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Bipolar topological pre-closed neutrosophic sets

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Abstract

We extend the topological spaces to bipolar neutrosophic study. The concept of topological spaces in neutrosophic theory recently discussed by many authors on semi generalized conditions of open and closed sets. In this present manuscript the topological space is defined on bipolar pre-closed neutrosophic sets (BPCNS).

Keywords

Bipolar neutrosophic closure, Bipolar neutrosophic interior, Bipolar pre-closed neutrosophic sets (BPCNS).

AMS Subject Classification

18B30, 03E72.

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Contents

1. Introduction

Neutrosophic technique gives a exact information about neutral reports and this technique was derived by Florentin Smarandache [5] in 1998on sets, logic and probability are reciprocated with neutrosophic technique. Neutrosophic technique is consequent obtained from Fuzzy and intuitionistic Fuzzy technique. Neutrosophic technique means to discover ordinary features on non ordinary values. T, I, F are known as neutrosophic values they stand for the truth membership, indeterminacy membership and false membership.The technique in this every intention is projected to contain the entitlement of T, the entitlement of $I$and entitlement of F with $0 \leq T+I+F \leq 3$as they are many neutrosophic rules of inference.

J. Dezert [8] demonstrated fuzzy concept while a numerical instrument intended for considering all uncertainties of elements with every input to obtain. Fuzzy logic and Intuitionistic Fuzzy logic was derived by L. Zadeh[11] and Atanassov[9] respectively, when the overview of fuzzy any whereas well as the contribution with each component. A. A. Salama et.al [1] established the model of topological spaces on neutrosophic sets. R. Chang [3] introduced neutrosophic generalized closed sets with generalized neutrosophic continuous mapping.In this article,to evaluate the bipolar pre-closed neutrosophic sets (BPONS) in bipolar neutrosophic topological spaces.

Notations:

1. Neutrosophic set (NS)
2. Bipolar Neutrosophic set (BNS)
3. Bipolar pre-open neutrosophic sets (BPONS)
4. Bipolar pre-closed neutrosophic sets (BPCNS).
5. Bipolar Neutrosophic topology (BNT)
6. Bipolar neutrosophic closure (BNC)
7. Bipolar neutrosophic interior (BNI)
8. Bipolarneutrosophic open set (BNOS)
9. Bipolar neutrosophic closedset(BNCS)
10. Bipolar neutrosophic semi-closed(BNSC)
11. Bipolar neutrosophic semi open(BNSO)

This article based on the soft neutrosophic topology. Here we start with some basic definitions.

2. Preliminaries

We remember several essential terminology exacting study of Smarandache[18],terminology based on Atanassov in [10] and notations based on A. A. Salama[2].
The number of types of Neutrosophic topology problems contained to residential through a lot of researchers who assign the weights to edges are not accurate in that an uncertainty is there for that see the references [4, 6, 7, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22]. In this section, we recall some definitions and basic results of fractional calculus which will be used throughout the paper.

Definition 2.1. *A Neutrosophic set (NS) is triplet structure (T,I,F) which is the degree of contribution functions with the circumstance $0 \leq T+I+F \leq 3$ and also lies among 0 and 1.**[5]**.*

Definition 2.2. *A Bipolar Neutrosophic set (BNS) is the structure having six values in that three are positive and remaining three are negative. The structure is $\left(T^N, I^N, F^N, T^P, I^P, F^P\right)$ and also the negative values lies between [-1,0] and positive values lies between [0,1].*

Definition 2.3. *Let $S_1 = (T_{S_1}, I_{S_1}, F_{S_1})$ and $S_2 = (T_{S_2}, I_{S_2}, F_{S_2})$ are the BNS in that consider two possible cases for subsets.*

$$1. S_1 \subseteq S_2 \Leftrightarrow T_{s_1} \leq T_{s_2}, I_{s_1} \leq I_{s_2} \text{ and } F_{s_1} \geq F_{s_2}.$$

$$2. S_1 \subseteq S_2 \Leftrightarrow T_{s_1} \leq T_{s_2}, I_{s_1} \geq I_{s_2} \text{ and } F_{s_1} \geq F_{s_2}.$$

Definition 2.4. *Let $S_1 = (T_{S_1}, I_{S_1}, F_{S_1})$and $S_2 = (T_{S_2}, I_{S_2}, F_{S_2})$ are the BNS then following may be defined as*

$$(I_1) S_1 \cap S_2 = T_{S_1} \wedge T_{S_2}, I_{S_1} \wedge I_{S_2}, F_{S_1} \vee F_{S_2}$$

$$(I_1) S_1 \cup S_2 = T_{S_1} \vee T_{S_2}, I_{S_1} \wedge I_{S_2}, F_{S_1} \wedge F_{S_2}$$

Theorem 2.5. *Let $S_1 = (T_{S_1}, I_{S_1}, F_{S_1})$ and $S_2 = (T_{S_2}, I_{S_2}, F_{S_2})$ are the BNS then the sub sequent situation are holds.*
$(1) BNC(S_1 \cap S_2) = BNC(S_1) \cup BNC(S_2)$
$(2) BNC(S_1 \cup S_2) = BNC(S_1) \cap BNC(S_2)$

Definition 2.6. *A BNT is a non-empty set W is a family $B\tau_N$ of a BNS in W holding the sub sequent situation.*
(BNT_1) $0,1 \in B\tau_N$
(BNT_2) $S_1 \cap S_2 \in B\tau_N$ for any $S_1, S_2 \in B\tau_N$
(BNT_3) $\cup S_i \in B\tau_N$ for every $\{S_i : i \in I\} \subseteq B\tau_N$
Therefore $(W, B\tau_N)$is BNT.

Example 2.7. *Let $W = \{w\}$ and*

$$P = \{< w, -0.2, -0.1, -0.5 : 0.4, 0.4, 0.3 >: w \in W\}$$

$$Q = \{< w, -0.2, -0.6, -0.4; 0.3, 0.4, 0.7 >: w \in W\}$$

$$R = \{< w, -0.6, -0.3, -0.2; 0.4, 0.5, 0.3 >: w \in W\}$$

$$S = \{< w, -0.7, -0.2, -0.5; 0.4, 0.6, 0.8 >: w \in W\}$$

Then the family $\tau = \{0_N, 1_N, P, Q, R, S\}$ of bipolar neutrosophic set in W is bipolar neutrosophic topology W.

Definition 2.8. *Let $(W, B\tau_N)$ be BNT space $S_1 = \langle T_{S_1}, I_{S_1}, F_{S_1} \rangle$ be a BNS in W then BNC and BNI of S_1 are defined as follows*

$$BNcl(S_1) = \cap\{R_1,\ r_1 \text{ is a BNcs in} W \text{ and } S_1 \subseteq R_1\}$$

$$BNint(S_1) = \cup\{R_2,\ r_2 \text{ is a BNos in} W \text{ and } R_2 \subseteq S_1\}$$

It preserves as well show that $BNcl(S_1)$ is BNC set and $BNint(S_1)$ is BNO set in W.
(i) S_1 is BNOS$\Leftrightarrow S_1 = BNint(S_1)$
(ii) S_1 is BNCS$\Leftrightarrow S_1 = BNcl(S_1)$.

Theorem 2.9. *Let$(W, B\tau_N)$ be BNT space and $S_1 \in (W, B\tau_N)$ we have*

$$(a)\ BNcl\left(C\left(S_1\right)\right) = C\left(BNint\left(S_1\right)\right)$$

$$(b)\ BNint\left(C\left(S_1\right)\right) = C\left(BNcl\left(S_1\right)\right)$$

Theorem 2.10. *Let $(W, B\tau_N)$ be BNT space and S_1, S_2 are two BNS in W then the subsequent rules are hold.*

$$a)\ BNint\left(S_1\right) \subseteq S_1$$

$$b)\ S_1 \subseteq BNcl\left(S_1\right)$$

$$c)\ S_1 \subseteq S_2 \Rightarrow BNint\left(S_1\right) \subseteq BNint\left(S_2\right)$$

$$d)\ S_1 \subseteq S_2 \Rightarrow BNcl\left(S_1\right) \subseteq BNcl\left(S_2\right)$$

$$e)\ BNint\left(BNint\left(S_1\right)\right) = BNint\left(S_1\right)$$

$$f)\ BNcl\left(BNcl\left(S_1\right)\right) = BNcl\left(S_1\right)$$

$$g)\ BNint\left(S_1 \cap S_2\right) = BNint\left(S_1\right) \cap BNint\left(S_2\right)$$

$$h)\ BNcl\left(S_1 \cup S_2\right) = BNcl\left(S_1\right) \cup BNcl\left(S_2\right)$$

$$i)\ BNint\left(0_N\right) = BNcl\left(0_N\right) = 0_N$$

$$j)\ BNint\left(1_N\right) = BNcl\left(1_N\right) = 1_N$$

$$k)\ S_1 \subseteq S_2 \Rightarrow BNC\left(S_1\right) \subseteq BNC\left(S_2\right)$$

$$l)\ BNCL\left(S_1 \cap S_2\right) \subsetneq BNcl\left(S_1\right) \cap BNcl\left(S_2\right)$$

$$m)\ BNint\left(S_1 \cup S_2\right) \supseteq BNint\left(S_1\right) \cup BNint\left(S_2\right)$$

Definition 2.11. *Let $(W, B\tau_N)$ be BNT space and $S_1 = (T_{S_1}, I_{S_1}, F_{S_1})$ be a BNS in W then S_1 is a BNSO if $S_1 \subseteq BNcl\left(BNint\left(S_1\right)\right)$*
and also
BNSC if $BNint\left(BNcl\left(S_1\right)\right) \subseteq S_1$.
The complement of BNSO set is a BNSC set.

3. Bipolar pre-closed neutrosophic sets (BPCNS)

Definition 3.1. *Let S_1 be BNS of a BNT spaces W. Then S_1 is known as BPCNS of W if there exists a BNC set such that $BNcl(BNC(S_1)) \subseteq S_1 \subseteq BNC(S_1)$.*

Theorem 3.2. *Any subset S_1 in a $BNTS$ W is BNCS set iff $BNcl(BNint(S_1)) \subseteq S_1$* ***Proof:*** *consider $BNcl(BNint(S_1)) \subseteq S_1$*
Then $BNC = BNcl(S_1)$
clearly $BNcl(BNint(S_1)) \subseteq S_1 \subseteq BNC(S_1)$
Therefore $S_1$is BNPC set
On the other hand, assume that Let S_1 be BNPC contained in W
Then $BNC(BNint(S_1)) \subseteq S_1 \subseteq BNC(S_1)$ for some BNS closed set BNC.
but $BNcl(S_1) \subseteq BNC(S_1)$.
Hence the theorem proved.

Theorem 3.3. *Let $(W, B\tau_N)$ be $BNTS$ and S_1be a BNS of W then S_1 is a BPCNS iff $BNC(S_1)$ is BPONS in W.*
Proof: *Let S_1 be a BPCNS subset of W.*
Clearly $BNcl(BNint(S_1)) \subseteq S_1$
By applying complement on together sides

$$BNC(S_1) \subseteq BNC(BNcl(BNint(S_1)))$$

$$BNC(S_1) \subseteq BNint(BNcl(BNC(S_1)))$$

Hence $BNC(S_1)$is BNPO set
On the contrary,assume that $BNC(S_1)$ is BNPO set
i.e. $BNC(S_1) \subseteq BNint(BNcl(BNC(S_1)))$
By applying complement on together sides obtained
$BNcl(BNint(S_1)) \subseteq S_1$ is BPCNS.
Hence the theorem proved.

Theorem 3.4. *Consider $(W, B\tau_N)$ be a BNT spaces. Then conjunction of two BPCNS is also a BPCNS.*
Proof: *Consider S_2 and S_2 are bipolar neutrosophic pre-closed sets on $(W, B\tau_N)$.*
Then $BNcl(BNint(S_1)) \subseteq S_1$, $BNcl(BNint(S_2)) \subseteq S_2$
Consider $S_1 \cap S_2 \supseteq BNcl(BNint(S_1)) \cap BNcl(BNint(S_2))$

$$S_1 \cap S_2 \supseteq BNcl(BNint(S_1)) \cap (BNint(S_2))$$

$$S_1 \cap S_2 \supseteq BNcl(BNint(S_1 \cup S_2))$$

$$BNcl(BNint(S_1 \cap S_2)) \subseteq S_1 \cap S_2$$

Hence $S_1 \cap S_2$ is BPCNS.

Remark 3.5. *The union of any two BPCNS need not be a BPCNS on $(W, B\tau_N)$.*

Theorem 3.6. *Let $\{S_1\}_{\alpha\in\Delta}$ be a collection of BPCNS on $(W, B\tau_N)$ then $\bigcap\limits_{\alpha\in\Delta} S_{1\alpha\in\Delta}$ BPCNS on $(W, B\tau_N)$.*
Proof: *A neutrosophic set BNC_α such that*
$BNC_\alpha(BNint(S_1)) \subseteq S_{1\alpha} \subseteq BNC_\alpha(S_1)$ for all $\alpha \in \Delta$
Then $\bigcap\limits_{\alpha\in\Delta} BNC_\alpha(BNint(S_1)) \subseteq \bigcap\limits_{\alpha\in\Delta} S_{1\alpha} \subseteq \bigcap\limits_{\alpha\in\Delta} BNC_\alpha(S_1)$

$$\bigcap_{\alpha\in\Delta} BNcl_\alpha(BNint(S_1)) \subseteq \bigcap_{\alpha\in\Delta} S_{1\alpha}$$

Hence $\bigcap\limits_{\alpha\in\Delta} S_{1\alpha}$ is BPCNS on$(W, B\tau_N)$.

Theorem 3.7. *Every BNCS in the BNT spaces $(W, B\tau_N)$ is BPCNS in $(W, B\tau_N)$.*
Proof: *Let S_1 be BNCS means $S_1 = BNcl(S_1)$ and also $BNint(S_1) \subseteq S_1$*
From that
$BNcl(BNint(S_1)) \subseteq BNcl(S_1)$, $BNcl(BNint(S_1)) \subseteq S_1$,
since $S_1 = BNcl(S_1)$
Hence S_1 is a BPCNS.

Theorem 3.8. *Let S_1 be a BNCS in BNT spaces $(W, B\tau_N)$ and suppose $BNint(S_1) \subseteq S_2 \subseteq S_1$then S_2 is BPCNS on $(W, B\tau_N)$.*
Proof: *Let S_1 be a BNS in BNT spaces $(W, B\tau_N)$*
Suppose $BNint(S_1) \subseteq S_2 \subseteq S_1$
There exist a BNCS, such that
$BNC(BNint(S_1)) \subseteq S_2 \subseteq S_1 \subseteq BNC$.
Then $S_2 \subseteq BNC$ and also $BNint(S_2) \subseteq S_2 \subseteq BNC$.
Thus, $BNcl(BNint(S_2)) \subseteq S_2$
Hence $S_2$is bipolar neutrosophic pre-closed set on $(W, B\tau_N)$.

Theorem 3.9. *Let W_1 and W_2 are BNT spaces with the conditions W_1 is BN multiplication associated to W_2 then the BN multiplication associated to $S_1 \times S_2$ is a BPCNS of the BN multiplication associated to topological space $W_1 \times W_2$.Where BPCNS S_1 of W_1 and a BPCNS S_2 of W_2.*
Proof:*Let S_1 and S_2 are BPCNS.*
$BNC_1(BNint(S_1)) \subseteq S_1 \subseteq BNC_1$ and
$BNC_2(BNint(S_2)) \subseteq S_2 \subseteq BNC_2$
Form the above,
$BNC_1(BNint(S_1)) \times BNC_2(BNint(S_2))$
$\subseteq S_1 \times S_1 \subseteq BNC_1 \times BNC_2$
$(BNC_1 \times BNC_2)(BNint(S_1 \times S_2)) \subseteq S_1 \times S_2 \subseteq BNC_1 \times BNC_2$
Hence $S_1 \times S_2$ is BPCNS in BNT space $W_1 \times W_2$.

4. Conclusion

In this article, Bipolar topological Pre-closed neutrosophic sets explained on pre closed bipolar neutrosophic theory. we discussed about some basic definitions about neutrosophic topological space, bipolar pre-closed neutrosophic set etc.,. Further we obtained the results based on pre-closed sets with similar results

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