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Some peculiar dominations in intuitionistic anti-fuzzy graphs

R. Muthuraj[1*], V. V. Vijesh[2] and S. Sujith[3]

Abstract

In this paper we define the notion of regularity of intuitionistic anti-fuzzy graphs. Also illustrated the path in an intuitionistic anti-fuzzy graph with some properties. The domination is a distinctive characteristic of an intuitionistic anti-fuzzy graph, hence which can be discussed with specific norms. The connected strong domination and multi-connected domination in intuitionistic anti-fuzzy graph with their dominating set and domination number are some peculiar terms in this paper. Also we derive some theorems based on these domination parameters of intuitionistic anti-fuzzy graph.

Keywords

Intuitionistic Anti-Fuzzy Graph (IAFG), regular IAFG, anti-path, domination number, dominating set, connected strong domination, multiple domination.

[1] *PG and Research Department of Mathematics, H. H. The Rajah's College, Pudukkottai-622001, Tamil Nadu, India.*

[2] *Research Scholar, PG and Research Department of Mathematics, H. H. The Rajah's College, Pudukkottai-622001, Tamil Nadu, India.*

[2] *Department of Mathematics, Sree Narayana Gurukulam College of Engineering, Kolenchery, Ernakulam-682311, Kerala, India.*

[3] *Department of Applied Science, College of Engineering, Vadakara, Calicut-673105, Kerala, India.*

[1,2] Affiliated to Bharathidasan University, Tiruchirappalli-620024, Tamil Nadu.

*__Corresponding author__: [1]rmr1973@yahoo.co.in; [2]v.vijesh14@gmail.com; [3]suji.svn@gmail.com

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Contents

1. Introduction

Fuzzy graph is defined by Kaufmann [2] as an extension of the definition of fuzzy sets and fuzzy relations introduced by L. A. Zadeh [15]. Rosenfeld [11] introduced the concept of graph theoretical terms like path, cycle and connecetdness in 1998. The theory of intuitionistic fuzzy relation has been introduced by Atanassov [1] using membership values and non-membership values. But this theory of intuitionistic fuzzy relation evoke the development of intuitionistic fuzzy graph. In the theory of IFG, each node and edge are defined by using the membership value and non-membership value which are in between 0 and 1 inclusive. A. Somasundaram and S. Somasundaram [13] gave an introductory discussion on domination in fuzzy graph. They had defined the notion of domination by using effective edges in fuzzy graph. But NagoorGani and Chandrasekharan [9] defined domination and independent domination in fuzzy graph using strong arcs. These theories led to the introduction of domination in intuitionistic fuzzy graph. But anti fuzziness exist in many situations of our day to day life. So the theory of IFG and its domination was a thought provoking idea to develop the intuitionistic anti-fuzzy graph and its properties. R. Muthuraj, Vijesh V. V. et al. [7], defined the concept of intuitionistic anti-fuzzy graph and their properties including operations like anti-union, anti-join etc. But domination is an essential term in the discussion of every fuzzy graphs, so we discussed domination in intuitionistic anti-fuzzy graphs too. Here we are introducing some more special kind of domination parameters and discussing their properties.

2. Preliminaries

Definition 2.1. *An intuitionistic anti-fuzzy graph is of the form* $G = \langle V, E \rangle$ *where* *(i)* $V = \{v_1, v_2, \ldots \ldots v_n\}$ *such that* $\mu_1 : V \to [0,1]$ *and* $\gamma_1 : V \to [0,1]$ *denote the degree of membership and non-membership of the element* $v_i \in V$ *respectively and*

$$0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1 \tag{2.1}$$

for every $v_i \in V, (i = 1, 2, \ldots n)$.

(i) $E \subseteq V \times V$ *where* $\mu_2 : V \times V \to [0,1]$ *and* $\gamma_2 : V \times V \to [0,1]$ *are such that*

$$\mu_2(v_i, v_j) \ge \max\{\mu_1(v_i), \mu_1(v_i)\}, \tag{2.2}$$

$$\gamma_2(v_i, v_j) \ge \min\{\gamma_1(v_i), \gamma_1(v_j)\} \tag{2.3}$$

and

$$0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1 \tag{2.4}$$

for every $(v_i, v_j) \in E, (i, j = 1, 2, \ldots n)$.

Note 2.2. *If one of the inequalities (2.1) or (2.2) or (2.3) or (2.4) is not satisfied, then the graph G is not an intuitionistic anti-fuzzy graph.*

Note 2.3. *An intuitionistic anti-fuzzy graph* $\langle V, E\rangle$ *is denoted by* $G_A\langle V, E\rangle$.

Note 2.4. *The triple* $\langle v_i, \mu_{1i}, \gamma_{1i}\rangle$ *represent the degree of membership and non − membership of vertex* v_i. *Also the triple* $\langle e_{ij}, \mu_{2ij}, \gamma_{2ij}\rangle$ *represent the degree of membership and non - membership of edge* $e_{ij} = (v_i, v_j)$ *on V*

$$\text{Here } \mu_{1i} = \mu_1(v_i), \gamma_{1i} = \gamma_1(v_i)$$
$$\text{and } \mu_{2ij} = \mu_2(v_i, v_j), \gamma_{2ij} = \gamma_2(v_i, v_j)$$

Note 2.5. *If* $\mu_{2ij} = \gamma_{2ij} = 0$, *for some i and j, then there is no edge between the vertices* v_i *and* v_j.

Example 2.6. *See Fig. 1.*

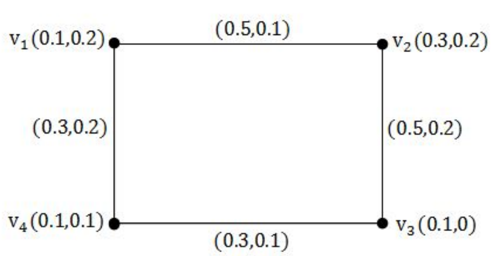

Figure 1. Intuitionistic anti-fuzzy graph $G_A\langle V, E\rangle$

Definition 2.7. *An intuitionistic anti-fuzzy graph* $H_A(V', E')$ *is an intuitionistic anti-fuzzy sub graph of* $G_A\langle V, E\rangle$ *if* $V' \subseteq V, E' \subseteq E$ *such that* $\mu'_{1i} \le \mu_{1i}$, $\gamma'_{1i} \ge \gamma_{1i}$ *and* $\mu'_{2ij} \le \mu_{2ij}$, $\gamma'_{2ij} \ge \gamma_{2ij}$.

Definition 2.8. *An intuitionistic anti-fuzzy sub graph* $H_A(V', E')$ *is called a spanning intuitionistic anti-fuzzy sub graph of* $G_A\langle V, E\rangle$ *if* $(i) V' = V, E' = E$ *and* $(ii) \mu'_{1i} = \mu_{1i}, \quad \gamma'_{1i} = \gamma_{1i}, \forall i, j$.

Definition 2.9. *Let* $G_A = \langle V, E\rangle$ *be an intuitionistic anti-fuzzy graph. Then the vertex cardinality of V is defined by*

$$|V| = \sum_{v_i \in V} \left(\frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2}\right)$$

Definition 2.10. *Let* $G_A = \langle V, E\rangle$ *be an intuitionistic anti-fuzzy graph. Then the edge cardinality of E is defined by*

$$|E| = \sum_{(v_i, v_j) \in E} \left(\frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2}\right) = \sum_{e_i \in E} \left(\frac{1 + \mu_2(e_i) - \gamma_2(e_i)}{2}\right)$$

Definition 2.11. *Let* $G_A = \langle V, E\rangle$ *be an intuitionistic anti-fuzzy graph. Then the cardinality of* G_A *is defined by*

$$|G_A| = ||V| + |E|| = \left| \sum_{v_i \in V} \left(\frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2}\right) + \sum_{(v_i, v_j) \in E} \left(\frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2}\right) \right|$$

Definition 2.12. *The vertices u and v are said to be the neighbours in an intuitionistic anti-fuzzy graph* G_A *if either of the following conditions hold.*

(i) $\mu_2(u, v) > 0, \quad \gamma_2(u, v) > 0$

(ii) $\mu_2(u, v) = 0, \quad \gamma_2(u, v) > 0$

(iii) $\mu_2(u, v) > 0, \quad \gamma_2(u, v) = 0.$

Definition 2.13. *An edge* $e = (u, v)$ *of intuitionistic anti-fuzzy graph* $G_A = \langle V, E\rangle$ *is said to be an effective edge if* $\mu_2(u, v) = \max\{\mu_1(u), \mu_1(v)\}$ *and* $\gamma_2(u, v) = \min\{\gamma_1(u), \gamma_1(v)\}$.

Definition 2.14. *An intuitionistic anti-fuzzy graph* $G_A = \langle V, E\rangle$ *is said to be complete if* $\mu_{2ij} = \max\{\mu_{1i}, \mu_{1j}\}$ *and* $\gamma_{2ij} = \min\{\gamma_{1i}, \gamma_{1j}\}, \forall v_i, v_j \in V$.

Note 2.15. *The underlying graph of a complete intuitionistic anti-fuzzy graph is complete.*

Definition 2.16. *An intuitionistic anti-fuzzy graph* $G_A = \langle V, E\rangle$ *is said to be bipartite intuitionistic antifuzzy graph if the vertex set V can be partitioned into two non-empty sets* V_1 *and* V_2 *such that*

(i) $\mu_{2ij} = 0$ *and* $\gamma_{2ij} = 0$, *if* $v_i, v_j \in V_1$ *or* $v_i, v_j \in V_2$

(ii) $\mu_{2ij} > 0$ *and* $\gamma_{2ij} > 0$, *if* $v_i \in V_1$ *or* $v_j \in V_2$ *for some i and j*
(or) $\mu_{2ij} = 0$ *and* $\gamma_{2ij} > 0$, *if* $v_i \in V_1$ *or* $v_j \in V_2$ *for some i and j*
(or) $\mu_{2ij} > 0$ *and* $\gamma_{2ij} = 0$, *if* $v_i \in V_1$ *or* $v_j \in V_2$ *for some i and j*

Definition 2.17. *A bipartite IAFG graph $G_A = \langle V, E\rangle$ is said to be complete if $\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}$ for all $u \in V_1$ and $v \in V_2$.*

Definition 2.18. *An IAFG graph $G_A = \langle V, E\rangle$ is said to be strong if $\mu_{2ij} = \max\{\mu_{1i}, \mu_{1j}\}$ and*

$$\gamma_{2ij} = \min\{\gamma_{1i}, \gamma_{1j}\}, \forall (v_i, v_j) \in E.$$

3. Regularity in IAFG

Definition 3.1. *Let $G_A = \langle V, E\rangle$ be an IAFG. The degree of a vertex u is denoted by $d_{G_A}(u)$ and defined as*

$$d_{G_A}(u) = \sum_{(u,v)\in E}^{n} \left(\frac{1+\mu_2(u,v)-\gamma_2(u,v)}{2}\right) = \sum_{v\neq u}^{n} \left(\frac{1+\mu_2(u,v)-\gamma_2(u,v)}{2}\right)$$

Definition 3.2. *The minimum degree of an intuitionistic anti-fuzzy* graph $G_A = \langle V, E\rangle$ *is* $\delta(G_A) = \min\{d_{G_A}(u)/u \in V\}$.

Definition 3.3. *The maximum degree of an intuitionistic anti-fuzzy* graph $G_A = \langle V, E\rangle$ *is* $\Delta(G_A) = \max\{d_{G_A}(u)/u \in V\}$.

Definition 3.4. *The total degree of a vertex u in an intuitionistic anti-fuzzy graph$G_A = \langle V, E\rangle$ is defined*

$$td(u) = d_{G_A}(u) + \left(\frac{1+\mu_1(u)-\gamma_1(u)}{2}\right)$$

Example 3.5. *See Fig 2.*

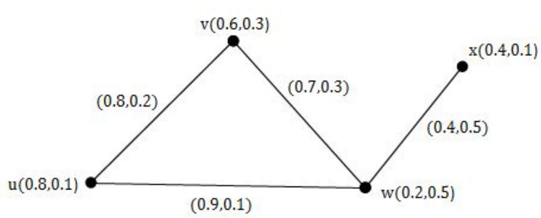

Figure 2. Intuitionistic Anti-Fuzzy Graph G_A

In Figure 2, degree of vertex u is 1.7, degree of vertex v is 1.5, degree of vertex w is 2.05 and degree of vertex x is 0.45. Thus $\delta(G_A) = 0.45$ and $\Delta(G_A) = 2.05$. Also $\text{td}(u) = 2.55, \text{td}(v) = 2.15, \text{td}(w) = 2.4$ $\text{td}(x) = 1.1$.

Definition 3.6. *An IAFG $G_A = \langle V, E\rangle$ is said to be regular if every vertex adjacent to vertices with same degree.*

Example 3.7. *See Fig 3.*

In Figure 3, degree of each vertex is 1.15. Hence it is a regular intuitionistic anti-fuzzy graph.

Definition 3.8. *An IAFG $G_A = \langle V, E\rangle$ is said to be irregular IAFG, if there is a vertex which is adjacent to vertices with distinct degree.*

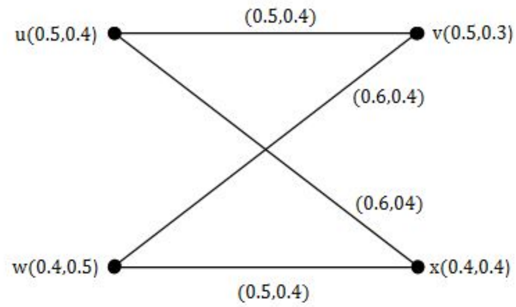

Figure 3. Intuitionistic Anti-Fuzzy Graph G_A

Definition 3.9. *Let $G_A = \langle V, E\rangle$ be a connected IAFG. Then G_A is said to be a neighbourly irregular IAFG if every two adjacent vertices of G_A has distinct degree.*

Definition 3.10. *Let $G_A = \langle V, E\rangle$ be a connected IAFG. Then G_A is said to be a highly irregular IAFG if every vertex of G_A is adjacent to vertices with distinct degree.*

Definition 3.11. *The number of vertices in an intuitionistic anti-fuzzy graph G_A is called the order of G_A and is denoted by $o(G_A)$ or p_A.*

Definition 3.12. *The number of edges in an intuitionistic anti-fuzzy graph G_A is called the size of G_A and is denoted by $s(G_A)$ or q_A.*

Example 3.13. *See Fig 4. and Fig 5.*

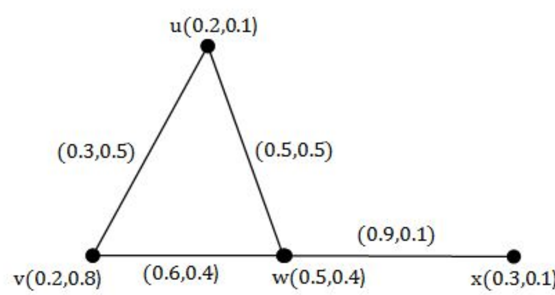

Figure 4. IAFG G_{A1}

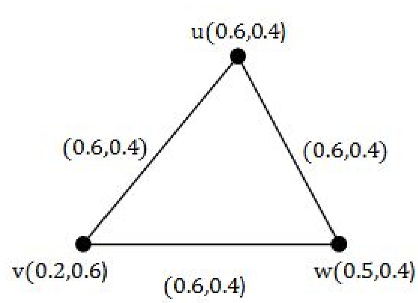

Figure 5. IAFG G_{A2}

Figure 4 is a neighbourly irregular IAFG G_{A1} and Figure 5 is a regular IAFG G_{A2}. But the following Figure 6 is a highly irregular IAFG G_{A3} .

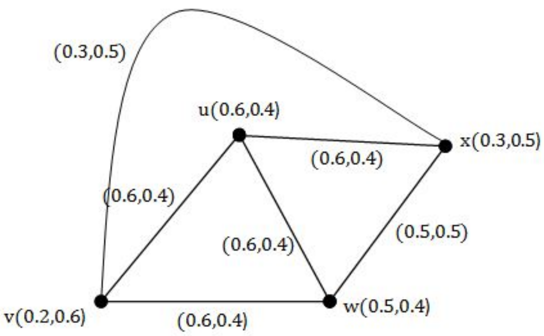

Figure 6. IAFG G_{A3}

Theorem 3.14. *In an IAFGG $_A = \langle V,E\rangle$ with n vertices,*

$$\sum_{i=1}^{n} d(v_i) = 2\left(s(G_A)+\sum_{i=1}^{n}\mu_2(v_i,v_{i+1})-\sum_{i=1}^{n}\gamma_2(v_i,v_{i+1})\right)\div 2$$
$$= s(G_A)+\sum_{i=1}^{n}\mu_2(v_i,v_{i+1})-\sum_{i=1}^{n}\gamma_2(v_i,v_{i+1})$$

Proof. Consider an IAFG $G_A = \langle V,E\rangle$ with vertex set $V = \{v_1, v_2, \ldots\ldots\ldots, v_n\}$. An edge is formed by joining two vertices such that at least one of μ_2, γ_2 value of that edge is non zero. While finding the degree of a vertex in IAFG, the membership value μ_2 and non-membership value γ_2 of incident edges are considered.

To find the sum of degrees of all the vertices of an IAFG $G_A = \langle V,E\rangle$, these μ_2 and γ_2 values of a particular edge is taken twice for the incident vertices together with a count of twice the number of edges by the terms in the formula of degree of a vertex. Thus the sum of degrees of all the vertices of an IAFG is same as twice the sum of the size of IAFG G_A, μ_2 and γ_2 values of each edge. That is

$$\sum_{i=1}^{n} d(v_i) = 2\left(s(G_A)+\sum_{i=1}^{n}\mu_2(v_i,v_{i+1})-\sum_{i=1}^{n}\gamma_2(v_i,v_{i+1})\right)\div 2$$
$$= s(G_A)+\sum_{i=1}^{n}\mu_2(v_i,v_{i+1})-\sum_{i=1}^{n}\gamma_2(v_i,v_{i+1})$$

□

Definition 3.15. *An anti-path P in an IAFG $G_A = \langle V,E\rangle$ is a sequence of distinct vertices $v_1, v_2, \ldots\ldots\ldots, v_n$ such that either of the following conditions satisfied:*

(i) $\mu_2(v_i,v_j) > 0, \quad \gamma_2(v_i,v_j) > 0$ *for some i and j*

(ii) $\mu_2(v_i,v_j) = 0, \quad \gamma_2(v_i,v_j) > 0$ *for some i and j*

(iii) $\mu_2(v_i,v_j) > 0, \quad \gamma_2(v_i,v_j) = 0$ *for some i and j*

Note 3.16. *The length of the anti-path $P = v_1v_2\ldots v_{n+1}(n > 0)$ is n.*

Definition 3.17. *Two vertices in an IAFG that are joined by an anti-path are called anti-connected vertices.*

Definition 3.18. *In an IAFGG $_A = \langle V,E\rangle$, a vertex $v \in V$ is called an isolated vertex if $\mu_2(u,v) = \gamma_2(u,v) = 0$, for all $u \in V$.*

Definition 3.19. *Let $G_A = \langle V,E\rangle$ be an IAFG. Let $u,v \in V$, we say that u dominates v in G_A if $\mu_2(u,v) = \max\{\mu_1(u),\mu_1(v)\}$ and $\gamma_2(u,v) = \min\{\gamma_1(u),\gamma_1(v)\}$. That is (u,v) is an effective edge of IAFGG $_A = \langle V,E\rangle$. A subset D of V is called a dominating set in an IAFG G_A if, for every vertex $v \notin D$, there exist $u \in D$ such that u dominates v. A dominating set D of IAFGG$_A = \langle V,E\rangle$ is said to be minimal dominating set if no proper subset of D is a dominating set of G_A.*

Theorem 3.20. *In an IAFG $G_A = \langle V,E\rangle$, an isolated vertex does not dominate any other vertex of G_A.*

Proof. Let $v \in V$ be an isolated vertex in IAFGG $_A = \langle V,E\rangle$. So there does not exist any edge to an arbitrary vertex $u \in V\backslash\{v\}$ such that

$$\mu_2(u,v) \geq \max\{\mu_1(u),\mu_1(v)\} \text{ and } \gamma_2(u,v) \geq \min\{\gamma_1(u),\gamma_1(v)\}$$

Thus any path between two vertices of this IAFG does not passes through the isolated vertex v. Hence we cannot find an effective edge from v to any other vertex in V and hence the neighbourhood of v became an empty set. So, in an IAFG, an isolated vertex does not dominate any other vertex of G_A.

4. Connected strong domination number in IAFG

□

Definition 4.1. *Let $G_A = \langle V,E\rangle$ be an IAFG without any isolated vertex. Then $D_{cS} \subseteq V$ is said to be an intuitionistic anti-fuzzy connected strong domination set if both induced sub graphs $\langle D_{cs}\rangle$ and $\langle V\backslash D_{cs}\rangle$ are connected. The intuitionistic anti-fuzzy connected strong domination number $\gamma_{cs}(G_A)$ is the maximum intuitionistic anti-fuzzy cardinality taken over all connected strong dominating sets of G_A.*

Example 4.2. *Consider the following IAFG G_A, see Fig. 7.*

In figure 7, minimal dominating sets are $D_1 = \{x,w\}$, $D_2 = \{x,v\}$ and $D_3 = \{u,w\}$. If we choose $D_{cs} = D_1 = \{x,w\}$, graphs $\langle D_{cs}\rangle$ and $\langle V\backslash D_{cs}\rangle$ are connected. So, D_1 is an intuitionistic anti- fuzzy connected strong domination set and D_1 is the only intuitionistic anti-fuzzy strong domination set of above G_A. Hence intuitionistic anti-fuzzy connected strong domination number $\gamma_{cs}(G_A)$ is 1.1.

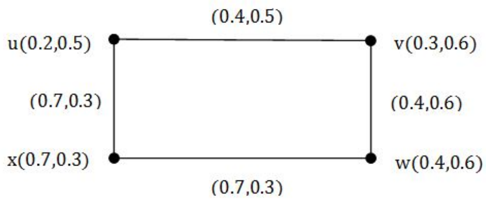

Figure 7. IAFG G_A

Definition 4.3. *Let $G_A = \langle V, E\rangle$ be an IAFG. Then $D_{cs} \subseteq V$ is said to be an intuitionistic anti-fuzzy total connected strong domination set if*

(i) D_{cs} is connected strong domination set, and

(ii) $N[D_{cs}] = V$

The intuitionistic anti-fuzzy total connected strong domination number $\gamma_{tcs}\,(G_A)$ is the maximum intuitionistic anti-fuzzy cardinality taken over all total connected strong dominating sets of G_A.

Theorem 4.4. *A connected strong dominating set D_{cs} of an IAFG is a minimal dominating set if and only if for each vertex $u \in D_{cs}$ one of the following conditions hold:*

(i) $N(u) \cap D_{cs} = \varphi$

(ii) there exist$c \in V \backslash D_{cs}$ such that $N(c) \cap D_{cs} = \{u\}$

Proof. Assuming the condition that the connected dominating set D_{cs} of an IAFG is minimal and there exist a vertex $v \in D_{cs}$ such that v does not satisfy any of the conditions (i) $N(v) \cap D_{cs} = \varphi$ or (ii)there exist$c \in V \backslash D_{cs}$ such that $N(c) \cap D_{cs} = \{v\}$. By these conditions (i) and (ii), $D' = D_{cs} \backslash \{v\}$ is a dominating set of IAFG G_A. Hence D' is a connected strong domination set of G_A. Which is a contradiction to the assumption and hence the theorem is proved. □

Theorem 4.5. *For any $IAFGG_A = \langle V, E\rangle$,*

$$\gamma_{cs}\,(G_A) \leq \gamma_{tcs}\,(G_A) \leq 2\gamma_{cs}\,(G_A)$$

Proof. By the definition of a total connected strong dominating set D_{cs} of an IAFG satisfy two conditions

(i) D_{cs} is connected strong dominating set and

(ii) $N[D_{cs}] = V, \quad \gamma_{cs}\,(G_A) \leq \gamma_{tcs}\,(G_A)$.

Suppose D_{cs} contains finite number of vertices with vertex set $V = \{v_1, v_2, \ldots\ldots\ldots, v_n\}$. Then to each $v_i \in D_{cs}$, we can choose $u_i \in V \backslash D_{cs}$ such that v_i and u_i are adjacent, by the definition of connected domination in intuitionistic anti-fuzzy graphs. This is happening as G_A has no isolated vertex. So $\{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$ is a fuzzy total connected strong domination set of IAFG G_A. □

Theorem 4.6. *An intuitionistic anti-fuzzy path$G_A = \langle V, E\rangle$ has at most two connected strong dominating sets.*

Proof. Let $V = \{v_1, v_2, \ldots\ldots\ldots, v_n\}$ be an intuitionistic anti-fuzzy vertex set of G_A. Consider the intuitionistic anti-fuzzy path as $P = v_1 v_2 \ldots\ldots\ldots v_{n-1} v_n (n > 1)$. If there is no effective edge (v_i, v_{i+1}), then there does not exist any connected strong dominating set in G_A. If there exist effective edges (v_i, v_{i+1}) for, then there we have two possibilities,

(i) $i = 1$ or $n-1$: By the definition of minimal dominating set, there set $D_1 = \{v_1, v_2, \ldots, v_{n-1}\}$ and $D_2 = \{v_2, v_3, \ldots, v_n\}$ as the intuitionistic anti-fuzzy connected strong domination sets.

(ii) $1 < i < n$: In this case the dominating set is not connected as the effective edge is obtained by two internal vertices. So there does not exist any connected dominating set.

Thus the intuitionistic anti-fuzzy path G_A has at most two connected strong dominating sets. □

5. Multiple domination in IAFG

Definition 5.1. *Let $G_A = \langle V, E\rangle$ be an IAFG $D \subseteq V$. Then $v \in V \backslash D$ is said to be an intuitionistic anti-fuzzy K -dominated vertex if it is dominated by at least K vertices in D.*

Definition 5.2. *In an IAFG G_A, every vertex in D is anti-fuzzy K-dominated, then D is called an antifuzzy K-dominating set.*

Definition 5.3. *The minimum cardinality of an intuitionistic anti-fuzzyK-dominating set on an IAFG $G_A = \langle V, E\rangle$ is called the intuitionistic anti-fuzzy K -domination number $\gamma_k\,(G_A)$.*

Theorem 5.4. *An intuitionistic anti-fuzzy 1-domination number of an IAFG $G_A = \langle V, E\rangle$ and the intuitionistic anti-fuzzy domination number of G_A are equal. That is $\gamma_1\,(G_A) = \gamma(G_A)$.*

Proof. Let D be a 1-dominating set of an intuitionistic anti-fuzzy graph G_A. So every vertex in D is an anti-fuzzy 1 dominating vertex. Therefore, there exist at least one vertex $u \in D$ such that u dominates a vertex $v \in V \backslash D$. Thus D become a dominating set of intuitionistic anti-fuzzy graph G_A. Hence these domination numbers are also same as the minimum cardinality of all such dominating sets and which is written as $\gamma_1\,(G_A) = \gamma(G_A)$. □

5.1 Algorithm to find K-dominating set of an IAFG

Let $G_A = \langle V, E\rangle$ be an IAFG,
Step 1: Identify strong edges (u, v) in IAFG G_A.
Step 2: Eliminate all the remaining edges in G_A and take the resultant IAFG as G_1.
Step 3: Choose a vertex v_1 in G_1 such that $d\,(v_1) = \Delta(G_1)$ and find $N_1 = N\,(v_1)$. Take the IAFG $G_1 - \{v_1\}$ as G_2.
Step 4: Choose another vertex v_2 in G_2 such that $d\,(v_2) = \Delta(G_2)$ and find $N_2 = N\,(v_2)$. Take the IAFG $G_2 - \{v_2\} asG_3$
Step 5: Continue this process until isolated vertices are obtained.
Step 6: These isolated vertices form a K-dominating anti-fuzzy set for an IAFG where $K = \min\{|N_1|, |N_2|, \ldots\}$.

Definition 5.5. *Let $G_{A1} = \langle V_1, E_1 \rangle$ and $G_{A2} = \langle V_2, E_2 \rangle$ be two intuitionistic anti-fuzzy graphs. Then the anti-cartesian product of G_{A1} and G_{A2} is denoted by $G_{A1} \times G_{A2}$ is denoted by $G_{A1} \times G_{A2} = \langle V, E' \rangle$ where $V = V_1 \times V_2$ and*

$$E' = \{(u, u_2)(u, v_2) : u \in V_1, (u_2, v_2) \in E_2\} \\ \cup \{(u_1, w)(v_1, w) : w \in V_2, (u_1, v_1) \in E_1\}$$

and defined by
(i)

$$(\mu_1 \times \mu_1')(u_1, u_2) = \max\{\mu_1(u_1), \mu_1'(u_2)\}, \forall u_1, u_2 \in V \text{ and}$$
$$(\gamma_1 \times \gamma_1')(u_1, u_2) = \min\{\gamma_1(u_1), \gamma_1'(u_2)\}, \forall u_1, u_2 \in V$$

(ii)

$$(\mu_2 \times \mu_2')(u, u_2)(u, v_2) = \max\{\mu_1(u), \mu_2'(u_2, v_2)\}, \forall u \in V_1 \text{ and } (u_2, v_2) \in E_2$$
$$(\gamma_2 \times \gamma_2')(u, u_2)(u, v_2) = \min\{\gamma_1(u), \gamma_2'(u_2, v_2)\}, \forall u \in V_1 \text{ and } (u_2, v_2) \in E_2$$
$$(\mu_2 \times \mu_2')(u_1, w)(v_1, w) = \max\{\mu_1'(w), \mu_2(u_1, v_1)\}, \forall w \in V_2 \text{ and } (u_1, v_1) \in E_1$$
$$(\gamma_2 \times \gamma_2')(u_1, w)(v_1, w) = \min\{\gamma_1'(w), \gamma_2(u_1, v_1)\}, \forall w \in V_2 \text{ and } (u_1, v_1) \in E_1$$

Example 5.6. *Let $V_1 = \{u_1, u_2, u_3\}$ and $V_2 = \{v_1, v_2\}$ such that $V_1 \cap V_2 = \varphi$. See Fig. 8 and Fig. 9.*

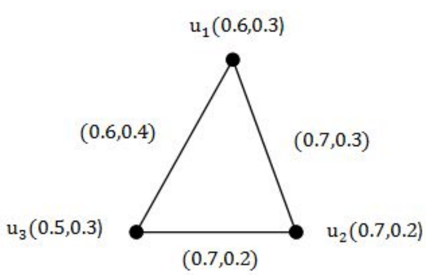

Figure 8. IAFG G_{A1}

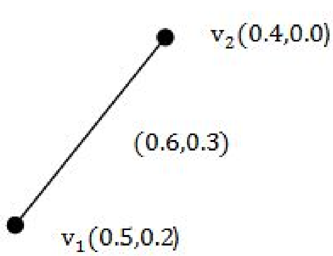

Figure 9. IAFG G_{A2}

Then $G_{A1} \times G_{A2}$ can be obtained as follows:

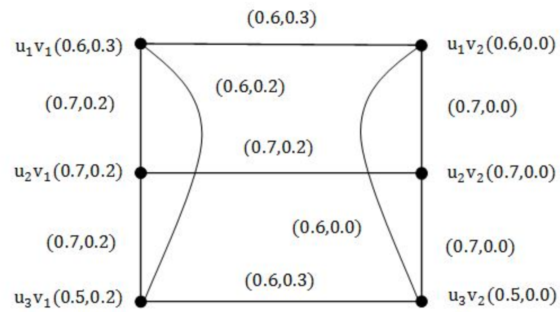

Figure 10. IAFG $G_{A1} \times G_{A2}$

Theorem 5.7. *If $G_{A1} = \langle V_1, E_1 \rangle$ and $G_{A2} = \langle V_2, E_2 \rangle$ are two intuitionistic anti-fuzzy graphs, then the number of edges in $G_{A1} \times G_{A2}$ is $N = mq + np$, where m and n are the number of vertices, p and q are the number of edges in G_{A1} and G_{A2}, respectively.*

Proof. According to the definition of anti-cartesian product $G_{A1} \times G_{A2}$, the edges are in the form of $(u, u_2)(u, v_2)$, for allu $\in V_1, (u_2, v_2) \in E_2$ and $(u_1, w)(v_1, w)$, for all $w \in V_2, (u_1, v_1) \in E_1$ Now consider the edges of the form (u, u_2) (u, v_2), for all $u \in V_1, (u_2, v_2) \in E_2$: The cardinality,

$$\begin{aligned} |(u, u_2)(u, v_2)| &= |V_1| \times |(u_2, v_2)|, \\ &\quad \text{for all } u \in V_1, (u_2, v_2) \in E_2 \\ &= |V_1| \times |E_1| \\ &= mq, \text{ for all } u \in V_1, (u_2, v_2) \in E_2 \end{aligned}$$

Similarly, we can find, $|(u_1, w)(v_1, w)| = np$, for all $w \in V_2, (u_1, v_1) \in E_1$ Therefore the number of edges in

$$\begin{aligned} G_{A1} \times G_{A2} &= |(u, u_2)(u, v_2)| + |(u_1, w)(v_1, w)| \\ &= mq + np \end{aligned}$$

for all $u \in V_1, (u_2, v_2) \in E_2; w \in V_2, (u_1, v_1) \in E_1$.
Thus $N = mq + np$. □

Theorem 5.8. *If D is a minimal vertex dominating set of an intuitionistic anti-fuzzy graph G_A, then there exist a vertex in $V \backslash D$ which is not dominated by multiple vertices.*

Proof. Let D be a minimal vertex dominating set of an intuitionistic anti-fuzzy graph G_A. Assume that every vertex in $V \backslash D$ is dominated by multiple vertices in D. Let $u \in V \backslash D$. Let v and w be two vertices in D which dominates u. Then by the assumption, every vertex in $V \backslash D$ is dominated by at least one vertex in $D \backslash \{v, w\}$. Therefore the set $D' = D \backslash \{v, w\} \cup \{u\}$ is the minimal dominating set of G_A. So, $|D'| < |D|$. This is a contradiction to the assumption that D is the dominating set in $V \backslash D$ and hence which is not dominated by multiple vertices. □

Theorem 5.9. *If $G_{A1} = \langle V_1, E_1 \rangle$ and $G_{A2} = \langle V_2, E_2 \rangle$ are two intuitionistic anti-fuzzy graphs and D is a K-dominating set of G_{A1}, then $D \times V_1$ is the minimum K-dominating set of $G_{A1} \times G_{A2}$.*

Proof. Let D be a K -dominating set of an IAFG G_{A1}. Then every vertex u in $V_1 \backslash D$ is dominated by K vertices in D. So there exist K strong arcs between $u \in V_1 \backslash D$ and vertices in D. Now we have to prove that (x, v_2) is dominated by K vertices in $D \times V_2$. There exist $v \in D$ such that $\mu_2(u,v) = \max\{\mu_1(u), \mu_1(v)\}$ and $\gamma_2(u,v) = \min\{\gamma_1(u), \gamma_1(v)\}$. Let $(u, v_2) \in (V_1 \backslash D) \times V_2$ and $(v, v_2) \in D \times V_2$. Therefore

$$
\begin{aligned}
&\left(\mu_2 \times \mu_2'\right)((u,v_2)(v,v_2)) \\
&\qquad = \max\left\{\mu_1'(v_2), \mu_2(u,v)\right\} \\
&\qquad = \max\left\{\mu_1(u), \mu_1(v), \mu_1'(v_2)\right\} \\
&\qquad = \max\left\{\mu_1(u), \mu_1'(v_2), \mu_1(v), \mu_1'(v_2)\right\} \\
&\qquad = \max\left\{\max\left\{\mu_1(u), \mu_1'(v_2)\right\}, \max\left\{\mu_1(v), \mu_1'(v_2)\right\}\right\} \\
&\qquad = \max\left\{\left(\mu_1 \times \mu_1'\right)(u,v_2), \left(\mu_1 \times \mu_1'\right)(v,v_2)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\left(\gamma_2 \times \gamma_2'\right)((u,v_2)(v,v_2)) \\
&\qquad = \min\left\{\gamma_1'(v_2), \gamma_2(u,v)\right\} \\
&\qquad = \min\left\{\gamma_1(u), \gamma_1(v), \gamma_1'(v_2)\right\} \\
&\qquad = \min\left\{\gamma_1(u), \gamma_1'(v_2), \gamma_1(v), \gamma_1'(v_2)\right\} \\
&\qquad = \min\left\{\min\left\{\gamma_1(u), \gamma_1'(v_2)\right\}, \min\left\{\gamma_1(v), \gamma_1'(v_2)\right\}\right\} \\
&\qquad = \min\left\{\left(\gamma_1 \times \gamma_1'\right)(u,v_2), \left(\gamma_1 \times \gamma_1'\right)(v,v_2)\right\}
\end{aligned}
$$

Thus (u, v_2) is dominated by (v, v_2). In D, there exist K vertices dominates u. Therefore, $(u, v_2) \in (V_1 \backslash D) \times V_2$ is dominated by K vertices in $D \times V_2$ by the definition of Cartesian productG $_{A1} \times G_{A2}$. So every vertex in $(V_1 \backslash D) \times V_2$ is dominated by K vertices in $D \times V_2$. Hence $D \times V_2$ is a K dominating set of $G_{A1} \times G_{A2}$. Now assume that $(D \times V_2) \backslash (u_1, v_2)$ is a minimum K dominating set of $G_{A1} \times G_{A2} \cdot u_1$ is dominated by K vertices of D. Since D is a K -dominating set of G_{A1}, then $(D \backslash u_1)$ is also K dominating set. This is a contradiction to the assumption that D is the minimum K -dominating set of $G_{\mathrm{A1.}}$ Therefore assumption is wrong and hence $(D \times V_2)$ is a minimum K -dominating set of $G_{A1} \times G_{A2}$. ☐

6. Conclusion

Intuitionistic anti-fuzzy graph is a valuable topic and we are facing many situations in day to day life to improve the membership value by the combined effect of two tasks or parameters. In this paper regular intuitionistic anti-fuzzy graph was discussed with their alternating situations and derived some results. Also introduced two different category of domination as connected strong domination and multiple connected domination in intuitionistic anti-fuzzy graphs. Again proved some theorems on these two dominations of IAFGs. This theory has numerous applications in communication networks, artificial intelligence, pattern clustering, image retrieval etc. In future, it is anticipated to do more work on other extensions of intuitionistic anti-fuzzy graphs with more application details.

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