



# Real world applications of discrete mathematics

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## Abstract

Discrete mathematics is an important branch of applied mathematics and graph theory is an important part of discrete mathematics. It have lot of applications in modern society. Graph colouring or graph labeling is an important branch of graph theory which can easily solve many real life problems. In this article, we have shown some direct applications of discrete mathematics, like applications of graph theory to scheduling problems, coloring of map in GSM mobile phone networks, google maps or *GPS*, traffic signal lights, Social networks, aircraft scheduling, etc.

## Keywords

colouring, scheduling problem, interval graph.

## AMS Subject Classification

05C85, 68R10.

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## 1. Introduction

Mathematics is classified into two branches, *Applied Mathematics* and *Pure Mathematics*. *Discrete Mathematics* is an essential branch of *Applied Mathematics* and *Graph Theory* is a crucial topic in the field of *Discrete Mathematics*. Graphs are significant since the graph is the only way of revealing information in pictorial form. A graph display information which is equivalent to many words. Computer science applications highly utilize graph-theoretical notions. Some well known graph theoretic problems are: Shortest path problem, job sequencing problem, conditional covering problem, traveling salesman problem, graph colouring problem, etc.

*Graph Colouring* is a significant problem in graph theory. This problem is widely need to solve many real problems, viz;

scheduling, resource allocation, traffic phasing, task assignment, etc [1, 2, 13].  $L(h_1, h_2, \dots, h_n)$ -labeling of the graph is the generalization of *Graph Colouring* problem and it has also widely applied so solve frequency assignment problem in wireless communication [3–12].

In this article, some applications of graph colouring have been studied. An introduction is given in section 1, some basic terms related to graph theory are provided in section 2. Graph colouring and extension of graph colouring are provided in section 3. Then, in section 4, some important graph theory applications and graph colouring to solve real life problems are presented. Finally in section 5, a brief conclusions has presented.

## 2. Basic Concept in Graph Theory

Earlier, we may realize the graph's application, so we must know graph theory related some definition. The author of this paper defined some terms related to graph theory most readily.

**Definition 2.1.** A graph usually denoted by  $G = (V, E)$  and consists of the set of vertices (nodes)  $V$  together with a set of edges  $E$ . The number of vertices in a graph is generally denoted by  $n$  while the number of edges is denoted by  $m$ .

**Example:** Let there be direct flights between the countries, India( $i$ ) and America( $a$ ); India( $i$ ) and China( $c$ ); America( $a$ )

and China( $c$ ); China( $c$ ) and Pakistan( $p$ ); Pakistan and Saudi Arabia( $s$ ); India( $i$ ) and Finland( $f$ ); Finland( $f$ ) and Saudi Arabia( $s$ ); Bangladesh( $b$ ) and America( $a$ ); Bangladesh( $b$ ) and Finland( $f$ ). Then the graph for the above problem is  $G = (V, E)$ , where the vertex set  $V = \{a, b, c, f, i, p, s\}$  and edge set  $E = \{(a, b), (a, c), (a, i), (b, f), (c, i), (c, p), (f, i), (f, s), (p, s)\}$ . The pictorial representation of the above graph are shown below.

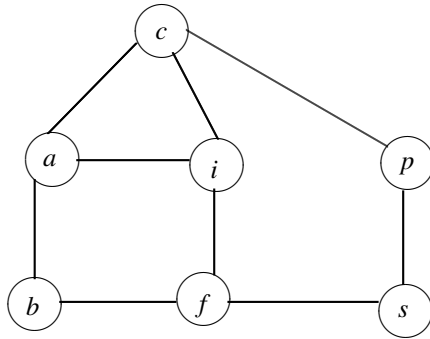


Figure 1. A graph with 7 vertices

**Definition 2.2.** The vertices are also known as points, nodes and (in social networks) as players or actors.

In Figure 1,  $a, b, c, i, f, s, p$  are the 7 vertices of the graph  $G$ .

**Definition 2.3.** An edge is a line (in social networks, it links) at which vertices are joined in the graph. Edges are denoted by  $e = (x, y)$  it is an unordered pair of two vertices.

In Figure 1,  $(a, b), (a, c), (a, i), (b, f), (c, i), (c, p), (f, i), (f, s), (p, s)$  are the edges of  $G$ .

**Definition 2.4.** A graph  $G$  is called undirected graph if its edges have no orientation. The edge  $(p, q)$  is identical to the edge  $(q, p)$ .

Figure 1 is an undirected graph.

**Definition 2.5.** A graph  $G = (V, E)$ , where  $V = \{v_1, v_2, \dots, v_n\}$  is called a path, denoted by  $P_n$ , if and only if  $(v_i, v_{i+1}) \in E$ , where  $1 \leq i \leq n - 1$ .

A path with 7 vertices are shown in Figure 2.

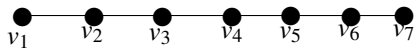


Figure 2. A path with 7 vertices

**Definition 2.6.** A graph  $G$ , where  $G = (V, E)$ ,  $V = \{v_1, v_2, \dots, v_n\}$  is called a cycle, denoted by  $C_n$ , if only  $(v_i, v_{i+1}) \in E$ , where  $1 \leq i \leq n - 1$  and  $(v_1, v_n) \in E$ .

A cycle with 6 vertices are shown below.

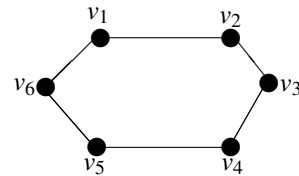


Figure 3. A cycle with 6 vertices

**Definition 2.7.** An Euler path in a graph  $G$  is defined by a walk, which passes through each vertex of  $G$  and traverses every edge of  $G$  exactly once.

**Definition 2.8.** In an undirected or directed graph a path is said to be Hamilton path is if the path that visits every node in the graph exactly once. A Hamilton path is called Hamilton circuit if the path forms a cycle.

**Definition 2.9.** A connected graph containing no cycle is called a tree. In a tree, every pair of points is connected by a unique path.

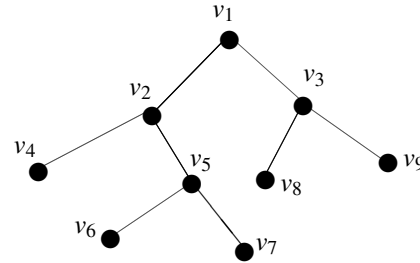


Figure 4. A Tree with 9 vertices

**Definition 2.10.** A simple undirected graph  $G = (V, E)$  is called complete if each pair of vertices of  $G$  are adjacent. A complete graph with  $n$  vertices is denoted by  $K_n$ . That is  $(u, v) \in E$  for all  $u, v \in V$ .

The Figure 5 presents a complete graph with 5 vertices

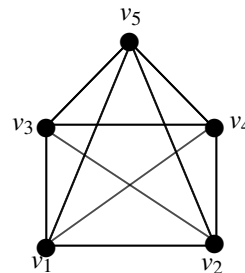


Figure 5. A complete graph with 5 vertices  $K_5$

**Definition 2.11.** A simple undirected graph  $G = (V, E)$  is called complete bipartite graph if the vertex set  $V$  can be partitioned into two non empty subsets  $V_1$  and  $V_2$  so that there is no such edge between any two vertices in  $V_1$  and also there is no such edge between any two vertices in  $V_2$  but every vertex in  $V_1$  is adjacent to every vertex in  $V_2$ . A



complete bipartite graph with  $p + q$  vertices is denoted by  $K_{p,q}$ . Obviously  $|V_1| + |V_2| = p + q$ .

A complete bipartite graph is given in Figure 6.

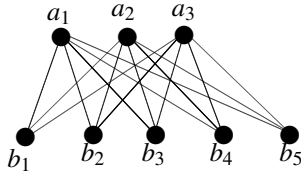


Figure 6. A complete bipartite graph  $K_{3,5}$

**Definition 2.12.** An undirected graph  $G = (V, E)$  is called an interval graph if there is a one to one correspondence between a vertex set  $V$  and a set of intervals  $I$  on the real line  $R$  such that two vertices are adjacent in  $G$  if and only if their corresponding interval have non empty intersection.

We draw a vertex  $v_j$  for the interval  $I_j, j = 1, 2, \dots, n$ . Two vertices  $v_i$  and  $v_j$  are connected by an edge if and only if there corresponding intervals have non-empty intersection. It is observed that an interval  $I_j$  and a vertex  $v_j$  are one and same thing. An interval graph and its corresponding interval representation are shown in Figure 7.

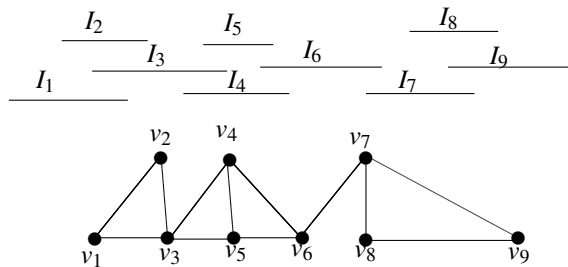


Figure 7. An interval representation and its corresponding interval graph

In Figure 7 the number of intervals is 9, so the number of vertices is also 9, namely  $v_1, v_2, \dots, v_9$ . Also, we see that the intervals  $I_1$  and  $I_2$  have non-empty intersection, so there is an edge between the vertices  $v_1$  and  $v_2$ . Also, the intervals  $I_1$  and  $I_3$  have non-empty intersection, so there is also an edge between the vertices  $v_1$  and  $v_3$ . But the intervals  $I_1$  and  $I_4$  have no common portion, so there is no edge between the vertices  $v_1$  and  $v_4$ . In the same manner the above intersection graph have drawn.

### 3. Graph Colouring

Let  $G = (V, E)$  be a simple graph. *Graph Colouring*, specially *Vertex Colouring* is the assignment of colours to the vertices of the graph so that no two adjacent vertices share the same colour. Therefore any graph having  $n$  vertices can easily be coloured using  $n$  colours (viz. one colour to one vertex). But, the main objective to colour a graph is to minimizing the number of colour used. The formal definition is presented below.

**Definition 3.1.** For any graph  $G = (V, E)$  is a graph the proper colouring means, colour the vertices of  $G$  using least number of colours in such a way that adjacent vertices have different colour.

If  $k$  colours are required to proper colour a graph  $G$  then the graph  $G$  is called  $k$ -colourable.

The least  $k$  for which  $G$  is  $k$ -colourable is called chromatic number and is denoted by  $\chi(G)$  is. Conversely, a graph  $G$  is  $k$ -chromatic if  $\chi(G) = k$ . A proper  $k$ -colouring of a  $k$ -chromatic graph is an optimal colouring.

If a graph  $G$  is  $k$ -chromatic, then to colour all the vertices of  $G$  needs  $k$  colours, it can not be coloured using  $k - 1$  colours or less colours.

Let us consider a graph with six vertices  $a, b, c, d, e$  and  $f$  (see Figure 8).

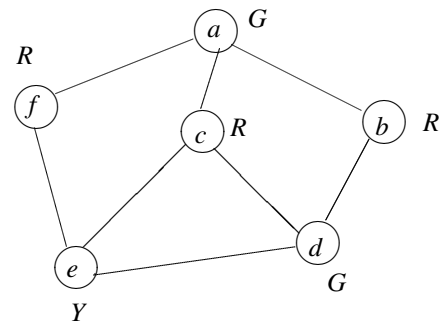


Figure 8. A three colourable graph

First we colour the vertex  $a$  by an arbitrary colour say  $G$ (green). Since, the vertex  $a$  is adjacent to the vertices  $b, c$  and  $f$  so, the colour of these vertices can never be green. So we colour these vertices by  $R$ (red). Note that the vertices  $b, c, f$  are not connected to each other. Now the colour of the vertex  $d$  can never be  $R$ (red) as it is adjacent to both the vertices  $b$  and  $c$  whose colour is  $R$ (red). But this vertex can be coloured by  $G$ . The vertex  $e$  is adjacent to the vertices  $d, c, f$  and they are coloured by either  $G$  or  $R$ . So, to colour the vertex  $e$  we need a new colour say  $Y$ (yellow). There is no vertices are left to colour the graph. Thus, the graph of Fig. 8 needs only three colours and hence this graph is 3-colourable.  $L(h_1, h_2, \dots, h_m)$ -labeling is an extension of vertex colouring problem.

An  $L(h_1, h_2, \dots, h_m)$ -labeling of a graph  $G = (V, E)$  is a function  $f$  from its vertex set  $V$  to the set of non-negative integers such that  $|f(x) - f(y)| \geq h_i$  if  $d(x, y) = i$  for  $i = 1, 2, \dots, m$ . The span of an  $L(h_1, h_2, \dots, h_m)$ -labeling  $f$  of  $G$  is  $\max\{f(v) : v \in V\}$ . The  $L(h_1, h_2, \dots, h_m)$ -labeling number  $\lambda_{h_1, h_2, \dots, h_m}(G)$  of  $G$  is the smallest non-negative integer  $p$  such that  $G$  has a  $L(h_1, h_2, \dots, h_m)$ -labeling of span  $p$ .

#### 3.1 Vertex Colouring

Now we discuss the colouring of some special types of graphs.

**Path  $P_n$ :** Let  $P_n$  be a path with  $n$  vertices  $v_1, v_2, \dots, v_n$ . Now, we can colour the vertices with odd indices by an arbitrary colour, say, blue( $B$ ) and the vertices with even indices by



yellow( $Y$ )( see Figure 9). Therefore,  $\chi(P_n) = 2$ , for  $n \geq 2$ .

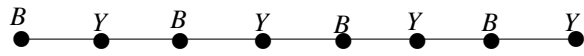


Figure 9.  $P_n$  is 2-colourable,  $n \geq 2$

**Cycle  $C_n$ :** Any cycle  $C_n$  with even number of vertices can be coloured by only two colours like  $P_n$ . But, if the number of vertices be odd then exactly three colours are needed to colour  $C_n$ . Let the vertices of  $C_{2k+1}$  be  $v_1, v_2, \dots, v_{2k}, v_{2k+1}$ . We assign the colour to the vertices  $v_1, v_3, \dots, v_{2k+1}$  by say violet( $V$ ) and  $v_2, v_4, \dots, v_{2k}$  by, say yellow( $Y$ ). The vertex  $v_{2k+1}$  is adjacent to both  $v_1$  (with colour  $V$ ) and  $v_{2k}$  (with colour  $Y$ ). So, to colour the vertex  $v_{2k+1}$  needs one more colour, say  $B$ . Thus  $\chi(C_{2k}) = 2$  and  $\chi(C_{2k+1}) = 3$  (see Figure 10).

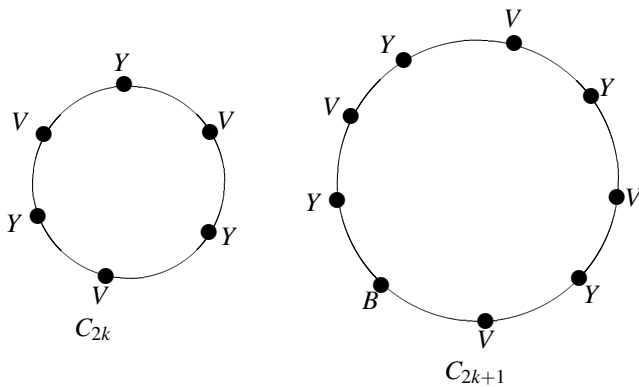


Figure 10. Colouring of  $C_n$

**Complete graph  $K_n$ :** Since in complete graph every vertices are adjacent with all other vertices. So, to colour  $n$  vertices of  $K_n$  needs exactly  $n$  colours. Thus,  $\chi(K_n) = n$ .

**Tree:** Tree is a very simple graph to colour it. Every tree is 2-colourable, explained in Figure 11

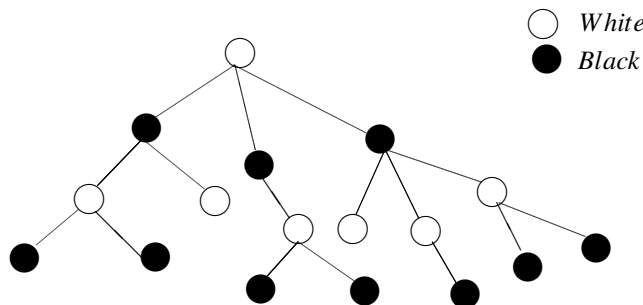


Figure 11. Tree is 2-colourable

**Complete bipartite graph  $K_{m,n}$ :** A  $K_{3,4}$  is shown in Fig. 12

In general  $\chi(K_{m,n}) = 2$ .

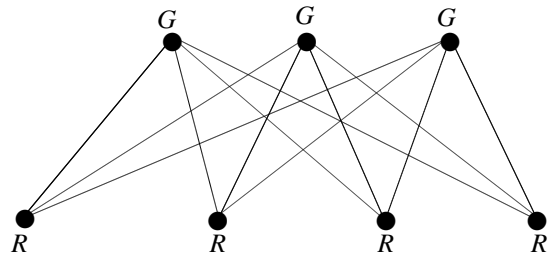


Figure 12.  $K_{3,4}$  is 2-chromatic

### 4. Real life applications of graph theory

Almost all real life problem can be solved by designing graph. Some important applications of graph theory in real life are presented below.

#### Application I:

A tropical fish hobbit had seven different types of fishers  $A, B, C, D, E, F$  and  $G$ . Because of prey-predator relationships, water conditions and size, some fishers can not kept in the same tank. The following table shows which fish can not be together.

Type	I	II	III	IV	V	VI
Can not be with	$A, B, E$	$B, C, D$	$E, G$	$D, F$	$C, F$	$A, G$

What is the minimum number of tanks needed to kept the fishers.

First we draw a graph  $G$  whose vertices are  $A, B, C, D, E, F$  and  $G$ . Two vertices are joined by an edge if they can not be kept in the same tank. The fishers  $A, B, E$  can not be with, so there is edges between the vertices  $A$  and  $B$ ;  $A$  and  $E$ ;  $B$  and  $E$ . Similarly, using this rule we obtain the graph  $G$  shown in Figure 13.

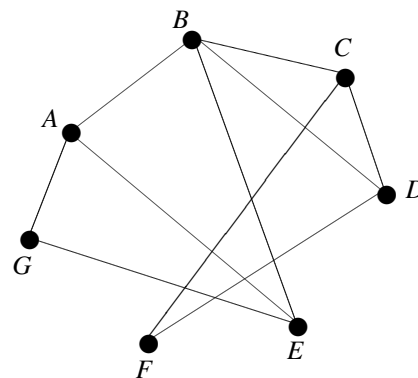


Figure 13. Relationship among the fisher

Now, we colour the vertices using  $V$  (Violet),  $P$  (Pink) and  $Y$  (Yellow) colours as shown in Figure 14.

Thus the graph of Figure 13 is three colourable and hence we conclude that three different tanks are required to the fishers, as per following schedule.



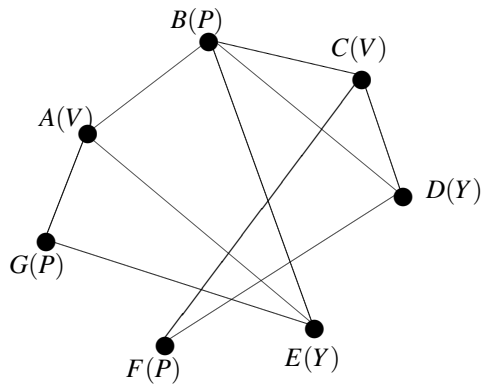


Figure 14. Vertices are coloured using  $V, P$  and  $Y$

Tank number	Fishers
Tank 1 V(violet)	$A, C$
Tank 2 P(pink)	$B, F, G$
Tank 3 Y(yellow)	$D, E$

**Application II:**

**Scheduling:** Suppose a university offer the following combinations of courses for the students. A student should choose one of the following group courses.

- Group 1: Biology, English, Computer Science,
- Group 2: Mathematics, Physics, Chemistry
- Group 3: Computer Science, Mathematics, Physics
- Group 4: Physics, English, Chemistry,
- Group 5: Mathematics, Chemistry, Biology
- Group 6: Biology, Psychology, Chemistry
- Group 7: Psychology, Biology, Mathematics
- Group 8: Computer Science, English, Chemistry

What is the least number of examination periods required in the seven course specified so that the students taking any of the given combinations of courses with out any conflicts. Also, find a possible schedule which uses this least number of periods.

We draw a graph with seven vertices  $E, B, CS, P, M, C$  and  $Ps$ , for English, Biology, Computer Science, Physics, Mathematics, Chemistry and Psychology respectively. Join two vertices with an edge if two subjects belong to the same group. For example, English, Biology, Computer Science belong to the same group 1, so their corresponding vertices  $E, B$  and  $CS$  are joined by edges. Physics, Mathematics, Chemistry belong to the same group 2, so their corresponding vertices  $P, M$  and  $C$  are joined by edges. In the same manner the have designed. The graph of this program is shown in Figure 15.

We colour the vertices  $M$  by  $V$  (violet), (so the colour of  $P, C, CS, B$  and  $Ps$  should not be  $V$ ),  $P$  by  $S$  (silver) (the colour of  $C$  can not be  $V$  and  $S$ ),  $C$  by  $R$  (red),  $CS$  by  $W$  (white),  $E$  by  $V$ ,  $B$  by  $S$  and  $Ps$  by  $W$  (see Figure 15).

Note that only four colours are needed to colour this graph. The vertices with the same colour are not adjacent and hence the corresponding subjects can be sheduled in the same period.

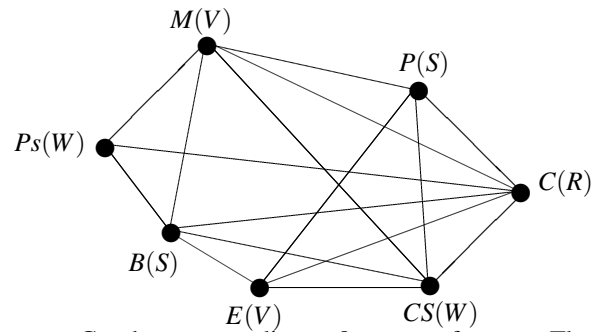


Figure 15. Graph corresponding to 8 groups of course. The alphabets within the parenthesis represent colours.

That is, the colours of  $M$  and  $E$  are same so the examination of Mathematics and English can take simultaneously. The computer schedule are shown below.

Period	Subjects
1(V)	Mathematics, English
2(S)	Physics, Biology
3(R)	Chemistry
4(W)	Computer Science, Psychology

**Application III:**

**Coloring of Map in GSM Mobile Phone Networks:**

Four colour theorem is used in the *GSM* network. In a Mobile phone network like Groups Special Mobile (*GSM*), the network’s geographical area is split up into hexagonal regions ,named cells. Mobile phones in the cell will be connected to the informing tower that exists in that cell. By searching the cells in the neighbors, mobile phones are connected to the *GSM* network. Four color theory is applied in *GSM*,since, it work only in four separate frequency ranges. The cellular regions are perfectly colored using four colors. The vertex coloring algorithm can also be used to determine a maximum four different frequencies for any *GSM* mobile phone network.

Map drawn on the plane which is coloured correctly by four color theorem using at most four distinct colors. The conventional coloring should be such that no two adjacent regions are assigned the identical color. For the given map, we develop the dual graph. The dual graph is prepared by imposing the vertex inside each region of a map. The edges will join the vertex inside two regions if they have a common edge forming the boundary. The chromatic number of the dual graph gives the chromatic number of the original map. The vertex coloring of the dual graph is same as the coloring of region in the map. So the four color theory can be used in *GSM* network where four colors refer to the four frequency ranges in which suvh a network operates. Figure 3(Map of India) shows the map of India colored using four colors. Figure 4(Dual map of India) shows the dual graph designed for the map. The vertex coloring is also used for the dual graph. A maximum of four colors are to be used to vertex coloring, so a maximum of four color is required for map colouring. So in a *GSM* network, a four color theorem can be used to assign four frequencies.





**Application IV:****Google Maps or GPS:**

Google Maps or *GPS* are to find out the shortest route from one destination to another. Here the goals are assumed as vertices, and their connections are edges comprising distance. The optimal path is determined by software. Colleges or Schools are also use this technique to gather students from their bus stop (or railway station) to college or school. To each one bus stop or station is treated as a vertex and the route as an edge. A Hamiltonian path presents the efficiency of including every vertex in the route.

**Application V:  
Traffic Signal Lights:**

Graph colouring is used in traffic signal lights problem. For studying traffic control problems at an arbitrary point of intersection, it has to be modeled mathematically using a simple graph for the traffic accumulation data problem. The rudimentary map edges will represent the communication link between the set of vertices at an intersection point. In the graph, set for the traffic control problem, an edge will join the traffic streams that may move simultaneously at a corner without any difference. An edge will not connect the streams that cannot move together. The function of traffic lights is turning Green/Red/Yellow lights and timing between them. Here vertex coloring technique is utilized to solve time and space by identifying the chromatic number for the number of cycles needed.

**Application VI:**

**Aircraft Scheduling:** Graph coloring can be used to schedule  $k$  aircrafts which has to assign to  $n$  flights. The  $p$ th flight is at the time interval  $(a_p, b_p)$ . If two flight overlap then we cannot assign the aircraft. In this case interval graph is used and flights are represented by vertices. The two vertices are connected by an edge if the time intervals of flights are overlap. The interval graph can be optimally coloured in polynomial time.

**Application VII:**

**To clear road blockage:** Graph theoretic idea is used to clear road blockage. When roads of any town/city is closed due to snowfall, planning is required so that the roads is clear to put salt on the roads. Then Euler paths or Euler circuits are used to traverse the streets efficiently.

**5. Conclusion**

The main objective of this paper is to present the significance of graph theory in our daily life. There are many real world problems that can be solved by graph theory and some real life problems exists where graph theory is the only alternative to solve these type of problems. In this article the author have discussed a lot of problems that is efficiently solved by

applying graph theory and most of them be solved by using colouring of graphs. There are many applications of graph theory in different branches like chemistry, physics, biology, economics etc. Therefore, graph theory has developed into a subject itself with variety of applications.

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