Malaya Journal of Matematik, Vol. 9, No. 1, 185-189, 2021

https://doi.org/10.26637/MJM0901/0031

Analysis for discrete/continuous particle's motion in flowing laminar heated fluids

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Abstract

The Integro-differential equation derived under turbulent flow hypothesis to study particle motion has been critically examined and shown as how the simplified trajectory equation possess several restrictions, when resolved along x direction. When such a theory has left conjecture, similar trajectory equation to describe motion has been derived from first principles and significance of particle effects over flow properties have been illustrated. The extremes of mathematical complexities are described on Eulerian scale, when multi-particles are interacting with flowing fluid medium.

Keywords

Fluid Mechanics, Boundary layer equations,ordinary differential equation

AMS Subject Classification

35Q35 37N10, 76A04, 78M15.

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Article History: Received **24** November **2020**; Accepted **09** January **2021**

Contents

Nomenclature:

A—Constant in Fourier Series for Fluid Motion
t—Time
a'—Acceleration Vector
t'—Time Step
B—Constant in Fourier Series for Fluid Motion
u'—Velocity of fluid vector
C—Constant in Fourier Series for particle Motion
u_0—Characteristic velocity of fluid
c_d—Drag Force
u—Vertical velocity of fluid
c_p—Specific heat of fluid at constant pressure
$\bar{u}$—Turbulence intensity of fluid
c_g—Specific heat of particle at constant pressure
v'—velocity of particle
D—Constant in Fourier Series for Particle motion
v—Vertical velocity of particle
d—Diameter of particle
$\bar{v}$—Turbulence intensity of particle
E_f—Energy Spectrum Function for Fluid
w—Relative velocity of u,v
E_p—Energy Spectrum Function for particle
w'—Relative velocity of u', v'
f'—Force Vector
x—Flow along x-direction
f—Mass concentration of particles
y—Flow along y-direction
g—Gravitational constant
y_f—RMS dispersion coefficient of fluid
K—Stokes Resistance Law
y_p—RMS dispersion coefficient of particle
k—Thermal conductivity of fluid
ρ—density of fluid
m—Mass of the fluid
ρ_p—density of particle
N—Number density of particles
μ—viscosity of fluid
P—Pressure

η −amplitude ration
P_r −Prandtl number
Ω −($2\pi n$)Angular frequency
q −Thermal interaction parameter
τ_v −Relaxation time for fluid
R −Gas Constant
τ_T −Relaxation time for temperature
R_t −Lagrange Correlation Coefficient for fluid
τ −Characteristic time
R_p −Lagrange Correlation Coefficient for Particle
$\in_t$ −Eddy diffusivity for fluid
T −Temperature of fluid
$\in_p$ −Eddy diffusivity for particle
T_0 −Characteristic Temperature of fluid.
λ_v −Relaxation Length
T' −Temperature of particle
λ_T −Thermal relaxation Length
$\overline{T}$ −Time period

1. Introduction

presence of solid particles composed of sand, ash, and dost in flowing fluids through several industrial equipment's has received researcher's attention to estimate erosion losses due to particle impingement on component surface. the performance characteristics that are obtained based on current designs are mostly empirical and not accounted the associated particulate laden flow parameters. Though the literature for theory, experiments and materials are available, still there is a need for well defined mathematical relations for particle flow analysis.

2. Particle motion on lagrangian scale:

According to basic theory , when a discrete particle is suspended in flowing fluid , the particle motion for homogeneous turbulence is defined by

$$\frac{\pi}{6}d^3\rho_p\frac{dv}{dt} = 3\pi\mu d(u-v) + \frac{\pi}{6}d^3\rho\frac{du}{dt} + \frac{\pi}{12}d^3\rho\left(\frac{du}{dt} + \frac{dv}{dt}\right) \tag{2.1}$$

$$+\frac{3}{2}d^2\sqrt{\rho\mu\pi}\int_{t_0}^{t}\left[\frac{\frac{du}{dt'} - \frac{dv}{dt'}}{\sqrt{t-t'}}\right]dt'$$

which physically states that force required to accelerate the particle is sum of forces acting on it . These forces includes viscous resistance , pressure gradient in fluid surrounding particle , acceleration force for mass of particle relative to fluid and the last term is due to Basset , which may become substantial , when particle is accelerated at high speed.

From the momentum balance , on substitution of pressure gradient , above eqn possess non-linearities with fluid inertia and discous forces . Considering particle velocity is much greater than fluid inertia and viscous terms , after algebraic manipulations in (1) reduces to

$$\frac{dv}{dt} = a(u-v) + b\frac{du}{dt} + c\int_{t_0}^{t'}\left[\frac{\frac{du}{dt'} - \frac{dv}{dt'}}{\sqrt{t-t'}}\right]dt' \tag{2.2}$$

$$a = \frac{36\mu}{(2\rho_p+\rho)d} : b = \frac{3\rho}{2\rho_p+\rho} : c = \frac{18}{(2\rho_p+\rho)}\sqrt{\frac{\rho\mu}{\pi}} \tag{2.3}$$

which is restricted to potential low and single discrete particle and contradicts other theories that particle presence will have significant effect on viscosity of fluids. Added to this, solution to equation (2) is still challenges the researchers without assumptions. However, few attempts have been made by considering

$$v = \int_0^\infty \eta\left[Acos(\Omega t+\beta) + Bcos(\Omega t+\beta)\right]d\Omega \tag{2.4}$$

$$\eta = \left[(1+f_1)^2 + f_2^2\right]^{1/2};\ \beta = tan^{-1}\left(\frac{f_2}{1+f_2}\right) \tag{2.5}$$

$$f_1 = \frac{\left[\Omega\left(\Omega + c\sqrt{\Omega\pi/2}\right)(b-1)\right]}{\left(a+c\sqrt{\pi\Omega/2}\right)^2 + \left(\Omega + c\sqrt{\pi\Omega/2}\right)^2} \tag{2.6}$$

$$f_2 = \frac{\left[\Omega\left(a + c\sqrt{\Omega\pi/2}\right)(b-1)\right]}{\left(a+c\sqrt{\pi\Omega/2}\right)^2 + \left(\Omega + c\sqrt{\pi\Omega/2}\right)^2} \tag{2.7}$$

To simplify equation (4), turbulent intensity of gas and particles are defined in terms of Fourier integrals as

$$\overline{u}^2 = \frac{\pi}{2}\int_0^\infty \frac{A^2+B^2}{T}d\Omega;\ \overline{v}^2 = \frac{\pi}{2}\int_0^\infty \frac{C^2+D^2}{T}d\Omega \tag{2.8}$$

which in energy spectrum function yields to

$$\overline{u}^2 = \int_0^\infty E_f(\Omega)d\Omega;\ \overline{v}^2 = \int_0^\infty E_p(\Omega)d\Omega \tag{2.9}$$

and can be equated to

$$R_f = \int_0^\infty E_f(\Omega)cos(\Omega t)d\Omega \tag{2.10}$$

$$E_f(\Omega) = \frac{2}{\pi}\int_0^\infty R_f(t')cos(\Omega t)d\Omega \tag{2.11}$$

$$R_f = \frac{\overline{u}^2}{\overline{v}^2}\int_0^\infty E_f(\Omega)cos(2\Omega t)d\Omega \tag{2.12}$$

so that diffusion coefficients for fluid and particles in terms of Lagrangian correlation coefficients as

$$y_f^2 = 2\bar{u}^2 \int_0^t (t-t')R_f(t')dt' \tag{2.13}$$

$$y_p^2 = 2\bar{v}^2 \int_0^t (t-t')R_p(t')dt' \tag{2.14}$$

under which, eddy diffusivities for fluid and particles can be defined as

$$\in_f = \bar{u}^2 \int_0^\infty \frac{sin(t)}{\Omega} E_f(\Omega) s d\Omega; \tag{2.15}$$

$$\in_p = \bar{v}^2 \int_0^\infty \frac{sin(t)}{\Omega} E_p(\Omega) s d\Omega; \tag{2.16}$$

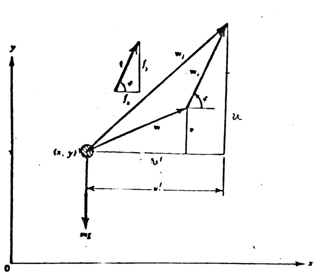
above equations for short diffusion times leads to $\in_p / \in_f = \bar{v}/\bar{u}$ and for long diffusion times, the particles falls under low frequency components and will not show any difference between fluid and particle. Also, resistance law have no role to play. Hence, at $n = 0$, $\in_p = \in_f$; leads to further complications, when n is different from zero. It can also be seen when ρ_p/ρ_f¿0, b will become finite and $a = c = 0$. But $a = 0$, $\in_p / \in_f \sim 1$ as $t > \infty$ and when a is finite as $t > \infty$; $\in_p / \in_f$ becomes undetermined. Therefore, it has been considered that ρ_p/ρ_f is so large that $b = c = 0$, which implies that eddy diffusivities decreases with the increase of frequency Under these conditions eqn (7) simplifies to

$$\frac{dv}{du} = a(\bar{u} - \bar{v}) \tag{2.17}$$

is an conjecture, because of initial hypothesis and several restrictions for it's use, when applied to multi-dimensional flow geometries.

3. Alternate approach for particle trajectory

To arrive at the conclusion reached in proceeding section, consider translation of particle suspended in fluid, whose geometrical description is shown in below figure.



By Newton's second law, the sum of forces acting on the particle are represented mathematically as

$$ma' = f' \tag{3.1}$$

Where a' in terms of velocities along x and y direction will results to

$$\frac{dv'}{dt} = f_1'(x, y, u', v', t) \tag{3.2}$$

$$\frac{dv}{dt} = f_1(x, y, u, v, t) \tag{3.3}$$

$$f_1' = \frac{1}{8}\pi\rho d c_d (u' - v') w_r' \tag{3.4}$$

$$f_2 = \frac{1}{8}\pi\rho d c_d (u - v) w_r \tag{3.5}$$

so that

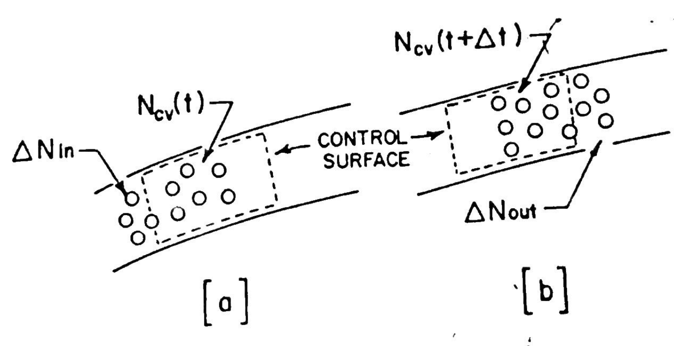
$$\frac{dv'}{dt} = \frac{3f}{4d} c_d \left[\frac{u' - v'}{1 + 0.5f} w_r'\right] \tag{3.6}$$

$$\frac{dv}{dt} = \frac{-(1-f)g + (3/4)(f/d)c_d(u-v)w_r}{1 + 0.5f} \tag{3.7}$$

Whose solution procedure for u, v, u', v' is described. Though it is quite possible to predict fluid-particle, and particle-wall interactions with above relations, but it will becomes tedious when fluid is induced to number of particles. Eulerian formulation for fluid and particles:

4. Eulerian formulation for fluid and particles

When the number of particles in fluid are entering and leaving. the channel as shown in below figure, the conservation equations in vector form will be written as:



$$\frac{D(\rho u)}{Dt} = 0 \tag{4.1}$$

$$\frac{D(\rho u)}{Dt} = -\nabla P - \nabla \times [\mu(\nabla \times u)] + \Delta[(2\mu + \lambda)\nabla . u] + KN(v - u)$$

$$(4.2)$$

$$\frac{D(\rho uT)}{Dt} = k\nabla^2 T + NQ(T' - T) \tag{4.3}$$

$$\frac{D(Nv)}{Dt} = 0 \tag{4.4}$$

$$\frac{D(mNV)}{Dt} = mNg + k(u - v) \tag{4.5}$$

$$\frac{D(NcT)}{Dt} = Q(T - T') \tag{4.6}$$

$$P = \rho RT \text{ and } \lambda = -2\mu/3 \tag{4.7}$$

$$\frac{D(\rho u)}{Dt} = \frac{\partial(\rho u)}{\partial t} + div(\rho u) \tag{4.8}$$

which for in compressible fluid-particle laminar flows, the conservation equations in two dimensional coordinates for constant flow properties leads to

$$\frac{\partial u'}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{4.9}$$

$$\frac{\partial u}{\partial t} + u'\frac{\partial u'}{\partial x} + u\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial \rho}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \frac{f}{\tau_v}(v - u) \tag{4.10}$$

$$\frac{\partial T}{\partial t} + u'\frac{\partial T'}{\partial x} + u\frac{\partial T}{\partial y} = -\frac{1}{\rho_T}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \frac{q}{\tau_T}(T' - T) \tag{4.11}$$

$$\frac{\partial f}{\partial t} + v'\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + f_u\frac{\partial v'}{\partial x} + f\frac{\partial v}{\partial y} = 0 \tag{4.12}$$

$$\frac{\partial v'}{\partial t} + v'\frac{\partial v'}{\partial x} + v\frac{\partial v'}{\partial y} = \frac{(u - v)}{\tau_v} \tag{4.13}$$

$$\frac{\partial T'}{\partial t} + v'\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y} = \frac{1}{\tau_T} \tag{4.14}$$

Equations $(34-35)$ are basically non-linear elliptic PDEs which can be simplified to parabolic linear/non-linear PDEs under boundary layer assumptions. Depending upon the requirements above equation can be modified for free and forced convection heat transfer additional body forces in momentum and energy equations. But last term and r h s in this equation and in $(36-38)$ give rise to coupling between fluid to particle and particle to fluid with the product of Mars consideration over relaxation time of particles. This is difficult to solve above system of equations forgiven flow situation for want of boundary conditions for particles and therefore the necessity rises to study the effect of interaction forces when compatible with τ.

5. Role of interaction forces

It is always advantage to solve equation $(34-38)$ simultaneously in time domain, but coupling terms pose serious complexities for both physics and mathematical solution aspects. Hence based on larger small relaxation Times the governing equations simplified in terms of interaction forces for homogeneous are heterogeneous flows. The dynamic equilibrium (relaxation time) defined as the ratio of the distance required for a particle to reach that of the fluid and mathematically represented as:

$$\tau_v = \frac{m}{3\pi\mu d}c_s; \quad \lambda_v = \tau_T u_0 \tag{5.1}$$

Similarly, relaxation time under thermal conditions are defined as the distance required for a particle temperature to reach that of the fluid is defined as

$$\tau_T = \frac{m}{2\pi kd}c_s; \quad \lambda_T = \tau_T T_0 \tag{5.2}$$

and mass concentration of particles are ratio of number density of particled to fluid density is written as

$$f = \frac{mN}{\rho} \tag{5.3}$$

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the particle motion depends upon its initial conditions that is by it's state at the time it enters the system (mostly particles are coarse)

the particle has time to adjust to local fluid motion before it has moved appreciable through this region (particles are fine).

the particle has a memory of events that took place throughout a region of relaxation length within which, significant changes in flow conditions takes place. When thermal relaxation time is compared with characteristic temperature similar observations can be made.

$f > 1$ single particle is moving through the fluid and fluid motion is effected very little.

$f \sim 1$ both the fluid and particle motions are affected by interaction forces.

6. Effect of fluid-particle motion over relaxation times

To understand the effect of relaxation time between fluid and particles, consider the equation of particle relative to fluid as a function of time as

$$\frac{dv}{dt} = \frac{u-v}{\tau_v} \tag{6.1}$$

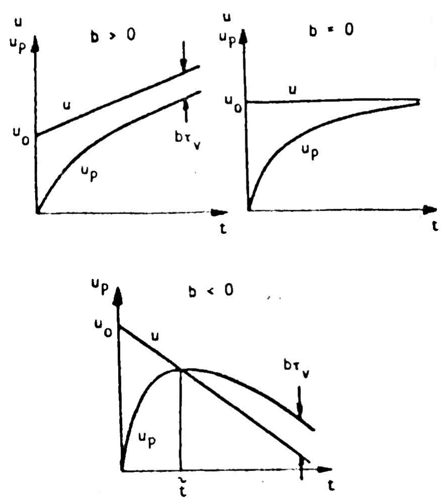
Let $u(t)$ be velocity which is instantaneous at the point where particle is located at the time t and $v = 0$ for $t = 0$ then solution for

$$v = e^{\frac{-t}{\tau_v}} \frac{1}{\tau_v} \int_0^t u(t') e^{\frac{t}{\tau_v}} dt'$$

Consider $u(t) = u_0 + b't$ where b' is positive and negative constant and u_0 is positive and constant. Eqn (43) after integration by parts will be rewritten as:

$$v = u(t) - b'\tau + u\left(b'\frac{\tau_v}{u_0-1}1\right) e^{\frac{-t}{\tau_v}}$$

whose graphical distributions over t for different b' are shown in Fig. 3. The Fig reveals.



when $b' = 0$, the motion of particle asymptotically approaches fluid velocity, for $b' > 0$ the particle reaches gas velocity with constant lag and when $b' < 0$ particle velocity exceeds the fluid velocity. However, these observations will be different when particle motion is incorporated with non-linear flow terms in time derivative. Hence the phenomena can be analyzed for wider aspects under limiting conditions $\tau_v < \tau$ and $\tau_T < \tau$, so that conservation equations maintains only single phase but involves interaction terms plus mass concentration of particles as a function of mixture flow velocity.

Solution to such homogeneous system of equations for several aspects, reveals that particle presence have pronounced effect on fluid properties, whose further study to the problem1 are addressed, it will provide valuable insights for practical applications.

7. Conclusions

When the discrete particle is present in flowing fluid, this paper has conjected it's motion derived from the integro-differential equation under turbulent flow hypothesis. Similar particle motion has been derived on Lagrangian scale from the first physical principles with flexibility to account particle-fluid interaction forces without additional assumptions. When multi-particles are interacting with flowing laminar fluids, structure of formulation has been examined after studying the interaction forces for physical aspects.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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