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Smart Materials production model with Costs associated with Decision Making on various perspectives

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Abstract
In recent times, the smart materials are gaining more momentum and this has channelized the attention of the production sectors towards producing at an optimal cost. This paper develops a production inventory model of smart materials with various trends of demands and different costs associated with decision making in the contexts of material selection, product propagation and customer procurement. An inventory model is modelled using differential equation to find the optimal order quantity. The proposed model is validated with numerical example. In this paper the production inventory model with conventional costs parameters are extended to smart production inventory model with the inclusion of new kinds of decision-making costs parameters in the perspective of material selection, machinery choice, screening, waste segregation, waste disposal, customer procurement, product propagation and carbon pricing measures. The model developed in this paper incorporates new types of costs to facilitate the formulation of comprehensive production inventory model to meet out the dynamic needs of the decision makers in this present era of information. The model can be discussed and extended with various demand patterns.

Keywords
Smart materials, production model, demand, decision making, material selection, customer procurement.

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Contents

1. Introduction

The industrial sectors are building its acquaintance with smart materials production as these materials possess the attributes of self-adaptation to environment property change, energy exchange, sensing stimulus, reversibility, absorption. The market demand for these smart materials is escalating at recent times as they are replacing the traditional materials. The smart materials market is likely to reach the target of $73.63 billion by 2022 according to a recent forecast. The smart materials are of low costs, flexible, reliable and robust in nature. The smart materials are of various kinds and the production processes are refurbished to obtain optimal profit. The significant role and the multidimensional applications of the smart materials has contributed to the industrial revolution in varied sectors such as automobile, constructors, transport, aerospace and also in many applications. [1]

Sameer Mittal et al [9] described the characteristics and the technology used in smart manufacturing systems. The paradigm shift towards smart systems, smart materials has unveiled the development of smart production inventory model. The inventory management is quite inevitable in the production environment. The optimal order quantity, time of placing the orders are to be planned well in advance to avoid the circumstances of product dearth.

The factors interrupting the persistent flow in production should be kept under control. Inventory model proposed by Harris and Taft [6] are the underlying inventory models comprising of the elementary costs such as costs of ordering, holding, purchasing. Researchers have modelled different production inventory models to handle the aspects of shortages, backlog, discount, price break-up, inflation.deteriorating items, ameliorating products, trade credit system in deterministic, probabilistic, fuzzy, stochastic environments. The other dimension of inventory models focusses on reverse manufacturing and supply chain models consisting of rework and recycle of imperfect items and waste disposal. These inventory models include the associated costs of refurbishing of defective items and disposing of waste. Maurice Bonney [8] developed the socially responsible inventory models with the inclusion of carbon emission costs and substantiated the inclusion of waste disposal costs to the fundamental Economic order quantity model. Eco-conscious inventory models were also developed with the discussion of different disposal methods and its related costs. The models proposed by Biswajit[3], Bazan [2]contributed to the development of Ecoconscious production inventory model. The changes in the production sectors must be reflected in the inventory models for better functioning of the production systems. As presently the manufacturing systems are shifting towards smart mechanisms, the development of smart production inventory models (SPIM) must be reinforced.

The foundation of SPIM was laid by Christian Decker et al [4]with the formulation of cost -benefit model for smart items. The EPQ-inventory model with smart elements was initiated by Igra Asghar et al [6]. Xue-Ming Yuan [12] presented the innovative research and development framework for modelling and optimizing the inventory systems. It is also suggested to incorporate the elements of automation and digitalization to inventory model for upgrading its status to smart systems.Rich Lee [10] discussed the integration of smart materials to production system. Mitali Sarkar [9] proposed an inventory model addressing the characteristics of the smart production system of multi-items together with the impact of energy and failure rate Zhongyuan Lyu et al [13] introduced the concept of zero warehousing smart manufacturing that begins from zero inventory level to just-in production systems. The smart production elements are creeping into inventory modelling. These smart production inventory models discuss about the intervention of smart elements, but not much focussed on new costs parameters. Also, the remarkable effects of these smart materials in the production system were highlighted but the production inventory model of smart materials was not developed to the best of our knowledge. This paper puts forth the smart material production inventory model with the integration of costs parameters in the domains of waste generation, decision-making, customer procurement and product propagation, waste disposal, carbon pricing. This proposed model encompasses almost all kinds of costs parameters incorporated during the time of product production and disposal. This model will certainly fit into the context of smart age and eco-consciousness and this is highly comprehensive in nature.

Generally, the profit maximization depends on the quality of the products and the quantity of customers. The smart production inventory model has to meet the demands of the customers with product enhancement with the integration of technology. The smart materials production inventory model comprises of the costs of decision-making, which is very essential in material, method and technology selection. The entities of technology in inventory management has become the integral part of the smart production system. The incorporation of the various new costs with the proposed inventory model is emphasized in this paper.

The paper is structured into the following sections, section 2 presents the model development; section 3 discusses the proposed model with different demand patterms; section 4 validates the proposed model with numerical illustrations and the last section concludes the work.

2. Model Development

This section presents the formulation of smart material production inventory model with new cost parameters with the following assumptions and notations.

2.1 Assumptions

- Deficit of the products are not taken into account.
- The nature of the demand is deterministic.
- Infinite planning horizon.

2.2 Notations

P - production rate per cycle
D - demand rate per cycle
General Costs
O - Ordering cost
Costs for time period $0 \leq t \leq t_2$

C_r - Carbon cost

DM - Decision Making Cost on Material Supplier Selection

DW - Decision Making Cost on Waste disposal Method

DS - Decision Making Cost on Storage Space

DT - Decision Making Cost on Technology

DC - Decision Making Cost on carbon pricing

C_p - Cost of Production

A - Customer procurement cost

MC - Total marketing cost for acquiring customers

WS - Wages Connected with Sales and Marketing

S - Marketing and Sales Associated Software Cost

PS - Professional Service costs in Marketing

M_s - Over heads Associated with Marketing and Sales.

C_s - Consultancy costs

D_c - Differential costs

OL - Online Ad costs

SE - Search Engine Optimization Costs

S_m - Social media advertising costs

T_c - Technology Integration costs

S_c - Sunk costs

I_c - Imputed costs

O_p - Opportunity Cost

R_c - Relevant Cost

Costs for time period $t_1 < t \leq T$

R - Waste generation Rate
W - Cost of Waste disposal
Costs common for both the time periods H - Holding costs

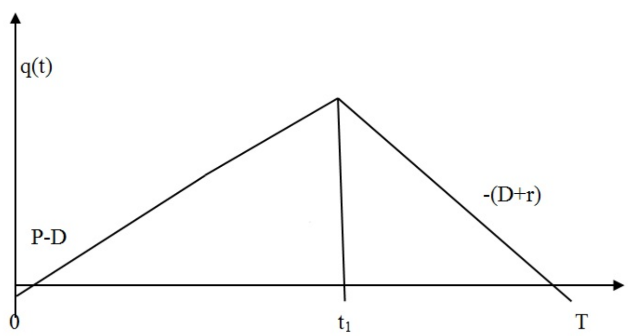

Figure 1

If $q\ (t)$ represents the inventory level at time $t \in [0,\ T]$, so the differential equation for the instantaneous inventory $q(t)$ at any time t over $[0,\ T]$ is

$$\frac{dq(t)}{dt} = P - D \quad 0 \leq t \leq t_1 \tag{2.1}$$

$$\frac{dq(t)}{dt} + rq(t) = -D \quad t_1 \leq t \leq T \tag{2.2}$$

With initial condition $q(0) = 0$ and Boundary condition $q(T) = 0$

$$\begin{aligned} \frac{dq(t)}{dt} =& P - D \\ dg(t) =& (P-D)dt \\ q(t) =& (P-D)t + c \end{aligned}$$

with initial condition $q(0) = 0$

$$q(0) = (P-D)0 + c = c$$

$$q(t) = (P-D)t \quad 0 \leq t \leq t_1 \tag{2.3}$$

solving equation (2.2)

$$q(t)e^{\int r(t)} = \int e^{rt}(-D)dt \quad t_1 \leq t \leq T$$

$$q(t) = \frac{-D}{r} + ce^{-rt}$$

With boundary condition $q(T) = 0$

$$q(T) = \frac{-D}{r} + ce^{-rT} \tag{2.4}$$

$$c = \frac{D}{r}e^{rt} \tag{2.5}$$

$$q(t) = \frac{D}{r}\left[e^{r(T-t)} - 1\right] \tag{2.6}$$

using equation (2.3), (2.4), we get

$$\begin{aligned} I_{max} =& (P-D)t_1 \\ I_{\max} =& \frac{D}{r}\left[e^{r(T-t_1)} - 1\right] \\ t_1 =& \frac{I_{\max}}{p-D} \\ T - t_1 =& \frac{I_{\max}}{D} \end{aligned}$$

We adding, we get

$$\begin{aligned} t_1 + T - t_1 =& I_{\max}\left[\frac{1}{(p-D)} + \frac{1}{D}\right] \\ T =& I_{\max}\left[\frac{1}{(p-D)} + \frac{1}{D}\right] \\ I_{\max} =& D\left[1 - \frac{D}{p}\right]T \end{aligned}$$

$$\begin{aligned} \text{Carbon cost } &= Cr\int_0^{t_1} q(t)dt \\ &= \text{Cr}\int_0^{t_1}(P-D)tdt \\ &= \frac{Cr}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D}\right] \end{aligned}$$

Decision Making Cost on Material Supplier Selection

$$\begin{aligned} &= DM\int_0^{t_1} q(t)dt \\ &= Dm\int_0^{t_1}(P-D)tdt \\ &= \frac{Dm}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right] \end{aligned}$$

Decision Making Cost on Waste disposal Method

$$\begin{aligned} &= DW\int_0^{t_1} q(t)dt \\ &= DW\int_0^{t_1}(P-D)tdt \\ &= \frac{Dw}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D}\right] \end{aligned}$$

Decision Making Cost on Storage Space

$$= \mathrm{Ds}\int_0^{t_1} q(t)dt$$
$$= DS\int_0^{t_1} (P-D)tdt$$
$$= \frac{Ds}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Decision Making Cost on Technology

$$= DT\int_0^{t_1} q(t)dt$$
$$= DT\int_0^{t_1} (P-D)tdt$$
$$= \frac{DT}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D}\right]$$

Decision Making Cost on carbon pricing

$$= Dp\int_0^{t_1} q(t)dt$$
$$= Dp\int_0^{t_1} (P-D)tdt$$
$$= \frac{Dp}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Cost of Production

$$= C_p\int_0^{t_1} q(t)dt$$
$$= Cp\int_0^{t_1} (P-D)tdt$$
$$= \frac{Cp}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Customer procurement cost

$$= A\int_0^{t_1} q(t)dt$$
$$= A\int_0^{t_1} (P-D)tdt$$
$$= \frac{A}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D}\right]$$

Total marketing cost for acquiring customers

$$= MC\int_0^{t_1} q(t)dt$$
$$= MC\int_0^{t_1} (P-D)tdt$$
$$= \frac{MC}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Wages Connected with Sales and Marketing

$$= WS\int_0^{t_1} q(t)dt$$
$$= WS\int_0^{t_1} (P-D)tdt$$
$$= \frac{WS}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Marketing and Sales Associated Software Cost

$$= S\int_0^{t_1} q(t)dt$$
$$= S\int_0^{t_1} (P-D)tdt$$
$$= \frac{S}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Professional Service cost in Marketing

$$= PS\int_0^{t_1} q(t)dt$$
$$= PS\int_0^{t_1} (P-D)tdt$$
$$= \frac{PS}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Over heads Associated with Marketing and Sales

$$= MS\int_0^{t_1} q(t)dt$$
$$= \mathrm{MS}\int_0^{t_1} (P-D)tdt$$
$$= \frac{MS}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Consultancy costs

$$= C_s\int_0^{t_1} q(t)dt$$
$$= C_s\int_0^{t_1} (P-D)tdt$$
$$= \frac{Cs}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D}\right]$$

Differential costs

$$= DC\int_0^{t_1} q(t)dt$$
$$= Dc\int_0^{t_1} (P-D)tdt$$
$$= \frac{Dc}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

$$\text{Online Ad costs } = OL\int_0^{t_1} q(t)dt$$
$$= OL\int_0^{t_1} (P-D)tdt$$
$$= \frac{OL}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Search Engine Optimization Costs

$$= SE\int_n^{t_1} q(t)dt$$
$$= SE\int_0^{t_1} (P-D)tdt$$
$$= \frac{SE}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D}\right]$$

Social media advertising costs

$$= Sm\int_0^{t_1} q(t)dt$$
$$= \mathrm{Sm}\int_0^{t_1} (P-D)tdt$$
$$= \frac{Sm}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Technology integration costs

$$= TC\int_0^{t_1} q(t)dt$$
$$= Tc\int_0^{t_1} (P-D)tdt$$
$$\frac{Tc}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D}\right]$$

$$\text{Sunk costs } = S_C\int_0^{t_1} q(t)dt$$
$$= \mathrm{Sc}\int_0^{t_1} (P-D)tdt$$
$$= \frac{Sc}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

$$\text{Imputed costs } = IC\int_0^{t_1} q(t)dt$$
$$= \mathrm{Ic}\int_0^{t_1} (P-D)tdt$$
$$= \frac{Ic}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

$$\text{Opportunity Cost } = Op\int_0^{t_1} q(t)dt$$
$$= Op\int_0^{t_1} (P-D)tdt$$
$$= \frac{op}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

$$\text{Relevant Cost } = Rc\int_0^{t_1} q(t)dt$$
$$= Rc\int_0^{t_1} (P-D)tdt$$
$$= \frac{Rc}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]$$

Waste generation Rate

$$= R\int_{t_1}^{T} q(t)dt$$
$$= R\int_{t_1 r}^{T} \frac{D}{r}\left[e^{r(T-t)}-1\right]dt$$
$$= \frac{RD}{r^2}e^{rT}\left[\frac{P}{r[p-D]T+P}+\frac{(P-D)T}{P}\right]$$

Cost of Waste disposal

$$= w\int_{t_1}^{T} q(t)dt$$
$$= W\int_{t_1}^{T} \frac{D}{r}\left[e^{r(T-t)}-1\right]dt$$
$$= \frac{WD}{r^2}e^{rT}\left[\frac{p}{r[p-D]T+P}+\frac{(p-D)T}{p}\right]$$

$$\therefore \text{ Holding cost } = C_1\left[\int_0^{t_1} q(t)dt+\int_{t_1}^{T} q(t)dt\right]$$
$$= c_1\left[\frac{1}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D}\right]\right.$$
$$\left.+\frac{D}{r^2}e^{rT}\left[\frac{p^2+(p-D)T[r(p-D)T+P]}{\mathrm{pr}[p-D]T+p^2}\right]\right]$$

∴ Total Cost = Ordering cost + Holding cost + Carbon cost + Decision Making Cost on Material Supplier Selection + Decision Making Cost on Waste disposal Method + Decision Making Cost on Storage Space +Decision Making Cost on Technology +Decision Making Cost on carbon pricing +Cost of Production + Customer procurement cost + Total marketing cost for acquiring customers + Wages Connected with Sales and Marketing + Marketing and Sales Associated Software Cost + Professional Service in Marketing + Over heads Associated with Marketing and Sales + Consultancy costs + Differential costs + Online Ad costs + Search Engine Optimization Costs + Social media advertising costs +Technology Integration costs + Sunk costs + Imputed costs y Cost + Relevant Cost + Waste generation rate + Cost of Waste disposal.

$$
\begin{aligned}
&= O + c_1 \left[\frac{1}{2} \left[\frac{\left[D\left(1-\frac{D}{p}\right)\right]^2 T^2}{p-D} \right] + \frac{D}{r^2} e^{rT} \left[\frac{p^2 + (p-D)T[r(p-D)T+P]}{\mathrm{pr}[p-D]T+p^2} \right] \right] + \frac{cr}{2} \left[\frac{\left[D\left(1-\frac{D}{p}\right)\right]^2 T^2}{p-D} \right] + \frac{DM}{2} \left[\frac{\left[D\left(1-\frac{D}{p}\right)\right]^2 T^2}{p-D} \right] \\
&+ \frac{Dw}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{Ds}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] + \frac{DT}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] + \frac{Dp}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] + \frac{Cp}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] \\
&+ \frac{A}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] + \frac{MC}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] + \frac{WS}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{S}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{PS}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] \\
&+ \frac{MS}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{Cs}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{Dc}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{OL}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{SE}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] \\
&+ \frac{SM}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{TC}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{Sc}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{IC}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{Op}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] \\
&+ \frac{Rc}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{P-D} \right] + \frac{RD}{r^2} e^{rt} \left[\frac{p}{r[p-D]T+p} + \frac{(p-D)T}{p} \right] + \frac{WD}{r^2} e^{rt} \left[\frac{p}{r[p-D]T+P} + \frac{(P-D)T}{p} \right] \\
&= O + c_1 \left[\frac{1}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] + \frac{D}{r^2} e^{rT} \left[\frac{p^2 + (p-D)T[r(p-D)T+P]}{\mathrm{pr}[p-D]T+P^2} \right] \right] + \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] \left[\frac{cr}{2} + \frac{DM}{2} + \frac{DW}{2} \right. \\
&+ \frac{DS}{2} + \frac{DT}{2} + \frac{Dp}{2} + \frac{Cp}{2} + \frac{A}{2} + \frac{MC}{2} + \frac{WS}{2} + \frac{S}{2} + \frac{PS}{2} + \frac{MS}{2} + \frac{Cs}{2} + \frac{Dc}{2} + \frac{OL}{2} + \frac{SE}{2} + \frac{SM}{2} + \frac{TC}{2} + \frac{Sc}{2} + \frac{Ic}{2} + \frac{Op}{2} + \left. \frac{Rc}{2} \right] \\
&+ \left[\frac{D}{r^2} e^{rT} \left[\frac{P}{r[p-D]T+P} + \frac{(P-D)T}{P} \right] \right] (R+w) \\
&= O + c_1 \left[\frac{1}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] + c \left[\frac{p^2 + (p-D)T[r(p-D)T+P]}{\mathrm{pr}[p-D]T+p^2} \right] \right] + \frac{1}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \quad (Cr + DM + DW + DS \right. \\
&+ DT + DP + Cp + A + MC + WS + S + PS + MS + Cs + Dc + OL + SE + SM + TC + SC + IC \\
&+ Op + Rc) + \left[c \left[\frac{p}{r[p-D]T+p} + \frac{(p-D)T}{p} \right] \right] (R+w).
\end{aligned}
$$

Total Average cost

$$
\begin{aligned}
&= \frac{1}{T} \left[\mathrm{O} + c_1 \left[\frac{1}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right] + c \left[\frac{p^2 + (p-D)T[r(p-D)T+P]}{\mathrm{pr}[p-D]T+p^2} \right] \right] + \frac{1}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T^2}{p-D} \right. \right. \\
&\qquad (\mathrm{Cr} + \mathrm{DM} + \mathrm{DW} + \mathrm{DS} + \mathrm{DT} + \mathrm{DP} + \mathrm{Cp} + \mathrm{A} + MC + WS + \mathrm{S} + PS + MS + Cs + Dc + OL + SE \\
&\qquad \left. + SM + TC + SC + IC + Op + Rc) + \left[c \left[\frac{p}{r[p-D]T+p} + \frac{(p-D)T}{p} \right] \right] (\mathrm{R}+\mathrm{w}) \right] \\
&= \frac{0}{T} + c_1 \left[\frac{1}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T}{p-D} \right] + \frac{c}{T} \left[\frac{p^2 + (P-D)T[r(P-D)T+P]}{\mathrm{pr}[P-D]T+P^2} \right] \right] + \frac{1}{2} \left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T}{P-D} \quad (\mathrm{Cr} + \mathrm{DM} \right. \\
&\qquad + \mathrm{DW} + \mathrm{DS} + \mathrm{DT} + \mathrm{DP} + \mathrm{C_p} + \mathrm{A} + MC + WS + \mathrm{S} + PS + MS + Cs + Dc + OL + SE + SM + TC + SC + IC \\
&\qquad + O_p + R_c) + \left[\frac{c}{T} \left[\frac{P}{r[p-D]T+P} + \frac{(P-D)T}{P} \right] \right] (\mathrm{R}+\mathrm{w}) \right]
\end{aligned}
$$

So the Classical EPQ model is

$$\begin{aligned}\text{Min}\,\text{TAC(T)} = \frac{0}{T} + c_1\left[\frac{1}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T}{p-D}\right] + \frac{c}{\tau}\left[\frac{p^2+(p-D)T[r(P-D)T+P]}{\Pr[p-D]T+p^2}\right]\right] + \frac{1}{2}\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2 T}{p-D}\right.(\text{Cr}\\ +\text{DM}+\text{DW}+\text{DS}+\text{DT}+\text{DP}+\text{Cp}+\text{A}+MC+WS+\text{S}+PS+MS+Cs+Dc+OL+SE+SM\\ \left.+\,TC+SC+IC+Op+Rc) + \left[\frac{c}{T}\left[\frac{p}{r[P-D]T+P}+\frac{(P-D)T}{P}\right]\right](\text{R}+\text{w})\right]\end{aligned}$$

Such that $T > 0$ We can show that TAC(T) will be minimum for

$$T^* = \sqrt{\frac{2O}{\begin{aligned}C_1\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2}{P-D}\right] + 2\text{c}\left[\frac{P^2+(P-D)[r(P-D)+P]}{\Pr[P-D]+P^2}\right] + \frac{\left[D\left(1-\frac{D}{P}\right)\right]^2}{P-D}(\text{Cr}+\text{DM}+\text{DW}\\ +\text{DS}+\text{DT}+\text{DP}+\text{Cp}+\text{A}+MC+WS+\text{S}+PS+MS+CS+DC+OL+SE+\\ \text{SM}+\text{TC}+SC+IC+Op+RC)+2\text{C}\left[\frac{P}{\text{r}[P-D]+P}+\frac{(P-D)}{P}\right](R+w)\end{aligned}}}$$

$$TAC^*(T^*) = \sqrt{\begin{aligned}2O+C_1\left[\frac{\left[D\left(1-\frac{D}{P}\right)\right]^2}{P-D}\right] + 2\text{c}\left[\frac{P^2+(P-D)[r(P-D)+P]}{\Pr[P-D]+P^2}\right] + \frac{\left[D\left(1-\frac{D}{P}\right)\right]^2}{P-D}(\text{Cr}+\text{DM}+\text{DW}\\ +\text{DS}+\text{DT}+\text{DP}+\text{Cp}+\text{A}+MC+WS+\text{S}+PS+MS+Cs+Dc+OL+SE\\ +\text{SM}+\text{TC}+SC+IC+Op+Rc)+2\text{c}\left[\frac{P}{\text{r}[P-D]+P}+\frac{(P-D)}{P}\right](R+w)\end{aligned}}$$

3. Numerical example

To illustrate the result obtained in this paper consider an inventory system with the following characteristics : production rate per cycle $= Rs.500$ unit/per month, demand rate per cycle $= Rs.250/month$ Ordering cost $= Rs.300$ /run, Carbon cost = Rs. 20 /unit, Decision Making Cost on Material Supplier Selection $= Rs.70/unit$, Decision Making Cost on Waste disposal Method = Rs.75/unit, Decision Making Cost on Storage Space $= Rs.65/unit$. Decision Making Cost on Technology $= Rs.50/unit$. Decision Making Cost on carbon pricing $= Rs.30/unit$. Cost of Production = Rs. 45/ unit, Customer procurement cost = Rs. 15/ unit, Total marketing cost for acquiring customers = Rs. 25/ unit, Wages Connected with Sales and Marketing = Rs. 35/ unit, e Marketing and Sales Associated Software Cost $= Rs.55/unit$, Professional Service costs in Marketing = Rs. 10 /unit, Over heads Associated with Marketing and Sales $= Rs.5/unit$ Consultancy costs $= Rs.7/unit$, Differential costs $= Rs.3/unit$, Online *Ad* costs $= Rs.30/unit$ Search Engine Optimization Costs $= Rs.15/unit$, Social media advertising costs = Rs.4/unit.Technology integration Costs $= Rs.5/unit$, Sunk costs $= Rs.2/unit$, Imputed costs $= Rs.1/unit$, Opportunity Cost $= Rs.15/unit$, Relevant Cost $= Rs.12/unit$, Waste generation rate $= Rs.5/unit$, Cost of Waste disposal $= Rs.7/unit$, Holding cost $= 1Rs.1/unit/year$. Find the time interval and find the total average cost.
Using 2.1 and 2.2 the respective values of T^* and $TAC^*(T^*)$ are 0.1269 and 194.42

4. Conclusion

In this paper a smart material production inventory model is proposed with the incorporation of various costs parameters pertinent to decision making and other contemporary perspectives. This research work throws light on the paradigm shift being experienced by the manufacturing sectors in the smart age. The production inventory model of smart materials is constructed with underlying costs to initialize the beginning of new entities of inventory models. The proposed model is distinct from the existing models in the dimension of costs incorporation and this model can be discussed under different kinds of demand patterns also these models can be extended to fuzzy inventory models to analyse the reflection of the changes in the demand patterns and the costs parameters.

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