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Some investigation on continuous functions in almost contra semi-generalized star b in topological spaces

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Abstract

This manuscript's main aim is to present some class of functions in topological spaces called almost contra semi generalized star b-continuous function (briefly almost contra sg^*b-continuous). Some characterizations and a variety of characteristics are obtained related to almost contra sg^*b-continuous functions.

Keywords

sg^*b-closed sets; sg^*b-closed map; sg^*b-continuous map; contra sg^*b-continuity.

AMS Subject Classification

54C05, 54C08, 54C10.

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1. Introduction

A novel class was organized and investigated by Jafari and Noiri in 2002 of functions called continuous contra-pre functions. The purpose of this paper is to implement and research almost against semi generalised star b-continuous functions through the semi generalised star b-closed sets. Also, near contra semi generalised star b-continuity characteristics are discussed about. In addition, fundamental properties and preservation are obtained. semi generalised star b-continuous theorems This paper defines the $((X,\tau)$ and (Y,σ) describing the non-empty topological spaces without axioms of separation Unless otherwise mentioned, this is presumed. Let $A \subseteq X$, the $cl(A)$ and will denote the closure of A and the interior of A $int(A)$ and the union of all X semi generalised star b-open sets found in A is classified as sg-b-interior of A and is marked by $sg^*bint(A)$, The intersection of all semi generalised star b-closed sets of X containing A is called semi generalised star b-closure of A and it is denoted by $sg^*bcl(A)$ [5].

2. Continuous functions Almost contra semi generalised star b

In this section, we present nearly contra semi generalised in this section Star b-continuous features and analyzed of some of their functions axioms.

Definition 2.1. *[11] A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is almost called contra semi generalised star b - continuous if $f^{-1}(V)$ is semi generalised star b - closed in (X,τ) for each regular open set V in (Y,σ).*

Example 2.2. *[11] Let $X=Y=\{l,m,n\}$ with $\tau=\{X,\varphi,\{l\},\{l,m\}\}$ and $\sigma=\{Y,\varphi,\{l\},\{m\},\{l,m\}.\{l,n\}\}$. Define a function $f:(X,\tau)\rightarrow(Y,\sigma)$ by $f(l)=m$, $f(m)=n$, $f(n)=l$. Clearly f is almost contra semi generalised star b - continuous.*

Theorem 2.3. *If $f:X\rightarrow Y$ is contra semi generalised star b - continuous then it is almost contra semi generalised star b - continuous.*

Proof. It's simple, because every open regular set is an open set. □

Remark 2.4. *Alternatively, the theorem alluded to above need not be valid as shown in the subsequent case in common.*

Example 2.5. *Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{n\}\}$ and $\sigma = \{Y,\varphi,\{l\},\{m\}, \{l,m\},\{m,n\}\}$. Defining a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = m$, $f(m) = n$, $f(n) = l$. Then f is almost contra semi generalised star b - continuous but not contra function semi generalised star b - continuous, because for the open set $\{l\}$ in Y and $f^{-1}\{l\} = \{n\}$ is not semi generalised star b - closed in X.*

Theorem 2.6. *1) Almost every contra pre - continuous function is nearly contra semi generalised star b - continuous function.*
2) Each semi-continuous almost contra function is almost contra semi generalised star b - continuous function.
3) Almost every contra α - the continuous function is almost contra semi generalised star b - continuous function.
4) Almost every contra αg - the continuous function is almost contra semi generalised star b - continuous function.
5) Almost every contra semi generalised star b - the continuous function is almost contra gsp - continuous function.
6) Almost every contra semi generalised star b -the continuous function is almost contra gb - continuous function.
7) Almost every contra sg - the continuous function is almost contra semi generalised star b - continuous function.

Remark 2.7. *As can be seen in the following example, the reverse of the above claims is not true.*

Example 2.8. *i) Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi, \{l\}, \{m\} ,\{l,m\}\}$ and $\sigma = \{Y,\varphi,\{l\},\{n\},\{l,n\}\}$. Defining a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = m$, $f(m) = l$, $f(n) = n$. Clearly f is almost contra semi generalised star b - continuous but f is not nearly contra pre - continuous. Because $f^{-1}(\{n\}) = \{m\}$ is not pre - closed in (X,τ) where $\{n\}$ is regular - open in (Y,σ).*
ii) Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{l,m\}\}$ and $\sigma = \{Y,\varphi,\{l\},\{m\},\{l,m\}\}$. Defining a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = n$, $f(m) = l$, $f(n) = m$. It's obvious that f is almost contra semi generalised star b - continuous but f is not almost contra semi - continuous. Because $f^{-1}(\{l\}) = \{m\}$ is not semi - closed in (X,τ) where $\{l\}$ is regular - open in (Y,σ).
iii) Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{m\},\{l,m\}\}$ and $\sigma = \{Y,\varphi,\{l\},\{m\},\{l,m\}, \{l,n\}\}$. Defining a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = m$, $f(m) = l$, $f(n) = n$. It's obvious that f is almost contra semi generalised star b - continuous but f is not almost contra α - continuous. Because $f^{-1}(\{l,n\}) = \{m,n\}$ is not α - closed in (X,τ) where $\{l,n\}$ is regular - open in (Y,σ).
iv) Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{l,m\}\}$ and $\sigma = \{Y,\varphi,\{l\},\{m\},\{l,m\},\{m,n\}\}$. Defining a functionDefine a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = n$, $f(m) = l$, $f(n) = m$. It's obvious that f is almost contra semi generalised star b - continuous but f is not almost contra αg - continuous. Because $f^{-1}(\{l\}) = \{m\}$ is not αg - closed in (X,τ) where $\{l\}$ is regular - open in (Y,σ).
v) Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{l\},\{l,n\}\}$ and $\sigma = \{Y,\varphi,\{l\},\{n\},\{l,n\}, \{l,m\}\}$. Defining a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = m$, $f(m) = n$, $f(n) = l$. It's obvious that f is almost contra gsp - continuous but f is not almost contra semi generalised star b - continuous. Because $f^{-1}(\{m,n\}) = \{l,m\}$ is not semi generalised star b - closed in (X,τ) where $\{m,n\}$ is regular - open in (Y,σ).
vi) Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{l\},\{l,m\}\}$ and $\sigma = \{Y,\varphi,\{l\},\{n\},\{l,n\}, \{m,n\}\}$. Defining a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = m$, $f(m) = l$, $f(n) = n$. It's obvious that f is almost contra gb - continuous but f is not almost contra semi generalised star b - continuous. Because $f^{-1}(\{m,n\}) = \{l,n\}$ is not semi generalised star b - closed in (X,τ) where $\{m,n\}$ is regular - open in (Y,σ).
vii) Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{m\},\{l,m\}\}$ and $\sigma = \{Y,\varphi,\{m\},\{n\},\{l,m\}, \{m,n\}\}$. Defining a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = m$, $f(m) = l$, $f(n) = n$. It's obvious that f is almost contra semi generalised star b - continuous but f is not almost contra sg - continuous. Because $f^{-1}(\{l,m\}) = \{l,m\}$ is not sg - closed in (X,τ) where $\{l,m\}$ is regular - open in (Y,σ).

Remark 2.9. *Near-contra semi generalised star b-continuous principle as seen in the following examples, almost contra sgb -continuous are independent.*

Example 2.10. *Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{m\},\{l, m\}\}$ and $\sigma = \{Y,\varphi, \{l\},\{m\},\{l,m\},\{m,n\}\}$. Defining a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = n$, $f(m) = m$, $f(n) = l$. It's obvious that f is almost contra semi generalised star b - continuous but f is not almost contra sgb - continuous. Because $f^{-1}(\{m,n\}) = \{l,m\}$ is not sgb - closed in (X,τ) where $\{m,n\}$ is regular - open in (Y,σ).*

Example 2.11. *Let $X = Y = \{l,m,n\}$ with $\tau = \{X,\varphi,\{l\},\{n\}, \{l,n\},\{m,n\}\}$ and $\sigma = \{Y,\varphi,\{m\},\{n\},\{m,n\}\}$. Define a function $f : (X,\tau) \to (Y,\sigma)$ by $f(l) = n$, $f(m) = l$, $f(n) = m$. It's obvious that f is almost contra sgb - continuous but f is not almost contra semi generalised star b - continuous. Because $f^{-1}(\{m\}) = \{n\}$ is not semi generalised star b - closed in (X,τ) where $\{m\}$ is regular - open in (Y,σ).*

Theorem 2.12. *The following are equivalent for a function $f : X \to Y$,*
(a) f is almost contra semi generalised star b - continuous.
(b) for every regular closed set F of Y, $f^{-1}(F)$ is semi generalised star b - open set of X.
(c) for each $x \in X$ and each regular closed set F of Y containing $f(x)$, there exists semi generalised star b - open U containing x such that $f(U) \subset F$.
(d) for each $x \in X$ and each regular open set V of Y not containing $f(x)$, there exists semi generalised star b - closed set K not containing x such that $f^{-1}(V) \subset K$.

Proof. (a) $\Rightarrow$ (b) : Let F be a regular closed set in Y, then $Y-F$ is a regular open set in Y. By (1), $f^{-1}(Y-F) = X - f^{-1}(F)$ is semi generalised star b - closed set in X. This implies $f^{-1}(F)$ is semi generalised star b - open set in X. Therefore, (b) holds.
(b) $\Rightarrow$ (a) : Let G be a regular open set of Y. Then $Y-G$ is a regular closed set in Y. By (b), $f^{-1}(Y-G)$ is semi generalised star b - open set in X. This implies $X-f^{-1}(G)$ is semi generalised star b - open set in X, which implies $f^{-1}(G)$ is semi generalised star b - closed set in X. Therefore, (a) hold.
(b) $\Rightarrow$ (c) : Let F be a regular closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F)$. By (b), $f^{-1}(F)$ is semi generalised star b - open in X containing x. Set $U = f^{-1}(F)$, which implies U is semi generalised star b - open in X containing x and $f(U) = f(f^{-1}(F)) \subset F$. Therefore (c) holds.
(c) $\Rightarrow$ (b) : Let F be a regular closed set in Y containing $f(x)$, which implies $x \in f^{-1}(F)$. From (c), there exists semi generalised star b - open U_x in X containing x such that $f(U_x) \subset F$. That is $U_x \subset f^{-1}(F)$. Thus $f^{-1}(F) = \{\cup U_x : x \in f^{-1}(F)$, which is union of semi generalised star b - open sets. Therefore, $f^{-1}(F)$ is semi generalised star b - open set of X.
(c) $\Rightarrow$ (d) : Let V be a regular open set in Y not containing $f(x)$. Then $Y-V$ is a regular closed set in Y containing $f(x)$. From (c), there exists a semi generalised star b - open set U in X containing x such that $f(U) \subset Y-V$ .This implies $U \subset f^{-1}(Y-V) = X - f^{-1}(V)$. Hence, $f^{-1}(V) \subset X-U$. Set $K = X-V$, then K is semi generalised star b - closed set not containing x in X such that $f^{-1}(V) \subset K$.
(d) $\Rightarrow$ (c) : Let F be a regular closed set in Y containing $f(x)$. Then $Y-F$ is a regular open set in Y not containing $f(x)$. From (d), there exists semi generalised star b - closed set K in X not containing x such that $f^{-1}(Y-F) \subset K$. This implies $X - f^{-1}(F) \subset K$. Hence, $X-K \subset f^{-1}(F)$, that is $f(X-K) \subset F$. Set $U = X-K$, then U is semi generalised star b - open set containing x in X such that $f(U) \subset F$. ☐

Theorem 2.13. *The following are equivalent for a function $f : X \to Y$,*
(I) f is almost contra semi generalised star b - continuous.
(II) $f^{-1}(Int(Cl(G)))$ is semi generalised star b - closed set in X to each open subset G of Y.
(III) $f^{-1}(Cl(Int(F)))$ is semi generalised star b - open set in X to each closed subset F of Y.

Proof. (I) $\Rightarrow$ (II) : Let G be an open set in Y. Then $Int(Cl(G))$ is regular open set in Y. By (I), $f^{-1}(Int(Cl(G)) \in sg^*b - C(X)$.
(II) $\Rightarrow$ (I) : Proof is obvious.
(I) $\Rightarrow$ (III) : Let F be a closed set in Y. Then $Cl(Int(G))$ is regular closed set in Y. By (I), $f^{-1}(Cl(Int(G)) \in sg^*b - O(X)$.
(III) $\Rightarrow$ (I) : Proof is obvious. ☐

Theorem 2.14. *Let $f : X \to Y$ be a contra semi generalised star b - continuous and $g : Y \to Z$ be semi generalised star b - continuous. If Y is Tsg^*b - space, then $g \circ f : X \to Z$ is an almost contra semi generalised star b - continuous.*

Proof. Let V be any regular open and hence open set in Z. Since g is semi generalised star b - continuous $g^{-1}(V)$ is semi generalised star b - open in Y and Y is Tsg^*b - space implies $g^{-1}(V)$ open in Y. Since f is contra semi generalised star b - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is semi generalised star b - closed set in X. Therefore, $g \circ f$ is an almost contra semi generalised star b - continuous. ☐

Theorem 2.15. *If $f : X \to Y$ is onto strongly semi generalised star b - open (or strongly semi generalised star b - closed) and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is an almost contra semi generalised star b - continuous, then g is an almost contra semi generalised star b - continuous.*

Proof. Let V be any regular closed (resp. regular open) set in Z. Since $g \circ f$ is an almost contra semi generalised star b - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi generalised star b - open (resp. semi generalised star b - closed) in X. Since f is surjective and strongly semi generalised star b - open (or strongly semi generalised star b - closed), $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semi generalised star b - open(or semi generalised star b - closed). Therefore g is an almost contra semi generalised star b - continuous. ☐

Definition 2.16. *A function $f : X \to Y$ is called weakly semi generalised star b - continuous if for every $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in sg^*b - O(X;x)$ such that $f(U) \subset cl(V)$.*

Theorem 2.17. *If a function $f : X \to Y$ is an almost contra semi generalised star b - continuous, then f is weakly semi generalised star b - continuous function.*

Proof. Let $x \in X$ and V be an open set in Y containing $f(x)$. Then $cl(V)$ is regular closed in Y containing $f(x)$. Since f is an almost contra semi generalised star b - continuous function by Theorem 3 (2), $f^{-1}(cl(V))$ is semi generalised star b - open set in X containing x. Set $U = f^{-1}(cl(V))$, then $f(U) \subset f(f^{-1}(Cl(V))) \subset cl(V)$. This shows that f is weakly semi generalised star b - continuous function. ☐

Definition 2.18. *A space X is called locally semi generalised star b - indiscrete if every semi generalised star b - open set is closed in X.*

Theorem 2.19. *If a function $f : X \to Y$ is almost contra semi generalised star b - continuous and X is locally semi generalised star b - indiscrete space, then f is almost continuous.*

Proof. Let U be a regular open set in Y. Since f is almost contra semi generalised star b - continuous $f^{-1}(U)$ is semi generalised star b - closed set in X and X is locally semi generalised star b - indiscrete space, which implies $f^{-1}(U)$ is an open set in X. Therefore f is almost continuous. ☐

Lemma 2.20. *Let A and X_0 be subsets of a space X. If $A \in sg^*b - O(X)$ and $X_0 \in \tau^{\alpha}$, then $A \cap X_0 \in sg^*b - O(X_0)$.*

Theorem 2.21. *If $f : X \to Y$ is almost contra semi generalised star b - continuous and $X_0 \in \tau^{\alpha}$ then the restriction $f/X_0 : X_0 \to Y$ is almost contra semi generalised star b - continuous.*

Proof. Let V be any regular open set of Y. By Theorem, we have $f^{-1}(V) \in sg^*b - O(X)$ and hence $(f/X_0)^{-1}(V) = f^{-1}(V) \cap X_0 \in sg^*b - O(X_0)$. By Lemma 1, it follows that f/X_0 is almost contra semi generalised star b - continuous. □

Theorem 2.22. *If $f : X \to \prod Y_\lambda$ is almost contra semi generalised star b - continuous, then $P_\lambda \circ f : X \to Y_\lambda$ is almost contra semi generalised star b - continuous for each $\lambda \in \nabla$, where P_λ is the projection of $\prod Y_\lambda$ onto Y_λ.*

Proof. Let Y_λ be any regular open set of Y. Since P_λ is continuous open, it is an R - map and hence $(P_\lambda)^{-1} \in RO(\prod Y_\lambda)$. By theorem, $f^{-1}(P_\lambda^{-1}(V)) = (P_\lambda \circ f)^{-1} \in sg^*b - O(X)$. Hence $P_\lambda \circ f$ is almost contra semi generalised star b - continuous. □

3. Semi generalised star b - regular graphs and strongly contra semi generalised star b - closed graphs

Definition 3.1. *A graph G_f of a function $f : X \to Y$ is said to be semi generalised star b - regular (strongly contra semi generalised star b - closed) if for each $(x,y) \in (X \times Y) \backslash G_f$, there exist a semi generalised star b - closed set U in X containing x and $V \in R - O(Y)$ such that $(U \times V) \cap G_f = \varphi$.*

Theorem 3.2. *If $f : X \to Y$ is almost contra semi generalised star b - continuous and Y is T_2, then G_f is semi generalised star b - regular in $X \times Y$.*

Proof. Let $(x,y) \in (X \times Y) \backslash G_f$. It is obvious that $f(x) \neq y$. Since Y is T_2, there exists $V, W \in RO(Y)$ such that $f(x) \in V$, $y \in W$ and $V \cap W = \varphi$. Since f is almost contra semi generalised star b - continuous, $f^{-1}(V)$ is a semi generalised star b - closed set in X containing x. If we take $U = f^{-1}(V)$, we have $f(U) \subset V$. Hence, $f(U) \cap W = \varphi$ and G_f is semi generalised star b - regular. □

Theorem 3.3. *Let $f : (X,\tau) \to (Y,\sigma)$ be a function and $g : (X,\tau) \to (X \times Y, \tau \times \sigma)$ the graph function defined by $g(x) = (x, f(x))$ to each $x \in X$. Then f is almost semi generalised star b - continuous iff g is almost semi generalised star b - continuous.*

Proof. **Necessary :** Let $x \in X$ and $V \in sg^*b - O(Y)$ containing $f(x)$. Then, we have $g(x) = (x, f(x)) \in R - O(X \times Y)$. Since f is almost semi generalised star b - continuous, there exists a semi generalised star b - open set U of X containing x such that $g(U) \subset X \times Y$. Therefore, we obtain $f(U) \subset V$. Hence f is almost semi generalised star b continuous.
Sufficiency : Let $x \in X$ and w be a regular open set of $X \times Y$ containing $g(x)$. There exists $U_1 \in RO(X,\tau)$ and $V \in RO(Y,\sigma)$ such that $(x, f(x)) \in (U_1 \times V) \subset W$. Since f is almost semi generalised star b - continuous, there exists $U_2 \in sg^*b - O(X,\tau)$ such that $x \in U_2$ and $f(U_2) \subset V$. Set $U = U_1 \cap U_2$. We have $x \in U_x \in sg^*b - O(X,\tau)$ and $g(U) \subset (U_1 \times V) \subset W$. This shows that g is almost semi generalised star b - continuous. □

Theorem 3.4. *If a function $f : X \to Y$ be a almost contra semi generalised star b - continuous and almost continuous, then f is regular set - connected.*

Proof. Let $V \in RO(Y)$. Since f is almost contra semi generalised star b - continuous and almost continuous, $f^{-1}(V)$ is semi generalised star b - closed and open. So $f^{-1}(V)$ is clopen. It turns out that f is regular set - connected. □

4. Connectedness

Definition 4.1. *A space X is called semi generalised star b - connected if X cannot be written as a disjoint union of two non - empty semi generalised star b - open sets.*

Theorem 4.2. *If $f : X \to Y$ is an almost contra semi generalised star b - continuous onto and X is semi generalised star b - connected, then Y is connected.*

Proof. Suppose that Y is not a connected space. Then Y can be written as $Y = U_0 \cup V_0$ such that U_0 and V_0 are disjoint non - empty open sets. Let $U = int(cl(U_0))$ and $V = int(cl(V_0))$. Then U and V are disjoint nonempty regular open sets such that $Y = U \cup V$. Since f is almost contra semi generalised star b - continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are semi generalised star b - open sets of X. We have $X = f^{-1}(U) \cup f^{-1}(V)$ such that $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint. Since f is onto, this shows that X is not semi generalised star b - connected. Hence Y is connected. □

Theorem 4.3. *The almost contra semi generalised star b - continuous image of semi generalised star b - connected space is connected.*

Proof. Let $f : X \to Y$ be an almost contra semi generalised star b - continuous function of a semi generalised star b - connected space X onto a topological space Y. Suppose that Y is not a connected space. There exist non - empty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is almost contra semi generalised star b - continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are semi generalised star b - open in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non - empty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not semi generalised star b - connected. This is a contradiction and hence Y is connected. □

Definition 4.4. *A topological space X is said to be semi generalised star b - ultra connected if every two non - empty semi generalised star b - closed subsets of X intersect.*

A topological space X is said to be hyper connected if every open set is dense.

Theorem 4.5. *If X is semi generalised star b - ultra connected and $f : X \to Y$ is an almost contra semi generalised star b - continuous onto, then Y is hyper connected.*

Proof. Suppose that Y is not hyperconnected. Then, there exists an open set V such that V is not dense in Y. So, there exist non - empty regular open subsets $B_1 = int(cl(V))$ and $B_2 = Y - cl(V)$ in Y. Since f is almost contra semi generalised star b - continuous, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint semi generalised star b - closed. This is contrary to the semi generalised star b - ultra - connectedness of X. Therefore, Y is hyperconnected. □

5. Separation axioms

Definition 5.1. *A topological space X is said to be $sg^*b - T_1$ space if for any pair of distinct points x and y, there exist a semi generalised star b - open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.*

Theorem 5.2. *If $f : X \to Y$ is an almost contra semi generalised star b - continuous one to one and Y is weakly Hausdorff, then X is $sg^*b - T_1$.*

Proof. Suppose Y is weakly Hausdorff. For any distinct points x and y in X, there exist V and W regular closed sets in Y such that $f(x) \in V$, $f(y) \notin V$, $f(y) \in W$ and $f(x) \notin W$. Since f is almost contra semi generalised star b - continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are semi generalised star b - open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $y \in f^{-1}(W)$ and $x \notin f^{-1}(W)$. This shows that X is $sg^*b - T_1$. □

Corollary 5.3. *If $f : X \to Y$ is a contra semi generalised star b - continuous one to one and Y is weakly Hausdorff, then X is $sg^*b - T_1$.*

Theorem 5.4. *If $f : X \to Y$ is an almost contra semi generalised star b - continuous one to one function from space X into a Ultra Hausdorff space Y, then X is $sg^*b - T_2$.*

Proof. Let x and y be any two distinct points in X. Since f is an one to one $f(x) \neq f(y)$ and Y is Ultra Hausdorff space, there exist disjoint clopen sets U and V of Y containing $f(x)$ and $f(y)$ respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint semi generalised star b - open sets in X. Therefore X is $sg^*b - T_2$. □

Theorem 5.5. *If $f : X \to Y$ is an almost contra semi generalised star b - continuous closed injection and Y is ultra normal, then X is semi generalised star b - normal.*

Proof. Let E and F be disjoint closed subsets of X. Since f is closed and one to one $f(E)$ and $f(F)$ are disjoint closed sets in Y . Since Y is ultra normal there exists disjoint clopen sets U and V in Y such that $f(E) \subset U$ and $f(F) \subset V$. This implies $E \subset f^{-1}(U)$ and $F \subset f^{-1}(V)$. Since f is an almost contra semi generalised star b - continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint semi generalised star b - open sets in X. This shows X is semi generalised star b - normal. □

Theorem 5.6. *If $f : X \to Y$ is an almost contra semi generalised star b - continuous and Y is semi - regular, then f is semi generalised star b - continuous.*

Proof. Let $x \in X$ and V be an open set of Y containing $f(x)$. By definition of semi - regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset V$. Since f is almost contra semi generalised star b - continuous, there exists $U \in sg^*b - O(X,x)$ such that $f(U) \subset G$. Hence we have $f(U) \subset G \subset V$. This shows that f is semi generalised star b - continuous function. □

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