13] A NS A of a NTS X is said to be*

*(i) a neutrosophic pre-open if $A \subseteq NInt(NClA)$*

*(ii) a neutrosophic Semi-open if $A \subseteq NCl(NIntA)$*

*(iii) a neutrosophic $\alpha$ -open if $A \subseteq NInt(NCl(NIntA))$.*

**Definition 2.14.** *[13] Let $(X, \tau)$ and $(Y, \sigma)$ be neutrosophic topological spaces. Then a map $f : (X, \tau) \to (Y, \sigma)$ is called neutrosophic continuous (in short N-continuous) function if the inverse image of every neutrosophic open set in $(Y, \sigma)$ is neutrosophic open set in $(X, \tau)$.*

## 3. Open sets in $\mu_N$ topological space

**Definition 3.1.** *A neutrosophic set in a $\mu_N$ topological space is said to be $\mu_N$ Semi Open if $A \subseteq \mu_N Cl(\mu_N IntA)$.*

**Theorem 3.2.** *Every $\mu_N$- open set is $\mu_N$-Semi Open .*

*Proof.* Let $U$ be a $\mu_N$- open set in $X$ which implies us that $\mu_N IntU = U$.
Hence, we get $U \subseteq \mu_N Cl(U) \Rightarrow U = \mu_N Cl(\mu_N IntU)$ .Thus, $U \subseteq \mu_N Cl(\mu_N IntU)$.Hence , it is $\mu_N$-Semi Open. □

**Remark 3.3.** *Converse of the theorem need not be true.*
*It is furnished by the following example.*
*Let $X = \{a\}$;$\mu_N = \{0_N, A, B, C\}$ where $A =< 0.3, 0.3, 0.5 >$ $, B =< 0.1, 0.2, 0.3 >, C =< 0.3, 0.2, 0.3 >, D =< 0.3, 0.6, 0.2 >, E =< 0.3, 0.8, 0.5 >$. Here, The $\mu_N$-Semi Open sets are $\{0_N, A, B, C, E\}$ which tell us that E is $\mu_N$-Semi Open but it is not $\mu_N$- open set .*

**Definition 3.4.** *A neutrosophic set in a $\mu_N$ topological space is said to be $\mu_N$ Pre-Open if $A \subseteq \mu_N Int(\mu_N ClA)$.*

**Theorem 3.5.** *Every $\mu_N$- open set is $\mu_N$-Pre Open.*

*Proof.* We have $A \subseteq \mu_N ClA$ which implies us obviously that $\mu_N IntA \subseteq \mu_N Int(\mu_N ClA)$.As we all know that $\mu_N IntA \subseteq A$. Hence , we get $A \subseteq \mu_N Int(\mu_N ClA)$. □

**Remark 3.6.** *The reversal concept of the above theorem need be necessarily true. The following problem gives us the crystal clear idea about it.*

**Example 3.7.** *Let $X = \{a,b\}$, $Y = \{u,v\}$ and $(X,\tau)$, $(Y,\sigma)$ be the $\mu_N$ TS where $\tau = \{A,B,C,D,0_N\}$ and $\sigma = \{D,E,0_N\}$, $A =< 0.7,0.3,0.8 >< 0.5,0.8,0.9 >, B =< 0.4,0.9,0.9 >< 0.3,0.9,0.9 >, C =< 0.5,0.8,0.7 >< 0.5,0.8,0.8 >, D =< 0.5,0.8,0.8 >< 0.5,0.8,0.7 >, E =< 0.3,0.9,0.9 >< 0.4,0.9,0.9 >$. Here $A,B,C,D,E,0_N$ are $\mu_N$-Pre Open sets of $(X,\tau)$. Particularly $E$ is $\mu_N$-Pre Open set of $(X,\tau)$ but it is not $\mu_N$-open in $(X,\tau)$.*

**Definition 3.8.** *A neutrosophic set in a $\mu_N$ topological space is said to be $\mu_N\alpha$-Open if $A \subseteq \mu_N Int(\mu_N Cl(mu_N Int A))$.*

**Theorem 3.9.** *Every $\mu_N$- open set is $\mu_N\alpha$- Open.*

*Proof.* Let $A$ be $\mu_N$- open set which yields us that $\mu_N Int A = A$. We have $A \subseteq \mu_N Cl(A)$, From this we obtain $\mu_N Int A \subseteq \mu_N Int(\mu_N Cl(\mu_N Int A)) \Rightarrow A \subseteq \mu_N Int(\mu_N Cl(\mu_N Int A))$.
Hence. It is $\mu_N\alpha$- Open. □

**Remark 3.10.** *Converse statement of the theorem need not be true. The Scenario is explained below.*
*Let $X = \{a\}$; $\mu_N = \{0_N,A,C\}$ where $A =< 0.7,0.8,0.9 >, B =< 0.3,0.4,0.6 >, C =< 0.9,0.7,0.6 >$. Here, The $\mu_N\alpha$-Open sets are $\{0_N,A,B,C,\}$. In this , $B$ is $\mu_N\alpha$- Open set but it is not $\mu_N$- open set .*

**Theorem 3.11.** *Every $\mu_N$- open set is $\mu_N\beta$- Open.*

*Proof.* Let $A$ be $\mu_N$- open set then that $\mu_N Int A = A$.We have $A \subseteq \mu_N Cl A$ which gives us that $\mu_N Int A \subseteq \mu_N Int(\mu_N Cl A) \Rightarrow A \subseteq \mu_N Int(\mu_N Cl A)$.
Now we have , $A \subseteq \mu_N Cl A$ and $A \subseteq \mu_N Int(\mu_N Cl A)$. Both togetherly gives us $A \subseteq \mu_N Cl(\mu_N Int(\mu_N Cl A))$. □

**Remark 3.12.** *Converse of the above theorem need not be true which explained with the help of an example as below. Let $X = \{a\}$; $\mu_N = \{0_N,A,B\}$ where $A =< 0.3,0.6,0.9 >, B =< 0.4,0.6,0.7 >, C =< 0.5,0.7,0.8 >$.Here, The $\mu_N\beta$-Open sets are $\{0_N,A,B,C\}$. In this , $B$ is $\mu_N\beta$- Open set but it is not $\mu_N$- open set .*

**Theorem 3.13.** *Every $\mu_N$ Pre-open set is $\mu_N\beta$- Open.*

*Proof.* Let $A$ be a $\mu_N$ pre-open set which leads us to have $A \subseteq \mu_N Int(\mu_N Cl A)$. Now let us remind the basic property , $A \subseteq \mu_N Cl A$. By making use of this we obtain that $\mu_N Cl A \subseteq \mu_N Cl(\mu_N Int(\mu_N Cl A))$ which obviously yields that $A \subseteq \mu_N Cl(\mu_N Int(\mu_N Cl A))$. □

**Remark 3.14.** *The reverse statement of the above theorem is not true. Let me picturize the concept using an example. Let $X = \{a\}$, $\mu_N = \{0_N,A,B,C\}$, $A =< 0.3,0.3,0.5 >, B =< 0.1,0.2,0.3 >, C =< 0.3,0.2,0.3 >, D =< 0.3,0.6,0.2 >, E =< 0.3,0.8,0.5 >$. The $\mu_N$ Pre-open sets are $\{0_N,A,B,C\}$ and $\mu_N\beta$- Open sets are $\{0_N,A,B,C,D,E,1_N\}$. Here $D,E,1_N$ are $\mu_N\beta$- Open sets but not $\mu_N$ Pre-open sets.*

**Theorem 3.15.** *Every $\mu_N\alpha$-open set is $\mu_N$ Semi- Open.*

*Proof.* Let $A$ be a $\mu_N\alpha$-open set which leads us to have $A \subseteq \mu_N Int(\mu_N Cl(\mu_N Int A))$.From this it can be easily derived as $A \subseteq (\mu_N Cl(\mu_N Int A))$. Hence, Every $\mu_N\alpha$-open set is $\mu_N$ Semi-Open. □

**Remark 3.16.** *The reversal statement of the above theorem need not be true. Let $X = \{a\}$, $\mu_N = \{0_N,A,B,C\}$, $A =< 0.3,0.3,0.5 >, B =< 0.1,0.2,0.3 >, C =< 0.3,0.2,0.3 >, D =< 0.3,0.6,0.2 >, E =< 0.3,0.8,0.5 >$.*
*The $\mu_N$ Semi-open sets are $\{0_N,A,B,C,E\}$ and $\mu_N\alpha$- Open sets are $\{0_N,A,B,C,\}$. Here, $E$ is $\mu_N$ Semi- Open set but not $\mu_N\alpha$-open set.*

**Definition 3.17.** *A neutrosophic set in a $\mu_N$ topological space is said to be*

*(i) $\mu_N$-Semi Closed if $\mu_N Int(\mu_N Cl A) \subseteq A$.*

*(ii) $\mu_N$-Pre Closed if $\mu_N Cl(\mu_N Int A) \subseteq A$.*

*(iii) $\mu_N$ $\alpha$- Closed if $\mu_N Cl(\mu_N Int(\mu_N Cl A)) \subseteq A$.*

*(iv) $\mu_N$ $\beta$-Closed sets if $\mu_N Int(\mu_N Cl(\mu_N Int A)) \subseteq A$.*

## 4. $\mu_N$- Continuous Functions

**Definition 4.1.** *A mapping $f : (X,\tau) \to (Y,\sigma)$ is said to be $\mu_N$- Continuous function if the inverse image of $\mu_N$- closed sets in $(Y,\sigma)$ is $\mu_N$- closed in $(X,\tau)$.*

**Example 4.2.** *Let $X = \{a,b,c\}$ and $Y = \{u,v,w\}$ and $\tau = \{A,B,0_N\}$, $\sigma = \{C,D,0_N\}$ where $A =< 0.5,0.8,0.9 >< 0.7,0.3,0.8 >< 0.2,0.5,0.7 >, B =< 0.7,0.2,0.4 >< 0.8,0.2,0.1 >< 0.8,0.4,0.3 >, C =< 0.7,0.3,0.8 >< 0.5,0.8,0.9 >< 0.2,0.5,0.7 >, D =< 0.8,0.2,0.1 >< 0.7,0.2,0.4 >< 0.8,0.4,0.3 >$. We define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = a, f(c) = c$.*
*Hence, we get $f^{-1}(C) =< 0.5,0.8,0.9 >< 0.7,0.3,0.8 >< 0.2,0.5,0.7 >= A$ and $f^{-1}(D) =< 0.7,0.2,0.4 >< 0.8,0.2,0.1 >< 0.8,0.4,0.3 >= B$.*
*Hence $f : (X,\tau) \to (Y,\sigma)$ is $\mu_N$- Continuous.*

**Theorem 4.3.** *Let $f : (X,\tau) \to (Y,\sigma)$ be a $\mu_N$- Continuous function if and only if the inverse image of $\mu_N$- open sets in $(Y,\sigma)$ is $\mu_N$- open in $(X,\tau)$.*

*Proof.* **Essential condition:** Let $f : (X,\tau) \to (Y,\sigma)$ be a $\mu_N$-Continuous function and $U$ be a $\mu_N$- open sets in $(Y,\sigma)$. Since $f$ is $\mu_N$- Continuous , $f^{-1}(Y-U) = X - f^{-1}(U)$ is $\mu_N$- closed set in $(X,\tau)$ and hence $f^{-1}(U)$ is $\mu_N$- open in $(X,\tau)$.
**Sufficient Condition:** Assume that $f^{-1}(V)$ is $\mu_N$- open in $(X,\tau)$ for each $\mu_N$- open set in $(Y,\sigma)$.Let $V$ be a $\mu_N$- closed set in $(Y,\sigma)$ which yields that $Y-V$ is $\mu_N$- open set in $(Y,\sigma)$.Now, $f^{-1}(Y-V) = X - f^{-1}(V)$ is $\mu_N$- closed set in $(X,\tau)$ which implies us that $f^{-1}(V)$ is $\mu_N$- open in $(X,\tau)$.
Hence, $f$ is $\mu_N$- Continuous . □

**Theorem 4.4.** *Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$- topological spaces. If $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\rho)$ are $\mu_N$- Continuous then $g\circ f:(X,\tau)\to(Z,\rho)$ is $\mu_N$- Continuous.*

*Proof.* Let $U$ be any $\mu_N$- open set in $(Z,\rho)$. Since $g$ is $\mu_N$-Continuous, $g^{-1}(U)$ is $\mu_N$- open and hence it is $\mu_N$- open in $(Y,\sigma)$ . Also ,Since $f$ is $\mu_N$- Continuous ,$f^{-1}(g^{-1}(U)) = (g\circ f)^{-1}(V)$ is $\mu_N$- open. Hence, $g\circ f$ is $\mu_N$- Continuous □

**Definition 4.5.** *A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be $\mu_N$-Semi Continuous function if the inverse image of $\mu_N$-closed sets in $(Y,\sigma)$ is $\mu_N$-Semi closed sets in $(X,\tau)$.*

**Theorem 4.6.** *Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N$-Semi Continuous function if and only if the inverse image of $\mu_N$- open sets in $(Y,\sigma)$ is $\mu_N$-Semi open in $(X,\tau)$. Proof is similar to the proof of theorem 4.4.*

**Theorem 4.7.** *Every $\mu_N$- Continuous is $\mu_N$-Semi Continuous.*

*Proof.* Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N$- Continuous mapping. Let $A$ be a $\mu_N$- open set in $(Y,\sigma)$. By hypothesis we get $f^{-1}(A)$ is $\mu_N$- open in $(X,\tau)$. Since, every $\mu_N$- open set is $\mu_N$-Semi Open, we get $f^{-1}(A)$ is $\mu_N$- Semi open in $(X,\tau)$. Thus we obtain that $f$ is $\mu_N$-Semi Continuous. □

**Remark 4.8.** *Converse of theorem 4.7 is not true. This condition can be explored by the following example.*

**Example 4.9.** *Let $X=\{a,b\}$ and $Y=\{u,v\}$, $\tau=\{A,B,C,D,0_N\}$ and $\sigma=\{D,E,0_N\}$ where $A=<0.7,0.3,0.8><0.5,0.8,0.9>, B=<0.4,0.9,0.9><0.3,0.9,0.9>, C=<0.5,0.8,0.7><0.5,0.8,0.8>, D=<0.5,0.8,0.8><0.5,0.8,0.7>, E=<0.3,0.9,0.9><0.4,0.9,0.9>$.*
*We define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Here, $f^{-1}(D)=D$ and $f^{-1}(E)=E$ where D and E are $\mu_N$-Semi Open sets of $(X,\tau)$ but E is not $\mu_N$- Open in $(X,\tau)$.Thus we conclude that Every $\mu_N$- Semi Continuous function need not be $\mu_N$- Continuous.*

**Definition 4.10.** *A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be $\mu_N$-Pre Continuous function if the inverse image of $\mu_N$- closed sets in $(Y,\sigma)$ is $\mu_N$-Pre closed sets in $(X,\tau)$.*

**Theorem 4.11.** *Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N$-Pre Continuous function if and only if the inverse image of $\mu_N$- open sets in $(Y,\sigma)$ is $\mu_N$-Pre open in $(X,\tau)$. Proof is similar to the proof of theorem 4.4.*

**Theorem 4.12.** *Every $\mu_N$- Continuous is $\mu_N$-Pre Continuous.*

*Proof.* Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N$- Continuous mapping. Let $A$ be a $\mu_N$- open set in $(Y,\sigma)$. Since $f$ is $\mu_N$- Continuous we get $f^{-1}(A)$ is $\mu_N$- open in $(X,\tau)$. Since, every $\mu_N$- open set is $\mu_N$-Pre Open. Thus , that $f$ is $\mu_N$-Pre Continuous. □

**Remark 4.13.** *The reverse statement of the above theorem need not be true .The Scenario will be emanated in the forthcoming example. Let $X=\{a,b\}$ and $Y=\{U,V\}$, $\tau=\{A,B,C,D,0_N\}$ and $\sigma=\{D,E,0_N\}$ where $A=<0.7,0.3,0.8><0.5,0.8,0.9>, B=<0.4,0.9,0.9><0.3,0.9,0.9>, C=<0.5,0.8,0.7><0.5,0.8,0.8>, D=<0.5,0.8,0.8><0.5,0.8,0.7>, E=<0.3,0.9,0.9><0.4,0.9,0.9>$ we define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Here, $f^{-1}(D)=D$ and $f^{-1}(E)=E$ where D and E are $\mu_N$-Pre Open sets of $(X,\tau)$ but E is not $\mu_N$- Open in $(X,\tau)$. Thus we deduced that Every $\mu_N$- Pre Continuous function need not be $\mu_N$- Continuous.*

**Definition 4.14.** *A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be $\mu_N-\alpha$ Continuous function if the inverse image of $\mu_N$- closed sets in $(Y,\sigma)$ is $\mu_N-\alpha$ closed sets in $(X,\tau)$.*

**Theorem 4.15.** *Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N-\alpha$ Continuous function if and only if the inverse image of $\mu_N$- open sets in $(Y,\sigma)$ is $\mu_N-\alpha$ open in $(X,\tau)$. Proof is similar to the proof of theorem 4.4.*

**Theorem 4.16.** *Every $\mu_N$- Continuous is $\mu_N-\alpha$ Continuous.*

*Proof.* Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N$- Continuous mapping. Let $A$ be a $\mu_N$- open set in $(Y,\sigma)$. Since $f$ is $\mu_N$- Continuous we get $f^{-1}(A)$ is $\mu_N$- open in $(X,\tau)$. Since, every $\mu_N$- open set is $\mu_N-\alpha$ Open, $f^{-1}(A)$ is $\mu_N-\alpha$ open in $(X,\tau)$. Thus , that $f$ is $\mu_N-\alpha$ Continuous. □

**Remark 4.17.** *Converse of the above theorem need not be true.It is explained by the example given below.*

**Example 4.18.** *Let $X=\{a,b\}$ and $Y=\{U,V\}$, $\tau=\{A,B,C,D,0_N\}$ and $\sigma=\{D,E,0_N\}$ where $A=<0.7,0.3,0.8><0.5,0.8,0.9>, B=<0.4,0.9,0.9><0.3,0.9,0.9>, C=<0.5,0.8,0.7><0.5,0.8,0.8>, D=<0.5,0.8,0.8><0.5,0.8,0.7>, E=<0.3,0.9,0.9><0.4,0.9,0.9>$ we define a mapping $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=u$ and $f(b)=v$. Here, $f^{-1}(D)=D$ and $f^{-1}(E)=E$ where D and E are $\mu_N-\alpha$ Open sets of $(X,\tau)$ but E is not $\mu_N$- Open in $(X,\tau)$.Thus we conclude that Every $\mu_N-\alpha$ Continuous function need not be $\mu_N$- Continuous.*

**Definition 4.19.** *A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be $\mu_N-\beta$ Continuous function if the inverse image of $\mu_N$- closed sets in $(Y,\sigma)$ is $\mu_N-\beta$ closed sets in $(X,\tau)$.*

**Theorem 4.20.** *A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be $\mu_N-\beta$ Continuous function if and only if the inverse image of $\mu_N$- open sets in $(Y,\sigma)$ is $\mu_N-\beta$ open in $(X,\tau)$. Proof is similar to the proof of theorem 4.4.*

**Theorem 4.21.** *Every $\mu_N$- Continuous is $\mu_N-\beta$ Continuous.*

*Proof.* Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N$- Continuous mapping. Let $A$ be a $\mu_N$- open set in $(Y,\sigma)$.The inverse image of $A$ is $\mu_N$-open set in $(X,\tau)$.We know that every $\mu_N$- open set is $\mu_N-\beta$ Open set in $(X,\tau)$.Hence , $f$ is $\mu_N-\beta$ Continuous. □

**Remark 4.22.** *Converse of the above theorem need not be true. The scenario is given in the preceeding exemplary.*

**Example 4.23.** *Let $X=\{a,b\}, Y=\{U,V\}, \tau=\{A,B,C,D,0_N\}$ and $\sigma=\{C,E,0_N\}$ be two $\mu_N$- topological spaces where $A=<0.6,0.4,0.8><0.8,0.6,0.9>, B=<0.6,0.3,0.8><0.9,0.2,0.7>, C=<0.5,0.4,0.9><0.7,0.8,0.9>, D=<0.4,0.6,0.9><0.6,0.8,0.9>, E=<0.3,0.7,0.9><0.5,0.9,0.9>$. We define a mapping $f(a)=u$ and $f(b)=v$. Hence we get $f^{-1}(C)=C$ and $f^{-1}(E)=E$ where C and E are $\mu_N-\beta$ Open sets of $(X,\tau)$ but the inverse image of $E\in\sigma$ is E which is not $\mu_N$- Open in $(X,\tau)$. Hence, Every $\mu_N-\beta$ Continuous need not be $\mu_N$- Continuous.*

**Theorem 4.24.** *Every $\mu_N-\alpha$ Continuous is $\mu_N$- Semi Continuous.*

*Proof.* Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N-\alpha$ Continuous mapping. Let $A$ be a $\mu_N-\alpha$ open set in $X$. By hypothesis we get $f^{-1}(A)$ is $\mu_N-\alpha$ open in $(X,\tau)$.Since, every $\mu_N-\alpha$ open set is $\mu_N$-Semi Open we get $f^{-1}(A)$ is $\mu_N$- Semi open in $(X,\tau)$. Thus we obtain that $f$ is $\mu_N$-Semi Continuous. □

**Remark 4.25.** *The reversal statement of the above theorem need not be true .It can be delineated below with the help of an example.*

**Example 4.26.** *Let $X=\{a,b\}, Y=\{U,V\}, \tau=\{A,B,C,D,0_N\}$ and $\sigma=\{C,E,0_N\}$ be two $\mu_N$- topological spaces where $A=<0.6,0.4,0.8><0.8,0.6,0.9>, B=<0.6,0.3,0.8><0.9,0.2,0.7>, C=<0.5,0.4,0.9><0.7,0.8,0.9>, D=<0.4,0.6,0.9><0.6,0.8,0.9>, E=<0.3,0.7,0.9><0.5,0.9,0.9>$. We define a mapping $f(a)=u$ and $f(b)=v$. Hence we get $f^{-1}(C)=C$ and $f^{-1}(E)=E$ where C and E are $\mu_N$-Semi Open sets of $(X,\tau)$ but the inverse image of $E\in\sigma$ is E which is not $\mu_N$- Open in $(X,\tau)$.Hence,every $\mu_N$- Semi Continuous need not be $\mu_N$- Continuous.*

**Theorem 4.27.** *Every $\mu_N$- Pre Continuous is $\mu_N-\beta$ Continuous.*

*Proof.* Let $f:(X,\tau)\to(Y,\sigma)$ be a $\mu_N$-Pre Continuous mapping. Let $A$ be a $\mu_N$-Pre open set in $X$. By hypothesis we get $f^{-1}(A)$ is $\mu_N$- Pre open in $(X,\tau)$.Since, every $\mu_N$- Pre open set is $\mu_N-\beta$ Open we get $f^{-1}(A)$ is $\mu_N-\beta$ open in $(X,\tau)$. Thus we obtain that $f$ is $\mu_N-\beta$ Continuous. □

**Remark 4.28.** *The reversal statement of the above theorem need not be true.It can be explained with the help of an example as below:*

**Example 4.29.** *Let $X=\{a,b\}, Y=\{U,V\}, \tau=\{A,B,C,D,0_N\}$ and $\sigma=\{C,E,0_N\}$ be two $\mu_N$- topological spaces where $A=<0.6,0.4,0.8><0.8,0.6,0.9>, B=<0.6,0.3,0.8><0.9,0.2,0.7>, C=<0.5,0.4,0.9><0.7,0.8,0.9>, D=<0.4,0.6,0.9><0.6,0.8,0.9>, E=<0.3,0.7,0.9><0.5,0.9,0.9>$. We define a mapping $f(a)=u$ and $f(b)=v$. Hence we get $f^{-1}(C)=C$ and $f^{-1}(E)=E$ where C and E are $\mu_N-\beta$ Open sets of $(X,\tau)$ but the inverse image of $E\in\sigma$ is E which is not $\mu_N$-pre Open in $(X,\tau)$.Hence, every $\mu_N-\beta$ Continuous need not be $\mu_N$- Continuous.*

**Theorem 4.30.** *Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$ topological spaces. If $f:(X,\tau)\to(Y,\sigma)$ is $\mu_N$- Semi Continuous and $g:(Y,\sigma)\to(Z,\rho)$ is $\mu_N$- Continuous then $g\circ f:(X,\tau)\to(Z,\rho)$ is $\mu_N$- Semi Continuous.*

*Proof.* Let $V$ be any $\mu_N$- open in $(Z,\rho)$. Since, $g:(Y,\sigma)\to(Z,\rho)$ is $\mu_N$- Continuous, $g^{-1}(V)$ is $\mu_N$- open in $(Y,\sigma)$. Since, $f$ is $\mu_N$- Semi Continuous, $f^{-1}(g^{-1}(U))$ is $\mu_N$- Semi open , $f^{-1}(g^{-1}(U))=(g\circ f)^{-1}(U)$ is $\mu_N$- Semi Continuous. □

Composition of two $\mu_N$- Semi Continuous need not be $\mu_N$- Semi Continuous. Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$ topological spaces. Let us assume $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\rho)$ be two $\mu_N$- Semi Continuous functions. Let $V$ be any $\mu_N$- open in $(Z,\rho)$.Since, $g:(Y,\sigma)\to(Z,\rho)$ is $\mu_N$-Semi Continuous, $g^{-1}(V)$ is $\mu_N$-Semi open in $(Y,\sigma)$. We know every $\mu_N$-Semi open sets need not be $\mu_N$- open. Since $f$ is $\mu_N$- Semi Continuous functions, we have to get the inverse image of $\mu_N$- open sets in $(Y,\sigma)$ must be $\mu_N$- Semi open in $(X,\tau)$ . But here all the element of $(Y,\sigma)$ are $\mu_N$-Semi open. So we cannot explore a $\mu_N$- Semi Continuous function.

**Theorem 4.31.** *Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$ topological spaces. If $f:(X,\tau)\to(Y,\sigma)$ is $\mu_N$- Pre Continuous and $g:(Y,\sigma)\to(Z,\rho)$ is $\mu_N$- Continuous then $g\circ f:(X,\tau)\to(Z,\rho)$ is $\mu_N$- Pre Continuous.*

*Proof.* Let V be any $\mu_N$- open in $(Z,\rho)$.Since, $g:(Y,\sigma)\to(Z,\rho)$ is $\mu_N$- Continuous, $g^{-1}(V)$ is $\mu_N$- open in $(Y,\sigma)$. Since, $f$ is $\mu_N$- Pre Continuous,$f^{-1}(g^{-1}(U))$ is $\mu_N$- Pre open in $(X,\tau)$, $f^{-1}(g^{-1}(U))=(g\circ f)^{-1}(U)$ is $\mu_N$- Pre Continuous. □

Composition of two $\mu_N$- Pre Continuous need not be $\mu_N$- Pre Continuous. Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$ topological spaces. Let us assume $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\rho)$ be two $\mu_N$- Pre Continuous functions. Let $V$ be any $\mu_N$- open in $(Z,\rho)$.Since, $g:(Y,\sigma)\to(Z,\rho)$ is $\mu_N$-Pre Continuous, $g^{-1}(V)$ is $\mu_N$-Pre open in $(Y,\sigma)$.We know every $\mu_N$-Pre open sets need not be $\mu_N$- open. Since $f$ is $\mu_N$-pre Continuous functions,we have to get the inverse image of $\mu_N$- open sets in $(Y,\sigma)$ must be $\mu_N$- Pre open in$(X,\tau)$ . But here all the element of $(Y,\sigma)$ are $\mu_N$-Pre open, every $\mu_N$-Pre open sets need not be $\mu_N$- open So we cannot explore a $\mu_N$-Pre Continuous fuction.

**Theorem 4.32.** *Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$ topological spaces. If $f:(X,\tau)\to(Y,\sigma)$ is $\mu_N-\alpha$ Continuous and $g:(Y,\sigma)\to(Z,\rho)$ is $\mu_N$- Continuous then $g\circ f:(X,\tau)\to(Z,\rho)$ is $\mu_N-\alpha$ Continuous.*

*Proof.* Let $V$ be any $\mu_N$- open in $(Z,\rho)$.Since,$g:(Y,\sigma)\to(Z,\rho)$ is $\mu_N$- Continuous, $g^{-1}(V)$ is $\mu_N$- open in $(Y,\sigma)$. Since, $f$ is $\mu_N-\alpha$ Continuous, $f^{-1}(g^{-1}(U))$ is $\mu_N-\alpha$ open in $(X,\tau)$, $f^{-1}(g^{-1}(U))=(g\circ f)^{-1}(U)$ is $\mu_N-\alpha$ Continuous. □

Composition of two $\mu_N - \alpha$ Continuous need not be $\mu_N - \alpha$ Continuous. Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$ topological spaces. Let us assume $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\rho)$ be two $\mu_N - \alpha$ Continuous functions. Let $V$ be any $\mu_N$-open in $(Z,\rho)$.Since, $g:(Y,\sigma)\rightarrow(Z,\rho)$ is $\mu_N - \alpha$ Continuous, $g^{-1}(V)$ is $\mu_N - \alpha$ open in $(Y,\sigma)$. Since $f$ is $\mu_N - \alpha$ Continuous functions,we have to get the inverse image of $\mu_N$-open sets in $(Y,\sigma)$ must be $\mu_N - \alpha$ open in$(X,\tau)$ . But here all the element of $(Y,\sigma)$ are $\mu_N - \alpha$ open, every $\mu_N - \alpha$ open sets need not be $\mu_N$- open So we cannot explore a $\mu_N - \alpha$ Continuous function.

**Theorem 4.33.** *Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$ topological spaces. If $f:(X,\tau)\rightarrow(Y,\sigma)$ is $\mu_N - \beta$ Continuous and $g:(Y,\sigma)\rightarrow(Z,\rho)$ is $\mu_N$- Continuous then $g\circ f:(X,\tau)\rightarrow(Z,\rho)$ is $\mu_N - \beta$ Continuous.*

*Proof.* Let $V$ be any $\mu_N$- open in $(Z,\rho)$. Since, $g:(Y,\sigma)\rightarrow(Z,\rho)$ is $\mu_N$- Continuous, $g^{-1}(V)$ is $\mu_N$- open in $(Y,\sigma)$. Since, $f$ is $\mu_N - \beta$ Continuous,$f^{-1}(g^{-1}(U))$ is $\mu_N - \beta$ open in $(X,\tau)$ , $f^{-1}(g^{-1}(U)) = (g\circ f)^{-1}(U)$ is $\mu_N - \beta$ Continuous. □

Composition of two $\mu_N - \beta$ Continuous need not be $\mu_N - \beta$ Continuous. . Let $(X,\tau),(Y,\sigma),(Z,\rho)$ be three $\mu_N$ topological spaces. Let us assume $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\rho)$ be two $\mu_N - \beta$ Continuous functions. Let $V$ be any $\mu_N$-open in $(Z,\rho)$. Since, $g:(Y,\sigma)\rightarrow(Z,\rho)$ is $\mu_N - \beta$ Continuous, $g^{-1}(V)$ is $\mu_N - \beta$ open in $(Y,\sigma)$. Since $f$ is $\mu_N - \beta$ Continuous functions,we have to get the inverse image of $\mu_N$ - open sets in $(Y,\sigma)$ must be $\mu_N - \beta$ open in $(X,\tau)$ . But here all the element of $(Y,\sigma)$ are $\mu_N - \beta$ open, every $\mu_N - \beta$ open sets need not be $\mu_N$- open So we cannot explore a $\mu_N - \beta$ Continuous function.

## 5. Conclusion

In this paper, we have explored some new open sets in $\mu_N$ topological spaces and their features were investigated. The continuous functions of $\mu_N$ topological spaces and the composite functions of $\mu_N$ topological spaces were discovered and their features were discussed. We made a comparison on continuous functions of $\mu_N$ topological spaces with different types of $\mu_N$ continuous functions and their nature were contemplated. Subsequently ,we build up our research towards $\mu_N$- compact, $\mu_N$- connected and so on.

## 6. Acknowledgment

The authors are grateful to the reviewers for their comments towards the betterment of this article.

## References

[1] Atanassov.K.T: Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1986), 87–96.

[2] Al-Omeri, W.; Smarandache, F.: *New Neutrosophic Sets via Neutrosophic Topological Spaces. In Neutrosophic Operational Research*, Smarandache, F., Pramanik, S., Eds.; Pons Editions: Brussels, Belgium, 2017; Volume I, pp. 189–209.

[3] Chang.C.L, : Fuzzy topological spaces, *Journal of Mathematical Analysis and Application*, 24(1968), 183–190.

[4] Dhavaseelan. R and Jafari,: Generalized Neutrosophic closed sets, *New Trends in Neutrosophic Theory and Applications*, 2(2018), 261–273.

[5] Dogan Coker: An introduction to intuitionstic fuzzy topological spaces, *Fuzzy Sets and Systems*, 88(1997), 81–89.

[6] Floretin Smarandache,: Neutrosophy and Neutrosophic Logic, *First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup*, NM 87301, USA, 2002.

[7] Floretin Smarandache, : Neutrosophic Set:- A Generalization of Intuitionistic Fuzzy set, *Journal of Defense Resourses Management,* 1(2010), 107–116.

[8] Floretin Smarandache,: *A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic set, Neutrosophic Probability*, Ameican Research Press, Rehoboth, NM, 1999.

[9] Iswarya .P, K. Bageerathi,: A Study on neutrosophic Frontier and neutrosophic semi frontier in Neutrosophic topological spaces, *Neutrosophic Sets and Systems,* 16(2017), 1–10.

[10] Raksha Ben .N, Hari Siva Annam.G.: Generalized Topological Spaces via Neutrosophic Sets, *J. Math. Comput. Sci.,* 11(2021), Accepted Manuscript.

[11] Salama A.A and Alblowi S.A, : Neutrosophic set and Neutrosophic topological space, *ISOR J. Mathematics,* 3(4)(2012), 31–35.

[12] Salama.A.A and Alblowi.S.A, : Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, *Journal computer Sci. Engineering,* 2(7)(2012), 12–23.

[13] Salama A.A, Florentin Smarandache and Valeri Kroumov, : Neutrosophic Closed set and Neutrosophic Continuous Function, *Neutrosophic Sets and Systems,* 4(2014), 4–8.

[14] Wadel Faris Al-Omeri and Florentin Smarandache, : New Neutrosophic Sets via Neutrosophic Topological Spaces, *New Trends in Neutrosophic Theory and Applications,* (2)(2016), 1-10.

[15] Zadeh. L.A: Fuzzy set, *Inform and Control*, 8(1965), 338–353.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆